Search for gravitational waves from Scorpius X-1 in the second Advanced LIGO observing run with an improved hidden Markov model

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We present results from a semi-coherent search for continuous gravitational waves from the low-mass x-ray binary Scorpius X-1, using a hidden Markov model (HMM) to track spin wandering. This search improves on previous HMM-based searches of LIGO data by using an improved frequency domain matched filter, the $J$-statistic, and by analyzing data from Advanced LIGO’s second observing run. In the frequency range searched, from 60 to 650 Hz, we find no evidence of gravitational radiation. At 194.6 Hz, the most sensitive search frequency, we report an upper limit on gravitational wave strain (at 95% confidence) of $h_0^{95\%} = 3.47 \times 10^{-25}$ when marginalizing over source inclination angle. This is the most sensitive search for Scorpius X-1, to date, that is specifically designed to be robust in the presence of spin wandering.

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I. INTRODUCTION

Rotating neutron stars with nonaxisymmetric deformations are predicted to emit persistent, periodic gravitational radiation. They are a key target for continuous-wave searches performed with gravitational wave (GW) detectors such as the second-generation Advanced Laser Interferometer Gravitational-Wave Observatory (Advanced LIGO) [1–5] and Virgo [4]. The time-varying quadrupole moment necessary for GW emission may result from thermal [6,7], or magnetic [8–10] gradients, $r$-modes [11–13], or nonaxisymmetric circulation of the superfluid interior [14–17]. These mechanisms produce signals at certain multiples of the spin frequency $f_s$ [1]. Of particular interest are accreting low-mass x-ray binaries (LMXB), such as Scorpius X-1 (Sco X-1), where a neutron star is spun up by accretion from its stellar companion. Electromagnetic observations of LMXBs to date imply $f_s \lesssim 620$ Hz [18], well short of the theoretical centrifugal break-up limit $f_c \lesssim 1.5$ kHz [19]. Regardless of the exact GW mechanism, the latter observation suggests an equilibrium between the spin-up accretion torque, and GW spin-down torque [20–22]. Torque balance also implies a relation between x-ray luminosity and the GW strain, making Sco X-1, the brightest LMXB x-ray source, the most promising known target.

Initial LIGO, a first-generation detector, started taking science data in 2002. It reached its design sensitivity in Science Run 5 (S5) starting 2005 [23], and exceeded it in Science Run 6 (S6) [24]. Following detector upgrades, the second-generation Advanced LIGO interferometer [2] began taking science data during Observing Run 1 (O1), which ran from September 2015 to January 2016. The strain noise in O1 is 3 to 4 times lower than S6 between 100 and 300 Hz [25]. During this period, LIGO observed three binary black hole mergers, GW150914 [26], GW151012, and GW151226 [27]. Observing Run 2 (O2) began in November 2016, and ran until August 26, 2017. From August 1, 2017, the two LIGO detectors were joined by Virgo, resulting in a three-detector network. As well as further binary black hole mergers [28], LIGO and Virgo made the first gravitational wave observation of a binary neutron-star merger during O2 [29].

No search has yet reported a detection of a continuous-wave source. To date, four searches for Sco X-1 have been conducted on Initial LIGO data, and three on Advanced LIGO data. The first search coherently analyzed the most-sensitive six hour segment from Science Run 2 (S2) using the $F$-statistic [30], a maximum-likelihood detection statistic [31]. The second was a directed, semi-coherent analysis using the $C$-statistic [32]. The third, also a directed analysis, used the TwoSpect algorithm on doubly Fourier transformed S5 data [33–35]. The fourth applied the radiometer algorithm [36] to conduct a directed search on S4 [37], S5 [38], and later O1 [39] data. Three LMXB searches have been performed with Advanced LIGO data, comprising the radiometer search [39], an analysis based on a hidden Markov model (HMM) [40], and a cross-correlation analysis [41–43]. The upper limits established by these searches are summarized in Table I.

Astrophysical modeling and x-ray observations suggest that the spin frequency of a LMXB wanders stochastically in response to fluctuations in the hydromagnetic accretion torque [46–49]. As no electromagnetic measurements of $f_s$ are available to guide a gravitational wave search for Sco X-1, such searches must either account for spin wandering or limit their observing times and/or coherence.

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times in accordance with the anticipated timescale and amplitude of the spin wandering [50]. For example, the sideband search described in Ref. [32] is restricted to data segments no longer than ten days. The HMM tracker, first applied to the search for Sco X-1 in Ref. [40], is an effective technique for detecting the most probable underlying spin frequency, $f_s(t)$ and thus accounting for spin wandering.

The signal from a binary source is Doppler shifted, as the neutron star revolves around the barycenter of the binary, dispersing power into orbital sidebands near the source-frame frequency. The separation of these sidebands and the source-frame frequency depends on the binary orbital parameters and $f_s$, but is typically within $0.05\%$ of the gravitational wave frequency for a source such as Sco X-1. Four maximum-likelihood matched filters have been developed to detect these sidebands: the $C$-statistic, which weights sidebands equally [32], the binary modulated $F$-statistic [51], the Bessel-weighted $F$-statistic [52], and the $J$-statistic, which extends the Bessel-weighted $F$-statistic to account for the phase of the binary orbit [53]. Any of these matched filters can be combined with the HMM to conduct a search for signals from a binary source that accounts for spin wandering.

In this paper, we combine the $J$-statistic described in Ref. [53] with the HMM described in that paper and Refs. [40, 50], and perform a directed search of Advanced LIGO O2 data for evidence of a gravitational wave signal from Sco X-1. In the search band 60–650 Hz, we find no evidence of a gravitational wave signal. The paper is organized as follows. In Sec. II, we briefly review the HMM and the $J$-statistic. In Sec. III, we discuss the search strategy and parameter space. In Sec. IV, we report on the results from the search and veto candidates corresponding to instrumental artifacts. In Sec. V, we discuss the search sensitivity and consequent upper limits on the gravitational wave strain.

### II. SEARCH ALGORITHM

In this section, we outline the two key components of the search algorithm: the HMM used to recover the most probable spin history $f_0(t)$, and the $J$-statistic, the matched filter that accounts for the Doppler shifts introduced by the orbital motions of Earth and the LMXB. The HMM formalism is the same as used in Refs. [40, 52, 53], so we review it only briefly. The $J$-statistic is described fully in Ref. [53]; again, we review it briefly.

#### A. HMM formalism

A Markov model describes a stochastic process in terms of a state variable $q(t)$, which transitions between allowable states $\{q_1, \ldots, q_{N_q}\}$ at discrete times $\{t_0, \ldots, t_{N_t}\}$. The transition matrix $A_{q_i,q_j}$ represents the probability of jumping from state $q_i$ at the time $t = t_n$ to $q_j$ at $t = t_{n+1}$ depending only on $q(t_n)$. A HMM extends the Markov model to situations where direct observation of $q(t)$ is impossible ($q(t)$ is called the hidden state). Instead one measures an observable state $o(t)$ selected from $\{o_1, \ldots, o_{N_o}\}$, which is related to the hidden state by the emission matrix $L_{o_i/q_j}$, which gives the likelihood that the system is in state $q_i$ given the observation $o_j$. In gravitational wave searches for LMXBs like Sco X-1, where the spin frequency cannot be measured electromagnetically, it is natural to map $q(t)$ to $f_0(t)$ and $o(t)$ to the raw interferometer data, some equivalent intermediate data product (e.g., short Fourier transforms), or a detection statistic (e.g., $F$-statistic, $J$-statistic).

