



UvA-DARE (Digital Academic Repository)

Essays on auctions

Hu, X.

Publication date

2010

Document Version

Final published version

[Link to publication](#)

Citation for published version (APA):

Hu, X. (2010). *Essays on auctions*. [Thesis, fully internal, Universiteit van Amsterdam]. Thela Thesis.

General rights

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

tinbergen *institute**Essays on Auctions*

Xianhua Hu

The past few decades have witnessed a remarkable expansion of auctions activities. From the sales of mobile-phone licenses and industrial goods, to the privatization of formerly state-owned enterprises, auction mechanisms have been employed to enhance revenue and efficiency in otherwise imperfectly competitive markets. This thesis incorporates the risk attitudes of the auction participants, as well as the possibility of collusion among the bidders, into the existing auction models and offers an in-depth analysis of several important topics. It shows how the seller's optimal reserve price is endogenously affected by the participants' risk preferences, how the use of premium tactics helps deter collusion, and how the outcome of an auction varies with the practical situations in terms of efficiency, expected revenue, and the expected utilities of the seller and buyers. While offering new insights into the competitive bidding behavior and the resulting performance of various auction policies in these more general situations, the thesis also contains some theoretical results that are of interest on their own.

Xianhua Hu (Audrey) obtained her master degree in finance at the University of Amsterdam (UvA) in 2005. She then continued her study at UvA/CREED and Tinbergen Institute as a doctoral researcher on license auctions. Some of her research was conducted at the California Institute of Technology in 2008. The theoretical and experimental results of her studies have been presented in various occasions, including the annual meetings of the Econometric Society, European Economic Association, and Royal Economic Society. Part of her work has been published in the *Journal of Economic Theory* and the *International Journal of Industrial Economics*. Audrey has recently been rewarded a Rubicon grant by the Netherlands Organization for Scientific Research (NWO) and begun her post-doctoral research at Tilburg University.

ESSAYS ON AUCTIONS

ISBN 978 90 3610 206 3

Cover design: Crasborn Graphic Designers bno, Valkenburg a.d. Geul

This book is no. **485** of the Tinbergen Institute Research Series, established through cooperation between Thela Thesis and the Tinbergen Institute. A list of books which already appeared in the series can be found in the back.

ESSAYS ON AUCTIONS

ACADEMISCH PROEFSCHRIFT

ter verkrijging van de graad van doctor

aan de Universiteit van Amsterdam

op gezag van de Rector Magnificus

prof. dr. D.C. van den Boom

ten overstaan van een door het college voor promoties ingestelde

commissie, in het openbaar te verdedigen in de *Agnietenkapel*

op woensdag 22 september 2010, te 14:00 uur

door

Xianhua Hu

geboren te Shanghai, China

Promotiecommissie

Promotor:

Prof. Dr. T. J. S. Offerman

Copromotor:

Dr. A. M. Onderstal

Overige leden:

Prof. Dr. J. Boone

Prof. Dr. P. J. J. Herings

Prof. Dr. T. Kittsteiner

Prof. Dr. A. J. H. C. Schram

Prof. Dr. R. Sloof

Faculteit Economie en Bedrijfskunde

Contents

Preface	vii
1 Introduction	1
1.1 Overview	1
1.2 Summary of Main Results	6
1.2.1 Chapter 2	6
1.2.2 Chapter 3	7
1.2.3 Chapter 4	8
2 Risk Aversion and Optimal Reserve Prices	11
2.1 Introduction	12
2.2 Model	16
2.3 Technical Preliminaries	17
2.4 Properties of the FPA Equilibrium	19
2.5 Seller Preferences over Auctions with the Same Reserve Price	23
2.6 Optimal Reserve Prices	24
2.7 Concluding Discussion	32

