Logic, algebra and topology: investigations into canonical extensions, duality theory and point-free topology
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## Contents

### Acknowledgments

#### 1 Introduction

1.1 Logic, algebra and topology ............................................ 1  
1.2 Mathematical surroundings ........................................... 1  
1.3 Survey of Contents .................................................. 5  

#### 2 Canonical extensions: a domain theoretic approach

2.1 Canonical extension via filters, ideals and topology ............ 10  
  2.1.1 The canonical extension of a lattice .......................... 11  
  2.1.2 Topologies on posets and completions ...................... 15  
  2.1.3 Characterizing the canonical extension via the δ-topology 18  
  2.1.4 Basic properties of the canonical extension ............... 24  
  2.1.5 Conclusions and further work ................................. 27  
2.2 Canonical extensions of maps I: order-preserving maps ....... 27  
  2.2.1 The lower and upper extensions of an order-preserving map 28  
  2.2.2 Operators and join-preserving maps ......................... 33  
  2.2.3 Canonical extension as a functor I: lattices only .......... 42  
  2.2.4 Conclusions and further work ................................. 44  
2.3 Canonical extensions via dcpo presentations .................... 45  
  2.3.1 Dcpo presentations ........................................... 47  
  2.3.2 A dcpo presentation of the canonical extension .......... 48  
  2.3.3 Extending maps via dcpo presentations ................... 50  
  2.3.4 Conclusions and further work ................................. 51  

#### 3 Canonical extensions and topological algebra

3.1 Topological algebra ................................................. 54  
  3.1.1 Compact Hausdorff algebras ................................ 55  
  3.1.2 Profinite algebras and profinite completions ............ 56
# Duality, profiniteness and completions

4.1 Dualities for distributive lattices with operators ........................................ 112
   4.1.1 Semi-topological DLO’s ........................................................................... 113
   4.1.2 Colimits of ordered Kripke frames .............................................................. 116
   4.1.3 Duality for profinite DLO’s ........................................................................ 120
   4.1.4 Profinite completion via duality ................................................................. 122
   4.1.5 Conclusions and further work .................................................................... 127

4.2 A brief survey of subcategories of \textit{DLO} ......................................................... 127
   4.2.1 Distributive lattices .................................................................................... 128
   4.2.2 Boolean algebras ....................................................................................... 129
   4.2.3 Heyting algebras ...................................................................................... 130

4.3 Duality for topological Boolean algebras with operators ................................. 132
   4.3.1 Duality for Boolean topological BAO’s ....................................................... 133
   4.3.2 Ultrafilter extensions of image-finite Kripke frames ................................. 137
   4.3.3 Conclusions and further work .................................................................... 139

# Coalgebraic modal logic in point-free topology

5.1 Introduction ........................................................................................................... 141
5.2 Preliminaries ......................................................................................................... 144
   5.2.1 Basic mathematics .................................................................................... 144
   5.2.2 Category theory ....................................................................................... 145
   5.2.3 Relation lifting ........................................................................................ 147
   5.2.4 Frames and their presentations ................................................................. 152
   5.2.5 Powerlocales via $\Box$ and $\Diamond$ ............................................................ 154

5.3 The $T$-powerlocale construction ..................................................................... 157
   5.3.1 Introducing the $T$-powerlocale ................................................................. 157