In a LMXB search, we divide the total observation (duration $T_{\text{obs}}$) into $N_T$ equal segments of length $T_{\text{drift}} = T_{\text{obs}}/N_T$. In practice, $T_{\text{drift}}$ is chosen on astrophysical grounds to give $N_T = \lceil T_{\text{obs}}/T_{\text{drift}} \rceil$ based on an estimate of plausible spin-wandering timescales [50]; in this paper we follow Ref. [40] in choosing $T_{\text{drift}} = 10d$. The tracker is able to track the signal even if the spin frequency occasionally jumps by two bins as it can catch up to the signal path, although with an attendant loss of sensitivity as the recovered must include a step that contains only noise.

In each segment, the emission probability $L_{o_i/q_j}$ is computed from some frequency domain estimator $G(f)$ such as the maximum likelihood $F$- or $J$-statistic (discussed in Sec. II B). The frequency resolution of the

<table>
<thead>
<tr>
<th>Search</th>
<th>Data</th>
<th>Upper limit</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$-statistic</td>
<td>S2</td>
<td>$h_0^{95%} \lesssim 2 \times 10^{-22}$ at 464–484 Hz, 604–626 Hz</td>
<td>[31]</td>
</tr>
<tr>
<td>$C$-statistic</td>
<td>S5</td>
<td>$h_0^{95%} \lesssim 8 \times 10^{-25}$ at 150 Hz</td>
<td>[32]</td>
</tr>
<tr>
<td>TwoSpect</td>
<td>S6, VSR2, VSR3</td>
<td>$h_0^{95%} \lesssim 2 \times 10^{-23}$ at 20–57.25 Hz</td>
<td>[34]</td>
</tr>
<tr>
<td>Radiometer</td>
<td>S4, S5</td>
<td>$h_0^{90%} \lesssim 2 \times 10^{-24}$ at 150 Hz</td>
<td>[38, 44]</td>
</tr>
<tr>
<td>TwoSpect</td>
<td>S6</td>
<td>$h_0^{95%} \lesssim 1.8 \times 10^{-24}$ at 165 Hz</td>
<td>[45]</td>
</tr>
<tr>
<td>Radiometer</td>
<td>O1</td>
<td>$h_0^{95%} \lesssim 6.7 \times 10^{-25}$ at 130–175 Hz</td>
<td>[39]</td>
</tr>
<tr>
<td>Viterbi 1.0</td>
<td>O1</td>
<td>$h_0^{95%} \lesssim 8.3 \times 10^{-25}$ at 106 Hz</td>
<td>[40]</td>
</tr>
<tr>
<td>Cross-correlation</td>
<td>O1</td>
<td>$h_0^{95%} \lesssim 2.3 \times 10^{-25}$ at 175 Hz</td>
<td>[43]</td>
</tr>
</tbody>
</table>
estimator is $\Delta f_{\text{drift}} = 1/(2T_{\text{drift}})$. The probability that an observation $O = \{o(t_1), \ldots, o(t_{N_f})\}$ is associated with a particular hidden path $Q = \{q(t_0), \ldots, q(t_{N_f})\}$ is then given by

$$P(Q|O) \propto L_{o(t_N)} q(t_N) A_{q(t_N)} q(t_{N-1}) \cdots L_{o(t_1)} q(t_1) \times A_{q(t_1)} q(t_0) \Pi q(t_0),$$

(1)

where $\Pi_q$ is the prior, i.e., the probability the system starts in state $q_i$ at $t = t_0$. For this search, we take a flat prior. [Note that there is no initial observation $o(t_0)$ as the initial state of the system is captured by the prior.] The task, then, is to find the optimal hidden path $Q^*$, that is, the path $Q^*$ that maximizes $P(Q|O)$ given $O$. We find $Q^*$ efficiently with the recursive Viterbi algorithm [54], which is discussed in detail in Appendix A of Ref. [40].

In this paper, we follow the convention in Ref. [40] of defining the Viterbi detection score $S$ for a path as the number of standard deviations by which that path’s log-likelihood exceeds the mean log-likelihood of all paths. Mathematically we have

$$S = \frac{\ln \delta Q(t_{N_f}) - \mu_{\ln \delta(t_{N_f})}}{\sigma_{\ln \delta(t_{N_f})}},$$

(2)

where

$$\mu_{\ln \delta(t_{N_f})} = N^{-1}_Q \sum_{i=1}^{N_Q} \ln \delta_q(t_{N_f}),$$

(3)

$$\sigma^2_{\ln \delta(t_{N_f})} = N^{-1}_Q \sum_{i=1}^{N_Q} [\ln \delta_q(t_{N_f}) - \mu_{\ln \delta(t_{N_f})}]^2,$$

(4)

$\delta_q(t_{N_f})$ denotes the likelihood of the most likely path ending in state $q_i$ at step $N_f$, and $\delta_q(t_{N_f}) = \max_i \delta_q(t_{N_f})$ is the likelihood of the optimal path overall.

**B. J-statistic**

The frequency domain estimator $G(f)$ converts the interferometer data into the likelihood that a signal is present at frequency $f$. For a continuous-wave search for an isolated neutron star, the maximum-likelihood $F$-statistic [30] is a typical choice for $G(f)$. The $F$-statistic accounts for the diurnal rotation of Earth, and its orbit around the Solar System barycenter. It is an almost optimal matched filter for a biaxial rotor [55].

For a neutron star in a binary system, such as a LMXB, the signal is frequency (Doppler) modulated by the binary orbital motion as well. Reference [40] used the Bessel-weighted $F$-statistic to account for this modulation, without using information about the orbital phase. Reference [53] introduced the $J$-statistic, which is a matched filter that extends the $F$-statistic to include orbital phase in the signal model. The orbital Doppler effect distributes the $F$-statistic power into approximately $2m + 1$ orbital sidebands separated by $P^{-1}$, with $m = \lceil 2\pi f_a a_0 \rceil$, where $\lceil \cdot \rceil$ denotes rounding up to the nearest integer, $P$ is the orbital period, and $a_0 = (a \sin i)/c$ is the light travel time across the projected semimajor axis (where $a$ is semimajor axis and $i$ is the inclination angle of the binary). For a zero-eccentricity Keplerian orbit, the Jacobi-Anger identity may be used to expand the signal $h(t)$ in terms of Bessel functions, suggesting a matched filter of the form [52,53]

$$G(f) = F(f) \otimes B(f),$$

(5)

with

$$B(f) = \sum_{s=-m}^{m} J_s(2\pi f_0 a_0) e^{-i\phi_a s} \delta(f - s/P),$$

(6)

where $J_s(z)$ is the Bessel function of the first kind of order $s$, $\phi_a$ is the orbital phase at a reference time, and $\delta$ is the Dirac delta function.