2.8	Appendix	33
3	Deterring Collusion using Premium Auctions	35
3.1	Introduction	36
3.2	Theory	42
3.3	Experimental Design and Procedure	50
3.4	Results	54
	3.4.1 Between auction comparisons	55
	3.4.2 Individual bidding behavior	65
	3.4.3 Explanation of the main results	70
3.5	Concluding Discussion	72
3.6	Appendix	75
4	Non-Quasilinear Preferences in Premium Auctions	89
4.1	Introduction	90
4.2	The Model	94
4.3	The EPA Symmetric Equilibrium	95
4.4	The Premium Effects	107
4.5	Concluding Discussion	119
5	Conclusion	121
6	Summary	125
7	Samenvatting (Summary in Dutch)	127
8	Bibliography	131

Preface

When Prof. Jacob Goeree and Prof. Jan Boone recruited me for a Ph.D. position in late 2005, I had little idea about what exactly I would do—except that I would pursue a career in the intellectual environment of University of Amsterdam and Tinbergen Institute. I then learned about the intriguing “Amsterdam Auctions” published by Prof. Jacob Goeree and Prof. Theo Offerman in *Econometrica*, and about my task to conduct research on issues related to “licence auctions.” Subsequently, my promoter Prof. Theo Offerman and copromoter Dr. Sander Onderstal deftly guided me into the field of auctions. I have especially benefited from Theo’s economic insight, knowledge in auctions and in other related disciplines such as behavioral economics, as well as his strong encouragement for me to push the boundaries.

While working under the excellent supervisions of Theo and Sander, I continued to receive gracious advice from Jacob and Jan as well. I was also fortunate enough to become a coauthor with Dr. Liang Zou, who had been my best teacher for many years during my Bachelor and Master studies.

Liang's rigorous approach has proven to be a valuable asset in our joint work indeed.

In the summer of 2008, I was given the opportunity to visit California Institute of Technology for two months. This visit not only allowed me to benefit from Jacob's supervision in person, but also enlarged my view about the academic world. Jacob's critiques on some of my previous work as well as his directional suggestions are exceptionally helpful. No sooner had I come back to Amsterdam from the Caltech visit, than I got the opportunity to write a new paper with Liang and Prof. Steve Matthews of University of Pennsylvania. Steve's world-class knowledge in economic theory impressed me greatly and his unyielding perfectionism has strongly influenced my own stance through our collaboration.

Looking back over the path I have gone through, and the work done that has now come to fruition, I feel enormously grateful to all these scholars mentioned above. Some other scholars have also given me encouragement and helpful comments on my work along the way, among them are Professors Zaifu Yang, Laixiang Sun, and Arthur Schram, to whom I am indebted as well. In addition, I wish to thank all my teachers who have taught me in the past—in particular, those who sharpened my understanding about incentive theory, auction theory, game theory, advanced calculus, industrial organization and so on in the courses offered at Tinbergen Institute for the MPhil and Ph.D students. From the first-rate teachers, such as Professors Bruno Biais, Chaim Fershtman, and Wen Quan, I acquired the knowledge and skills necessary to conduct independent research. I am also obliged to my teachers of mathematics at Fudan University and the Olympic School, who showed me the fun side of this fundamental and fascinating subject and instigated my passion for scientific discoveries. The warm and supportive research environment of UvA/CREED and Tinbergen Institute, as well as the friendship I enjoyed with my Ph.D colleagues such as Ailko and Michal, are well appreciated, too.

My deep gratitude also goes to my grandparents, who taught me the value of independent thinking at a young age; to my mother and uncles,

who made me understand the importance of persistence in scholarly pursuit through their own examples; to my father, who, being a businessman himself, urged me from time to time not to lose the touch of reality; and to my childhood friends—although receiving Ph.Ds from other disciplines, their courage when facing adversities has constantly served as a stimulus for me to strive for better.

Finally, this thesis would not have been completed without the financial support of the Netherlands Organization for Scientific Research (NWO) through a VICI grant (NWO-VICI 453-03-606).

I wish to dedicate this thesis to all those whose work has inspired me.