All else being equal, using the $J$-statistic instead of the Bessel-weighted $F$-statistic improves sensitivity by a factor of approximately 4. Reference [53], particularly Sec. IV of that paper, examines the difference between the two estimators in depth.

The Bessel-weighted $F$-statistic requires a search over $a_0$ but does not depend on $\phi_a$. By contrast, the more-sensitive $J$-statistic involves searching over $\phi_a$ too. In this paper we apply the $J$-statistic to search for Sco X-1. Details of the search and priors derived from electromagnetic measurements are discussed in Sec. III.

**III. LIGO O2 SEARCH**

**A. Sco X-1 parameters**

The matched filter described in Sec. II B depends on three binary orbital parameters: the period $P$, the projected semimajor axis $a_0$, and the phase $\phi_a$. The $F$-statistic depends on the sky location $\alpha$ (right ascension) and $\delta$ (declination), and optionally the source frequency derivatives. For this search, we assume there is no secular evolution in frequency. The other parameters have been measured electromagnetically for Sco X-1 and are presented in Table II.

For $\alpha$, $\delta$, and $P$, the uncertainties in the electromagnetic measurements are small enough that they have no appreciable effect on the sensitivity of the search [51,60,61], and a single, central value can be assumed. However, the uncertainties in $a_0$ and $\phi_a$ cannot be neglected. The time spent searching orbital parameters scales as the number of $(a_0, \phi_a)$ pairs. Careful selection of the ranges of $a_0$ and $\phi_a$ is essential to keep computational costs low.
TABLE II. Electromagnetic measurements of the sky position and binary orbital parameters of Sco X-1. The uncertainties represent one-sigma (68\%) confidence intervals, except for \(a_0\), for which hard limits are given. The search resolution for \(a_0\) and \(T_{\text{asc}}\) is different in each frequency subband, as discussed in Sec. III A. The search range for the time of ascension is the observed time of ascension propagated forward to the start of O2.

<table>
<thead>
<tr>
<th>Observed parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right ascension</td>
<td>(\alpha)</td>
<td>16 h 19 m 55.0850 s</td>
<td>[56]</td>
</tr>
<tr>
<td>Declination</td>
<td>(\delta)</td>
<td>(-15^\circ 38' 24.9'')</td>
<td>[56]</td>
</tr>
<tr>
<td>Orbital period</td>
<td>(P)</td>
<td>68023,86048 ± 0.0432 s</td>
<td>[57]</td>
</tr>
<tr>
<td>Projected semimajor axis</td>
<td>(a_0)</td>
<td>[1.45, 3.25] s</td>
<td>[57]</td>
</tr>
<tr>
<td>Polarization angle</td>
<td>(\psi)</td>
<td>234 ± 3°</td>
<td>[58]</td>
</tr>
<tr>
<td>Orbital inclination angle</td>
<td>(i)</td>
<td>44 ± 6°</td>
<td>[58]</td>
</tr>
<tr>
<td>Time of ascension</td>
<td>(T_{\text{asc}})</td>
<td>974416624 ± 50 s</td>
<td>[57,59]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Search parameter</th>
<th>Symbol</th>
<th>Search range</th>
<th>Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>(f_0)</td>
<td>60–650 Hz</td>
<td>5.787037 \times 10^{-7} Hz</td>
</tr>
<tr>
<td>Projected semimajor axis</td>
<td>(a_0)</td>
<td>1.450–3.250 s</td>
<td>Variable</td>
</tr>
<tr>
<td>Time of ascension</td>
<td>(T_{\text{asc}})</td>
<td>1164543014–1164543614 s</td>
<td>Variable</td>
</tr>
</tbody>
</table>

The previous analysis described in Ref. [40] used the Bessel-weighted \(\mathcal{F}\)-statistic in place of the \(\mathcal{J}\)-statistic, and searched over a uniformly gridded range of \(a_0\), where the grid resolution did not depend on frequency. However, the \(\mathcal{J}\)-statistic is more sensitive to mismatch in the binary orbital parameters, so a finer grid is required. We must also choose an appropriate grid for \(\phi_a\) (The Bessel-weighted \(\mathcal{F}\)-statistic is independent of \(\phi_a\)).

As the \(\mathcal{J}\)-statistic has a similar overall response to parameter mismatches as the binary \(\mathcal{F}\)-statistic, we follow the formalism in Ref. [51] to select an appropriate parameter space gridding. We choose a grid which limits the maximum loss in signal-to-noise ratio (mismatch) \(\mu_{\max}\) to \(\mu_{\max} = 0.1\). Equation (71) in Ref. [51] gives a general equation for the number of grid points needed for each search parameter. For the particular search considered in this paper, the number of choices for \(a_0\) and \(\phi_a\) are

\[
N_{a_0} = \left[ \frac{\pi}{2} \mu_{\max}^{-1/2} f_0 \Delta a_0 \right],
\]

\[
N_{\phi_a} = \left[ \frac{1}{2} \mu_{\max}^{-1/2} f_0 a_0 \left( \frac{2\pi}{P} \right) \Delta \phi_a \right],
\]

where \(\Delta a_0\) and \(\Delta \phi_a\) are the widths of the search ranges for \(a_0\) and \(\phi_a\), respectively. The number of orbital parameters to be searched depends on the search frequency. Accordingly for each search subband, we adopt a different grid resolution, with the grid refined at higher frequencies. In the subband beginning at 60 Hz, we have \(N_{a_0} = 768\) and \(N_{\phi_a} = 78\); in the subband beginning at 650 Hz, we have \(N_{a_0} = 8227\) and \(N_{\phi_a} = 824\). In principle we could achieve further computational savings by noting that \(N_{\phi_a}\) also depends on \(a_0\), but for safety we use the largest \(a_0\).

The search range for \(a_0\) is \(1.45 \leq a_0/(1 \text{ s}) \leq 3.25\), which matches the most recent electromagnetic measurement [57] and widens the error bars on the widely cited and previous best published measurement, \(a_0 = 1.44 \pm 0.18\) s [62].

The orbital phase \(\phi_a\) can be related to the electromagnetically measured time of ascension, \(T_{\text{asc}}\), given in Table II, by

\[
\phi_a = 2\pi T_{\text{asc}}/P \pmod{2\pi}.
\]

The one-sigma uncertainty in the published value for \(T_{\text{asc}}\) is \(\pm 50\) s [57,59] for a time of ascension at GPS time 974416624 s (in November 2010). As O2 took place significantly after this time, to make a conservative estimate on appropriate error bars for \(T_{\text{asc}}\), we advance \(T_{\text{asc}}\) by adding 3135 orbital periods to the time of ascension taken from Ref. [57]. As there is uncertainty associated with the measured orbital period, this widens the one-sigma uncertainty of \(T_{\text{asc}}\) to \(\pm 144\) s, which we round up to \(\pm 150\) s.

To cover a significant portion of the measured \(T_{\text{asc}}\) range while keeping the search computationally feasible, we search a two-sigma range around the central \(T_{\text{asc}}\), namely, 1164543014 \(\leq T_{\text{asc}}/(1 \text{ s}) \leq 1164543614\) (expressed for presentation purposes as the time of the last ascension before the start of O2).

As there is no electromagnetic measurement of \(f_\star\) for Sco X-1, we search the band \(60 \leq f_\star/(1 \text{ Hz}) \leq 650\), where LIGO is most sensitive, again adopting a uniform prior (see Sec. II A for a discussion of the HMM prior). The same band is analyzed in Ref. [40]. For computational convenience, we split the band into blocks of approximately 0.61 Hz (discussed further in Sec. III B).

The final electromagnetically measured parameter is the polarization angle, \(\psi\). Because the \(\mathcal{F}\)-statistic components of the \(\mathcal{J}\)-statistic are maximized over the polarization angle, the \(\mathcal{J}\)-statistic is insensitive to \(\psi\).
A summary of the search ranges flowing from the electromagnetically measured parameters of Sco X-1 is presented in Table II.

B. Workflow

The workflow for the search is displayed as a flowchart in Fig. 1.

The data from the detector are provided as short Fourier transforms (SFTs), each covering $T_{SFT} = 1800$ s. We divide the search into subbands, both to facilitate managing the volume of data, and to ensure that replacing the search frequency $f$ with the midpoint of the subband, $\bar{f}$, is a good approximation in Eq. (6). To achieve best performance from the fast Fourier transforms used to compute the convolution in (6), it is desirable to have a power of 2 number of frequency bins in the band, so we set the subband width to be $\Delta f_{\text{band}} = 2^{20} \Delta f_{\text{drift}} = 0.6068148$ Hz. This in turn sets the number of hidden states per subband per binary orbital parameter to be $N_Q = 2^{20}$.

For each subband, we divide the data into $N_f$ blocks, each with duration $T_{\text{drift}} = 10 \, d$. We then compute, from the SFTs, the $F$-statistic “atoms” [63] ($F_0$, $F_\phi$) for each block using the fixed parameters ($\alpha$, $\delta$, $P$) in Table II.

The next step is to compute the $J$-statistic for the $(a_0, \phi_a)$ search grid described in Sec. III A. The $F$-statistic atoms do not depend on the binary orbital parameters so they are not recomputed when calculating the $J$-statistic. The code to compute the $J$-statistic is based on the $F$-statistic subroutines contained in the LIGO Scientific Collaboration Algorithm Library [64].

After computing the $J$-statistic, we use the Viterbi algorithm to compute the optimal paths through the HMM trellis, i.e., the set of vectors $Q^*$. In principle, the tracking problem is three dimensional (over $f_0$, $a_0$, and $\phi_a$), but $a_0$ does not vary significantly over $T_{\text{obs}} \approx 1$ yr and $\phi_a$ varies deterministically, with the phase at time step $n$ given by $\phi_a(t_n) = \phi_a(t_{n-1}) + 2\pi T_{\text{drift}}/P$. Thus, it is convenient to search independently over $f_0$ and pairs $(a_0, \phi_a)$. This allows searches over $(a_0, \phi_a)$ pairs to be performed in parallel.

The result of this procedure is one log-likelihood for the optimal path through the trellis terminating at every 3-tuple $(f_0, a_0, \phi_a)$. Equation (2) converts these log-likelihoods to Viterbi scores. As the noise power spectral density (PSD) of the detector is a function of $f_0$, we compute $\mu$ and $\sigma$ separately for each band. By contrast, the PSD is not a function of $a_0$ and $\phi_a$. Therefore, we can recalculate $\mu$ and $\sigma$ for every $(a_0, \phi_a)$ pair (rather than calculating $\mu$ and $\sigma$ using every log-likelihood across the entire search), thereby considerably reducing memory use. This has no significant impact on the Viterbi scores.

For each subband that produces a best Viterbi score lower than the detection threshold (chosen in Sec. III C), we compute an upper limit on the gravitational wave strain for a source in that subband. For Viterbi scores that exceed the threshold, we apply the veto tests described in Sec. IVA. We claim a detection, if a candidate survives all vetoes.

For performance reasons, the most computationally intensive parts of the search (computing the $J$-statistic,
and the Viterbi tracking) were run using NVIDIA P100 graphical processing units (GPUs). Other steps were run using CPU codes on Intel Xeon Gold 6140 CPUs.

### C. Threshold and false alarm probability

It remains to determine a detection score threshold $S_{\text{th}}$ corresponding to the desired false alarm probability. Consider the probability density function (PDF) $p_n(S)$ of the Viterbi score in noise. For a given threshold $S_{\text{th}}$ and a fixed search frequency and set of binary orbital parameters, the probability that the score will exceed this threshold (i.e., produce a false alarm) is

$$
\alpha = \int_{S_{\text{th}}}^{\infty} dS p_n(S). \tag{10}
$$

In general, the search covers many frequency bins and choices of binary parameters. The probability $\alpha_N$ of a false alarm over a search covering $N$ parameter choices (number of frequency bins multiplied by number of binary parameter choices) is

$$
\alpha_N = 1 - (1 - \alpha)^N. \tag{11}
$$

This equation assumes that the Viterbi score in noise is an independent random variable at each point in the parameter space, which is not necessarily true, as the $J$-statistic calculated for two points nearby in parameter space is correlated to some degree. However, for $\mu_{\text{max}} = 0.1$ as used in this search, these correlations do not have a significant impact [65]. In practice, we fix $\alpha_N$ and $N$ and solve (10) and (11) for $\alpha$ and hence $S_{\text{th}}$.

As the noise-only PDF $p_n(S)$ of the Viterbi score is unknown analytically [40], we resort to Monte Carlo simulations. We generate $10^5$ Gaussian noise realizations in seven subbands of width $\Delta f_{\text{band}}$, namely those starting at 55, 155, 255, 355, 455, 555, and 650 Hz. The noise is generated using the standard LIGO tool lalapps_makefakedata_v4. These are the same subbands used in Sec. III C of Ref. [40], and the one-sided noise PSD $S_h(f)$ is set to match the O2 data. We then perform the search described in Sec. III B (including scanning over $a_0$ and $\phi_0$).

The results of this search produce an empirical version of $p_n(S)$. Plotting the tail of this distribution on a logarithmic plot suggests that a fit to a function of the form $e^{\lambda S}$ is an appropriate choice to allow the PDF to be extrapolated in order to solve (11).