Audrey X. Hu

February 2010, Amsterdam

1

Introduction

1.1 Overview

The past few decades have witnessed a remarkable expansion of auctions activities, along with a remarkable expansion of the literature on auctions. From the sales of mobile-phone licences and industrial goods, to the privatization of formerly state-owned enterprises, auction mechanisms have been employed to enhance revenue and efficiency in otherwise imperfectly competitive markets. The growth of e-commerce has greatly facilitated business-to-business transactions through auctions, inspiring at the same time research and innovations toward new auction designs.

“An auction is a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from the market participants.” (McAfee and McMillan, 1987, p. 701). Although the auction rules may vary – in practice or in design – they share a common objective to find out the highest possible value of the object for sale.¹ This is done

¹As usual, for expositional convenience I shall assume that there is a seller facing a number of competing buyers. The parallel conclusions of this thesis can be drawn straightforwardly for the cases

by encouraging or inducing competition among the potential buyers, who typically have private information that could influence the prices. As competition tends to go hand-in-hand with efficient allocation of resources, this explains the popularity of auctions in practice and the prominence of the auction related topics in the economics literature (e.g., Milgrom, 2004; Klemperer, 2004; Krishna, 2009; and the references contained therein). However, although “[e]conomists are proud of their role in pushing for auctions,” important issues remain as to why “many auctions—including some designed with the help of leading academic economists—have worked very badly.” (Klemperer, 2002, p. 169). Klemperer has identified various practical reasons that may cause an auction to fail, including bidder collusion, entry deterrence/predation by a bidding firm, inadequate reserve prices set by the seller, and other political or institutional pitfalls.

In this thesis, I will devote some in-depth analyses on the role of explicit or implicit reserve prices, as well as the role of risk attitudes of the participants, in a number of standard and non-standard auctions. Many of the practical concerns of the sellers, such as those mentioned above, can be mitigated by properly set reserve prices; whereas failure of such reserve prices could seriously damage the expected outcome of an auction. For example, in its sales of four third-generation mobile-phone licenses in November 2000, the Swiss government collected a total amount of revenue that was worth only one-fifteenth of what had been expected. The major reason for this disastrous outcome is that the Swiss government had set a ridiculously low reserve price prior to the auction (Klemperer, 2002, pp. 174-175).

Most of the studies in the auctions literature begin with the assumption that the bidders are risk neutral—or that they have a linear utility function for income. A well-known result obtained under this assumption is the revenue equivalence theorem (e.g., Vickrey, 1961; Myerson, 1981; Riley and Samuelson, 1981; Krishna and Maenner, 2001; and Milgrom and

where a single buyer faces a number of competing sellers, such as the cases with several firms bidding for a procurement contract with the government.

Segal, 2002). The theorem predicts that when bidders have independent private information or signals, the payment rules do not matter under the same allocation rule. Another well-known result under these circumstances is that for bidders who are ex ante symmetric, the optimal reserve price that maximizes the seller's expected revenue will be the same in the standard auctions such as the English auction, the second-price sealed bid auction, the first-price sealed bid auction, or the Dutch auction.² Because under the optimal reserve price the lowest bidder is indifferent between winning and losing, by the revenue equivalence theorem the allocation rule uniquely determines the expected payment by the buyers and hence the expected revenue to the seller. Clearly, these results, despite their elegance, are unable to explain the variety of auction policies adopted under different circumstances—let alone to answer the question why some auctions perform better or worse than the others in raising revenue.

Another well-known prediction derived under risk neutrality is due to Milgrom and Weber (1982) who predict that, when bidders have interdependent values and affiliated signals, the English auction generates a higher expected revenue than the second-price auction, which in turn generates a higher expected revenue than the Dutch and first-price auctions. None of the above predictions, however, is robust to changes in the bidders' risk preferences. Existing studies (e.g., Holt, 1980; Riley and Samuelson, 1981; Harris and Raviv, 1981; Milgrom and Weber, 1982; Matthews, 1983,

²The English (ascending) auction and the Dutch (descending) auction are open-bid auctions. In the former, the price for the object(s) for sale increases until all but one bidder remains, who will then be awarded the object(s) and pay for the stopping price. In the latter, the price decreases and stops as soon as one bidder shows willingness to pay for the object(s) to become the winner and pay the stopping price.