We first analyze each band independently to ensure that there is no frequency dependence in $p_n(S)$. Table III gives the best-fit $\lambda$, and the threshold $S_{\text{th}}$ obtained, for each band analyzed in isolation. We find that there is no significant dependence on the subband searched, nor any identifiable trend in $\lambda$ or $S_{\text{th}}$. Combining the realizations for all bands produces $\lambda = -3.28$ and hence $S_{\text{th}} = 13.66$ for $\alpha = 0.01$.

### TABLE III. Results of investigating the empirical PDF of the Viterbi score in seven subbands of Gaussian noise.

<table>
<thead>
<tr>
<th>Start of band (Hz)</th>
<th>$\lambda$</th>
<th>$S_{\text{th}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>-3.02</td>
<td>14.12</td>
</tr>
<tr>
<td>155</td>
<td>-3.24</td>
<td>13.63</td>
</tr>
<tr>
<td>255</td>
<td>-3.26</td>
<td>13.58</td>
</tr>
<tr>
<td>355</td>
<td>-3.27</td>
<td>13.61</td>
</tr>
<tr>
<td>455</td>
<td>-3.30</td>
<td>13.62</td>
</tr>
<tr>
<td>555</td>
<td>-3.29</td>
<td>13.66</td>
</tr>
<tr>
<td>650</td>
<td>-3.29</td>
<td>13.63</td>
</tr>
</tbody>
</table>

The empirical PDF and fitted exponential are shown in Fig. 2.

### D. Sensitivity

After selecting $S_{\text{th}}$, it remains to determine the lowest (as a function of frequency) characteristic wave strain, $h_0^{5\%}$, that can be detected with 95% efficiency (i.e., a 5% false dismissal rate). To do this, we generate Monte Carlo realizations of Gaussian noise with Sco X-1-like signals injected. We determine the proportion of signals recovered as a function of $h_0$ and double-check the false alarm probability quoted above.

For O2, the most sensitive subband of width $\Delta f_{\text{band}} = 0.6068148$ Hz is the one beginning at 194.6 Hz. Following a typical procedure used to find upper limits for continuous gravitational wave searches [66], we generate $10^2$ noise realizations and inject signals using the source parameters in Table II, with $T_{\text{obs}} = 230$ d (the duration of O2), $T_{\text{shift}} = 10$ d, $N_T = 23$, $h_0 = 7.058 \times 10^{-24}$ Hz$^{-1/2}$, and $\cos i = 1$. The remaining range-bound parameters, namely $f_{\text{min}}, a_{\text{min}}, T_{\text{asci}}, \text{and } \psi_{\text{inj}}$ are chosen from a uniform

![Empirical PDF](image)

**FIG. 2.** Tail of the PDF of the Viterbi score $S$ in noise. The purple histogram shows the empirical PDF derived from $10^2$ realizations of the noise analyzed in the seven 0.61 Hz subbands starting at 55, 155, 255, 355, 455, 555, and 650 Hz. The green curve is an exponential fitted to the histogram.
distribution within the range given by their one σ error bars. The source frequency \( f_{0_{\text{inj}}} \) is chosen from a uniform distribution on the interval \([194.6 \, \text{Hz}, 194.7 \, \text{Hz}]\). For each realization, the signal is injected with progressively lower \( h_0 \) until it can no longer be detected. We denote by \( h_{0,\text{min},i} \) the lowest \( h_0 \) that can be detected in realization \( i \). To obtain \( h_{0,95\%} \), we take the 95th highest \( h_{0,\text{min},i} \). The simulations return the threshold \( h_{0,95\%} = 1.46 \times 10^{-25} \) at 194.6 Hz.

In general, the signal-to-noise ratio is strongly affected by the inclination angle \( \iota \), not just \( h_0 \). We follow Ref. \[59\] and define an effective \( h_0 \) that absorbs the dependence on \( \iota \):

\[
h_0^{\text{eff}} = h_0 2^{-1/2} \left( \left( 1 + \cos^2 \iota \right) / 2 + \cos^2 \iota \right)^{1/2},
\]

allowing us to generalize results from the simulations above, where all injections were done with \( \cos \iota = 1 \). Thus, the result obtained above corresponds to circular polarization. The electromagnetically measured inclination of Sco X-1’s orbit is \( \iota \approx 44^\circ \pm 6^\circ \) \[58\]. Although it is not necessarily the case, if we assume that the orbital inclination equals the inclination angle \( \iota \) of the putative neutron star’s spin axis, we obtain \( h_0^{\text{eff}95\%} = 1.35 h_0^{95\%} \).

The search in Ref. \[40\] found a scaling relation of the form \( h_{0,95\%} \propto S_h^{1/2} f_0^{1/4} \) to hold for fixed \( T_{\text{obs}} \). The \( f_0^{1/4} \) dependence arises because the latter search added sidebands incoherently. In the case of the \( J \)-statistic, which adds sidebands coherently, we expect the scaling to depend just on \( h_0 \), with

\[
h_{0,95\%} \propto S_h^{1/2}.
\]  

We verify this scaling in Gaussian noise by repeating the injection procedure described above in frequency bands beginning at 55, 355, and 650 Hz. The scaling is the final ingredient needed to produce the blue dashed curve in Fig. 3, which shows the expected sensitivity of a search over the full search band, assuming Gaussian noise, a 100% duty cycle, and a circularly polarized signal.

There is no simple scaling similar to (13) that can be used to account for the effect of non-Gaussian noise and the detector duty cycle. Hence we introduce a multiplicative correction factor \( \kappa_j \) for a selection of subbands indexed

![Graph showing sensitivity of a search for Sco X-1 across the frequency band searched in this work.](image)

**FIG. 3.** Sensitivity of a search for Sco X-1 across the frequency band searched in this work. The horizontal axis shows the search frequency. The vertical axis shows the wave strain \( h_{0,95\%} \) needed for a 95% detection efficiency, on the assumption \( \cos \iota = 1 \), namely, a circularly polarized signal. The blue dashed curve is based on simulations of Gaussian noise, while the red solid curve is corrected for the non-Gaussian statistics of the noise and the interferometer duty cycle, through multiplying by \( \kappa_j(f) \) (see Sec. III D). The diamonds show \( h_{0,95\%} \) derived through injections into subbands, again assuming a circularly polarized signal (see Sec. III D).
by $j$, following Ref. [40]. We determine $\kappa_j$ by doing $10^2$ injections (drawing parameters as described above) into the detector data for the $j$th subband, again using progressively lower $h_0$ until we determine the minimum $h_0$ detected. Then, $\kappa_j$ equals $h_0^{\text{eff,95\%}}$ for injections into real noise, divided by $h_0^{\text{eff,95\%}}$ for injections into Gaussian noise.

Producing $\kappa_j$ in this way for a random selection of subbands in the search band suggests that $\kappa$ depends weakly on frequency, most likely due to the $J$-statistic not perfectly summing sidebands [40]. A linear fit to the computed $\kappa_j$ values suggests a frequency-dependent correction factor

$$\kappa_{\text{freq}}(f) = 1.944 + 4.60 \times 10^{-4} f/(1 \text{ Hz}).$$  \hfill (14)

We use $\kappa_{\text{freq}}(f)$ to adjust the blue dashed curve in Fig. 3, producing the expected sensitivity across the full search band, where the noise is realistic (i.e., not Gaussian). The 50 subbands sampled are shown on the plot as gray diamonds.

IV. O2 ANALYSIS

We now analyze the data from LIGO’s O2, using the full dataset from November 30, 2016 to August 26, 2017, including data from the LIGO Livingston (L1) and Hanford (H1) observatories. The Virgo interferometer also participated in the last two months of O2, but we do not use any Virgo data in this analysis.

There are two notable pauses in data gathering: an end-of-year break starting on December 22, 2016 lasting for 13 days, and a commissioning break starting on May 7, 2017 lasting for 19 (L1) or 32 (H1) days.

Data stretches shorter than $T_{\text{SFT}}$ are discarded, as is a period of approximately one month where much of the band was contaminated due to a blinking light in the power system and a digital camera (used for detector diagnostics) that was inadvertently left on. A detailed discussion of Advanced LIGO detector noise can be found in Ref. [67]. Taking all these factors into account, the overall duty cycle (i.e., proportion of time spent gathering science-quality data) for O2 was 51.9% (L1) and 46.2% (H1).

Because of the commissioning break, one ten-day block has no data. We fill this block with a uniform log-likelihood, so that the HMM has no preference for remaining in the same frequency bin, or moving by one bin, during the break, while still allowing a maximum drift of $\Delta f_{\text{drift}}$ every ten days. An alternative, but equivalent, approach would be to remove the break entirely, and alter the transition matrix $A_{q_1q_2}$ for that step to allow the HMM to wander up to two frequency bins. The end-of-year break is also longer than ten days, but it is covered by two blocks. Both of the blocks that overlap with the end-of-year break contain data.

We search the same frequency band as Ref. [40], namely 60–650 Hz. The lower limit is set by LIGO’s poor sensitivity for signals $\lesssim 25$ Hz and the significant contamination from instrumental noise in the band 25–60 Hz. The sensitivity of the search falls as frequency increases, while compute time rises dramatically. We terminate the search at 650 Hz, as in Ref. [40].

The results of the search are presented in Fig. 4, which shows the frequency and recovered orbital parameters $a_0$ and $\phi_a$ for every path with $S > S_0$. The color of the points shows the Viterbi score associated with that path. As the
most a signal can wander during the observation is $N_T \Delta f_{\text{drift}} \approx 1.3 \times 10^{-3}$ Hz, which is small compared to $\Delta f_{\text{band}}$ (and what can be visually discerned on Fig. 4), we define $f_0$ for a given path to be equal to $f_0(t = N_T)$ for convenience.

To rule out false alarms, we apply the hierarchy of vetoes first described in Ref. [40]. The vetoes are (1) the known instrumental lines veto (described in Sec. IVA 1 below), (2) the single-interferometer veto (Sec. IVA 2), (3) the $T_{\text{obs}}/2$ veto (Sec. IVA 3), and (4) the $T_{\text{drift}}$ veto (ultimately not used, but discussed in Sec. IVA 4 of Ref. [40]). To ensure that the vetoes are unlikely to falsely dismiss a true signal, we perform the search on a dataset with synthetic signals injected into it, and ensure that those injections are not vetoed. These veto safety tests are described in Sec. IV B.

The number of candidates found in the initial search, and then vetoed at each step, are listed in Table IV.

### A. Vetoes

1. **Known lines veto**

There are a large number of persistent instrumental noise lines identified as part of LIGO’s detector characterization process [67,68]. These lines can arise from a number of sources, including interference from equipment around the detector, resonant modes in the suspension system, and external environmental causes (e.g., the electricity grid).

A noise line generally produces high $|F_a|$ and $|F_b|$ values. The convolution in (6) reduces the impact of this somewhat by summing bins near and far from the line, but in practice the noise lines are strong enough that they contaminate any candidate nearby. Accordingly, we veto any candidate whose Viterbi path $f_0(t)$ satisfies $|f_0(t) - f_{\text{line}}| < 2\pi a_i f_0/P$, for any time $t$ along the path and for any line frequency $f_{\text{line}}$. This veto is efficient, excluding 14 of the 20 candidates.

2. **Single interferometer veto**

During O2, L1 was slightly more sensitive than H1, but overall the sensitivities of the two interferometers were similar. Accordingly, any astrophysical signal that can be detected in the combined dataset should either be detected by the individual detector datasets when analyzed separately (for stronger signals) or in neither (for weaker signals). A signal that is detectable in one interferometer only is likely to be a noise artifact, so we veto it.

Following Ref. [40], we compare the Viterbi scores obtained from individual detectors to the original combined score $S_{\text{o}}$ to classify survivors of the known line veto into four categories discussed below, one of which is vetoed.

- **Category A.**—One detector returns $S < S_{\text{th}}$, while the other detector returns $S > S_{\text{th}}$, and the frequency estimated by the latter detector is close to that of the original candidate $f_{\text{o}}$, i.e., $|f_{\text{o}} - f_0| < 2\pi a_i f_{\text{o}}/P$, where the subscript $\cup$ denotes a quantity estimated by the search in both detectors. This category and the next represent signals where the score is dominated by one detector. We veto candidates in category A.

- **Category B.**—As with category A, one detector returns $S < S_{\text{th}}$, while the other detector returns $S > S_{\text{th}}$. Unlike category A, the frequency estimated by the latter detector is far from the original candidate, i.e., $|f_{\text{o}} - f_0| > 2\pi a_i f_{\text{o}}/P$. In this case, it is possible that there is signal at $f_{\text{o}}$, which is detectable when combining the data from both detectors but not from one detector, because an artifact masks its presence. Hence we keep the candidate for follow-up.

- **Category C.**—The candidate is seen with $S > S_{\text{th}}$ in both detectors. This could either be a relatively strong signal, or an artifact from a noise source common to both detectors. The single-interferometer veto cannot distinguish these possibilities. Again, we keep the candidate for follow-up.

- **Category D.**—The candidate is not seen by either detector, with $S < S_{\text{th}}$ in both detectors. This could be a signal that is too weak to see in either detector individually. We keep the candidate for follow-up.

Category A of the single-interferometer veto eliminates two of the remaining six candidates. The two eliminated candidates were stronger in H1 compared to L1.

3. **$T_{\text{obs}}/2$ veto**

We divide the observing run into two segments, the first covering 140 days from November 30, 2016 (GPS timestamp 1164562334) to April 19, 2017 (GPS timestamp 1176658334), and the second covering 90 days from January 19, 2017 (GPS timestamp 1168882334) to August 25, 2017 (GPS timestamp 1187731792). This division is chosen to get approximately equal effective observing time in the two segments. There is no forceful evidence to suggest that the gravitational wave strength of a LMXB varies significantly with time (and a signal with time-varying strength is likely to have a considerably more complicated form than assumed here); thus we do not expect a signal to appear preferentially in either segment. We search the segments separately for the candidates which survived both preceding vetoes. To determine whether to veto candidates at this stage, we apply the same set of categories as in veto 2.