The first-price and second-price (Vickrey) auctions refer to the sealed-bid auctions, both of which invite the bidders to submit sealed bids, with the understanding that the highest bidder will win the object(s). They differ only in the payment rule: the winner pays his own bid in the former and the second-highest bid in the latter. These auctions are conventionally viewed as the standard auctions (e.g., Maskin and Riley, 1984).

It can be shown that in the independent private values settings, the English and second-price auctions are strategically equivalent, so are the Dutch and first-price auctions (see Milgrom and Weber (1982) for detailed arguments).

1987; Maskin and Riley, 1984; Cox et al., 1982, 1988) show that in symmetric independent private values settings, bidders submit higher bids in the first-price auction when they are risk averse rather than risk neutral. This implies that the expected revenue is higher in the first-price auction than it is in the second-price auction, as the bids in the second-price auction are unaffected by the risk attitudes of the bidders. This unambiguous result has two immediate consequences. First, it suggests that a risk neutral seller would prefer the Dutch or first-price auctions to the English or second-price auctions when bidders are risk averse and have symmetric independent private values. Second, even if the bidders' values are interdependent and signals affiliated, by a continuity argument the seller's preference for the Dutch or first-price auctions may continue to hold, as long as the interdependence of the bidders' information and valuation is not "too" strong. Since these results are at variance with those of Milgrom and Weber's, they suggest that the risk preferences of the bidders can be an important factor that will affect an auction's outcome.

Although most of the attention has been directed to the seller's expected revenue in the auctions literature, there also exists some work on how risk aversion affects the expected utilities of the bidders under different auction policies. For instance, Matthews (1987) shows that in symmetric independent private values settings, the buyers who exhibit constant Arrow-Pratt absolute risk aversion have the same expected utilities in any of the standard auctions: English, Dutch, first-price, or second-price. This result suggests that "payoff equivalence" from the buyers' viewpoint is not necessarily a result of buyer risk neutrality, as it can hold for certain type of risk averse buyers (see Milgrom, 2004). Matthews further shows that if the buyers exhibit decreasing (increasing) risk aversion, then they have strictly higher (lower) expected utilities in the English and second-price auctions than in the Dutch and first-price auctions. Understanding the buyers' preferences over auction formats can be important because even though an individual bidder may not have much power to influence the auction design, he has nevertheless the choice to "vote with his feet." This

possibility is relevant to the seller especially where the potential bidders are few and they face certain costs of participating in the auction (e.g., Smith and Levin, 1996).

With few exceptions (e.g., Myerson, 1981; Riley and Samuelson, 1981; Levin and Smith, 1994; Waehrer et al., 1998), the existing studies of auctions assume either that there is no reserve price,³ or that the same reserve price is exogenously given under different auction formats. Departing from these assumptions, Chapter 2 of this thesis will study how the optimal reserve prices can be determined in the standard auctions, assuming that both the seller and the buyers are risk averse. An implicit assumption in this chapter is that the seller is capable of committing to a reserve price prior to the auction.

In some other situations, the seller may not have sufficient knowledge about, or the ability of commitment to, a proper reserve price. Implicit reserve prices could then be derived through a two-stage auction such as the Anglo-Dutch auction proposed by Klemperer (2002). Chapter 3 will study such cases in a non-standard premium auctions setting, where the seller pays a number of highest bidders a cash reward according to some pre-specified rule. Such auctions typically proceed in two stages (Goeree and Offerman, 2004). The first stage will generate a reserve price for the second stage, and the two highest bidders in the first stage will enter the second stage to continue bidding. The auctions we consider differ from the Anglo-Dutch auction mainly in that the second-stage bidders are entitled of the premium to be paid by the seller. The objective of this chapter is to study the possibility of using the premium tactics to deter collusion among the buyers.