### Table IV. Number of candidates found in the first pass, and number remaining after applying the vetoes described in Sec. IVA.

<table>
<thead>
<tr>
<th>After veto</th>
<th>Survivors</th>
</tr>
</thead>
<tbody>
<tr>
<td>First pass</td>
<td>20</td>
</tr>
<tr>
<td>Line</td>
<td>6</td>
</tr>
<tr>
<td>Single interferometer</td>
<td>4</td>
</tr>
<tr>
<td>$T_{\text{obs}}/2$</td>
<td>3</td>
</tr>
</tbody>
</table>
This veto eliminates one remaining candidate, which is much stronger in the first segment of the observing run than the second.

Reference [40] describes the $T_{\text{drift}}$ veto as a fourth veto that can be applied to candidates surviving the $T_{\text{obs}}/2$ veto. However, this veto is applicable to candidates with an observed spin-wandering timescale that is 20 days or longer. This is not the case for the surviving three candidates, so the $T_{\text{drift}}$ veto is not applicable to them.

The remaining candidates are in the subbands starting at 85.4, 503.6, and 507.2 Hz. The scores relevant to performing the veto procedure are given in Table V. All three candidates are stronger when analyzing the H1 detector data alone compared to analyzing L1 detector data alone, with the L1 results consistent with noise. The candidates in the subbands starting at 85.4 and 507.2 Hz are both stronger during the second half of O2 compared to the first half, while the candidate in the subband starting at 503.6 Hz is stronger in the analysis of the first half of O2. Particularly for the candidate in the 85.4 Hz subband, the asymmetry in score between the first and second half of the observing is extreme and suggestive of a detector artifact rather than an astrophysical signal. The asymmetry is less pronounced for the candidates in the subbands starting at 503.6 and 507.2 Hz, but both of these candidates are in a region of frequency space that is significantly contaminated by interferometer noise, particularly violin modes associated with the LIGO mirror suspension. For these reasons, it is most likely that these candidates are due to unknown instrumental noise in the H1 detector, although they are not formally ruled out by the veto procedure described above.

### B. Veto safety

To verify that the vetoes described previously do not unduly increase the false dismissal probability, we inject signals into the O2 data and perform the veto procedure described in the previous section. We inject a total of 50 signals into 50 subbands of width $\Delta f_{\text{band}}$ chosen to be comparable to the 200 injections used for the equivalent tests in Ref. [40] while having a large enough sample to be confident that false dismissals caused by the vetoes are rare in the context of the 5% false dismissal rate used in calculating sensitivity. The subbands and parameters chosen are selected randomly from the search band to achieve good frequency coverage, but excluding those subbands that contain a known line (and hence would be excluded by the known lines veto). Into these subbands, we inject a signal near the detection limit with $h_0$ typically at $h_0^{95\%}$ for that subband (although we inject a stronger signal if the signal turns out to be undetectable), and with $f_0$ drawn randomly from a uniform distribution over the interval $[f_{\text{start}}, f_{\text{start}} + 0.1$ Hz, $f_{\text{start}} + 0.1$ Hz], where $f_{\text{start}}$ is the lowest frequency in the subband. At each block, the signal is allowed to wander at most one frequency bin (i.e., by an amount drawn uniformly from $[-\Delta f_{\text{drift}}, +\Delta f_{\text{drift}}]$), and the signal frequency is constant within the block, following Ref. [40]. The other parameters are chosen in the same way as for the sensitivity tests described in Sec. III D.

We then apply vetoes 2 (single-interferometer veto) and 3 ($T_{\text{obs}}/2$ veto) to each candidate ($veto 1$ is inapplicable, as the injection bands avoid known lines, and veto 4 [$T_{\text{drift}}$ veto] was not used in this search). No injection was vetoed.

Because the veto safety procedure uses the O2 data as noise, it is possible that the safety results described above depend in some way on the specifics of O2. However, as the veto procedure copies the equivalent procedure in Ref. [40], which tests both S5 noise and O1 noise, we have confidence that the veto safety result is not specific to the peculiarities of O2.

### V. UPPER LIMITS

We can use the nondetection reported in the previous section, in concert with the approach outlined in Sec. III D, to place an upper limit on $h_0$ as a function of $f_0$ and compare the result to the indirect, torque-balance upper limit established by the x-ray flux [20].

#### A. Frequentist upper limit at 95% confidence

Failure to detect a gravitational wave signal allows us to place an upper limit on $h_0$ from a particular source, given a desired confidence level. In this section, we follow Ref. [40] in using a frequentist approach and setting 95% as the desired confidence level. The alternative, Bayesian approach in Ref. [61] is hard to adapt to the HMM-based search, because correlations between the Viterbi paths render the distribution of Viterbi scores difficult to calculate analytically.

We define $h_0^{95\%}$ to be the lowest amplitude signal for which we have a 95% probability or greater of detecting a signal with $h_0 \geq h_0^{95\%}$, that is, $\Pr(S \geq S_{\text{th}}|h_0 \geq h_0^{95\%}) \geq 0.95$. The value of $h_0^{95\%}$ depends on the inclination angle of the source, through Eq. (12). Figure 5 show the upper limit for three cases: assuming the neutron-star spin axis inclination

<table>
<thead>
<tr>
<th>Subband containing candidate</th>
<th>Original score</th>
<th>H1</th>
<th>L1</th>
<th>First part</th>
<th>Second part</th>
</tr>
</thead>
<tbody>
<tr>
<td>85.4</td>
<td>42.4</td>
<td>30.7</td>
<td>6.3</td>
<td>7.2</td>
<td>41.8</td>
</tr>
<tr>
<td>503.6</td>
<td>41.3</td>
<td>34.6</td>
<td>5.8</td>
<td>37.5</td>
<td>6.1</td>
</tr>
<tr>
<td>507.2</td>
<td>17.3</td>
<td>10.6</td>
<td>6.1</td>
<td>10.2</td>
<td>16.4</td>
</tr>
</tbody>
</table>
angle $i$ is equal to the electromagnetically constrained orbital inclination angle $i \approx 44^\circ$ (purple plus signs), a pure circularly polarized signal $|\cos i| = 1$ (green crosses), and a flat prior on $\cos i$ (blue asterisks). For subbands with no candidate path with a Viterbi score above the threshold, we take $h_0^{5\%}$ from Fig. 3 for the circularly polarized case, and determine $h_0^{5\%}$ for the two other cases using Eq. (12). No upper limit is established for subbands containing a vetoed candidate (because those bands are deemed to be contaminated by instrumental artifacts). Accordingly those subbands are excluded from Fig. 5.

The circularly polarized case produces the most stringent upper limit reflecting the fact that $|\cos i| = 1$ would be the most favorable configuration for producing gravitational waves. Conversely, assuming no knowledge of the inclination angle (the flat prior case) produces a looser upper limit. The lowest upper limit for this search is in the subband starting at 194.6 Hz, with upper limits of $h_0^{5\%} = 3.47 \times 10^{-25}, 1.93 \times 10^{-25}, 1.42 \times 10^{-25}$ for the unknown polarization, electromagnetically constrained, and circularly polarized cases, respectively. Previous work with the HMM in Ref. [40] found $h_0^{5\%} = 8.3 \times 10^{-25}, 4.0 \times 10^{-25}, 3.0 \times 10^{-25}$ for those cases in its most sensitive subband starting at 106 Hz.

**B. Torque-balance upper limit**

An indirect upper limit on gravitational wave strain can be obtained from x-ray observations. If the spin-down torque due to gravitational wave emission balances the accretion spin-up torque, with the latter inferred from the x-ray luminosity, one has $h_0 \geq h_0^{eq}$ with [20,22,60]

\[
h_0^{eq} = 5.5 \times 10^{-27} \left( \frac{F_X}{10^{-8} \text{ erg cm}^{-1} \text{s}^{-1}} \right)^{1/2} \left( \frac{R}{10 \text{ km}} \right)^{3/4} \times \left( \frac{1.4 M_\odot}{M_\star} \right)^{1/4} \left( \frac{300 \text{ Hz}}{f_\star} \right)^{1/2},
\]

where $F_X$ is the x-ray flux, $R$ is the length of the notional “lever arm” to which the accretion torque is applied, $M_\star$ is the stellar mass, and $f_\star$ is the (unknown) spin frequency.

To establish an upper limit, we take the electromagnetically measured $F_X = 4 \times 10^{-7} \text{ erg cm}^{-2} \text{s}^{-1}$ [49] of Sco X-1, and the common fiducial neutron-star mass $M_\star = 1.4 M_\odot$. The most conservative choice for the accretion torque lever arm is the stellar radius $R_\star = 10 \text{ km}$. We plot $h_0^{eq}$ as a function of frequency as the solid red curve in Fig. 5. Another physically reasonable choice of lever arm length is the Alfvén radius, $R_A$, i.e., the distance out to which outflowing material corotates with the star’s magnetic field. This is given by [40,48]

\[
R_A = 35 \left( \frac{B_\star}{10^3 G} \right)^{4/7} \left( \frac{R_\star}{10 \text{ km}} \right)^{12/7} \times \left( \frac{1.4 M_\odot}{M_\star} \right)^{1/7} \left( \frac{\dot{M}}{10^{-8} M_\odot \text{ yr}^{-1}} \right)^{2/7} \text{ km},
\]

where $B_\star$ is the polar magnetic field strength at the stellar surface, $G$ is Newton’s gravitational constant, and $\dot{M}$ is the accretion rate. The accretion rates in LMXBs can range from the Eddington limit, $2 \times 10^{-8} M_\odot \text{ yr}^{-1}$, down to about $10^{-11} M_\odot \text{ yr}^{-1}$ [69,70]. The magnetic fields on the neutron
stars in LMXBs are comparatively weak, lying in the range \(10^5 \text{ G} \lesssim B_* \lesssim 10^9 \text{ G}\) [20,70,71]. We substitute \(M = 10^{-8} M_\odot \text{ yr}^{-1}\) and \(B_* = 10^9 \text{ G}\) into Eq. (16) to maximize \(R_A\) and hence \(h_0\). The result is plotted as the orange curve in Fig. 5. Both torque-balance curves are plotted with \(f_0 = 2 f_*\), i.e., an orthogonal biaxial rotor, which is a conventional assumption [30].

At the most sensitive subband starting at \(f_0 = 194.6 \text{ Hz}\), the electromagnetically constrained upper limit is a factor of about 1.2 below (3.1 above) the torque balance for \(R = R_A\) \((R = R_\perp)\). The upper limits for a circularly polarized signal beat the \(R = R_A\) torque-balance upper limit between 60 and 223 Hz, and the upper limits assuming an electromagnetically constrained inclination angle beat the \(R = R_A\) torque-balance limit between 94 and 113 Hz.

The upper limits given in Fig. 5 are somewhat higher than those achieved by the most sensitive search to date, the O1 cross-correlation search, which has upper limits that are typically lower by a factor of approximately 1.5 [43]. A significant contributing factor to this is that the threshold \(S_{th}\) is set by assuming that the search at each binary orbital parameter is independent, while in fact there are significant correlations between adjacent points in search parameter space. These correlations are difficult to safely account for and so we make the conservative assumption that they are independent. Thus \(S_{th}\) is an overestimate of the threshold for a 1\% false alarm probability, in turn overestimating the upper limits and making a direct comparison of the upper limits difficult.

This search also uses updated binary orbital parameter ranges, taking advantage of a more recent analysis of electromagnetic observations to produce a search better targeted at Sco X-1. Similarly, while the detector design is fundamentally unchanged between O1 and O2, various detector improvements mean that some instrumental lines have been removed or ameliorated, making this search sensitive to signals that would have been obscured by instrumental noise in searches using earlier datasets. The hidden Markov model is also designed with particular emphasis on robustness to spin wandering. Together, these three reasons mean that the search covers a slightly different region of parameter space compared to previous Sco X-1 searches.

VI. CONCLUSION

In this paper, we search the LIGO O2 dataset for continuous gravitational waves from the LMXB Sco X-1, using a hidden Markov model combined with the \(J\)-statistic. We find no signal. The search band extends from 60 to 650 Hz. The sky location \(\alpha, \delta\) and orbital parameters \(P, a_0, \phi_p\) used for the matched filter are electromagnetically constrained; values are given in Table II. Monte Carlo simulations of spin-wandering signals injected into the LIGO O2 data imply frequentist 95\% upper limits of \(h_0^{95\%} = 3.47 \times 10^{-25}, 1.92 \times 10^{-25}, 1.42 \times 10^{-25}\) for unknown, electromagnetically restricted \((\cos i \approx 0.72)\), and circular polarizations respectively. The upper limits apply at 194.6 Hz, which is the most sensitive search frequency. For the electromagnetically restricted case, the limit is 3.1 times above, or 1.2 times below, the torque-balance limit, when the torque-balance lever arm is the stellar radius or the Alfvén radius respectively. Monte Carlo simulations are used to establish a detection threshold corresponding to a false alarm probability of \(\alpha = 0.01\).

These results improve on the results from the previous HMM search described in Ref. [40], by using data from LIGO’s second observing run, and by substituting the \(J\)-statistic for the Bessel-weighted \(F\)-statistic to track the phase of the orbital Doppler shift. As a result, the search in this paper is \(\approx 2\) times more sensitive compared to that in Ref. [40]. The analysis remains computationally efficient, requiring \(\lesssim 3 \times 10^5\) CPU-hr for the search itself and \(\lesssim 10^6\) CPU-hr for simulations to characterize the sensitivity and false alarm rate.

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