Chapter 4 continues the study of two-stage premium auctions in the preceding chapter, extending the analysis to the case in which the partici-

³The term “no reserve price” is in fact inaccurate because most of the auctions do have a reserve price, as pointed out by Milgrom and Weber (1982). It is more adequate to say that “the reserve price is not binding” instead, meaning that bidders with the lowest possible value (or willingness to pay) for the auctioned object is not prevented from bidding.

pants (both the seller and the buyers) can be risk averse or risk loving. The focus of this chapter is then placed on the interplay between the premium and the bidders' risk preferences in terms of how they affect the seller's and the bidders' expected utilities.

1.2 Summary of Main Results

1.2.1 Chapter 2

This chapter focuses on the effects of buyer *and* seller risk aversion on the seller's optimal reserve price in standard Dutch or first-price auctions (FPA) and English or second-price auctions (SPA). Sharp results are obtained by restricting attention to the otherwise simplest setting, that of symmetric and independent private values.

The main conclusions of this chapter are as follows.

- When the seller and/or the buyers are risk averse, the seller's optimal reserve price will be lower in the FPA than in the SPA. Risk aversion thus makes the FPA, in general, more ex post efficient than the SPA.
- In either auction, a more risk averse seller sets a lower reserve price. Thus, the more risk averse the seller, the more ex post efficient are both auctions.
- In fairly general cases, the seller sets a lower reserve price in the FPA if the bidders are more risk averse.

The general conclusion of this chapter is that risk aversion can be a disguised blessing in terms of ex post efficiency, as it induces the seller to lower the reserve price, resulting in a higher probability that the object is allocated to the one who values it most. It is worth noting that the comparative efficiency implications of the FPA and the SPA would be absent if the players were all risk neutral, in which case the seller would choose the same reserve price in both auctions (e.g., Myerson, 1981).

1.2.2 Chapter 3

This chapter examines the potential of the English premium auctions (EPA) in deterring bidder collusion. The main idea is that a premium auction can discourage “strong” bidders (e.g., those who have a serious interest in acquiring the object for sale) to form a cartel because “weak” bidders (e.g., “fortune hunters”), in quest of the premium, may be attracted to the auction and bid aggressively to spoil the potential profits of the cartel. For tractability the bidders in this chapter are assumed to be risk neutral. The collusive properties of the first-price, English, and the English premium auctions are then investigated using a laboratory experiment.

The main findings of this laboratory experiment reported in this chapter are as follows.

- When the cartel members do not defect, the FPA appears to be more conducive to collusion than other auctions. This result may be due to the possibility that the subjects were actually risk averse. Then, consistent with the results of Chapter 2, without collusion the competitive bidders would bid most aggressively in the FPA. The incentive to collude was therefore the highest under this auction policy.
- We also observe that the English auction is very conducive to collusion—almost as strongly as in the FPA.
- Whenever collusion is possible, the revenue dominance of the FPA over the English auction, as usually reported in experimental private value auctions, completely disappears.
- Even though the model considered in this chapter involves multiple equilibria in the EPA, the subjects (weak bidders) in the experiment are found to focus on the more aggressive equilibrium, which makes collusion unattractive.

Therefore, the experiment confirms the theoretical prediction that the EPA is less conducive to collusion than the other auction formats. The chapter concludes that the EPA can outperform the English and first-price auctions in avoiding collusion and generating higher expected revenue.

1.2.3 Chapter 4

This chapter provides a theory of the English premium auction (EPA) for the canonical case in which risk averse or risk loving bidders with symmetric private values compete. The aim of this chapter is to sharpen and enrich our understanding about the premium auctions, which have been studied mainly under the assumption of risk neutrality in the literature so far. The chapter shows how bidding behavior and the expected utilities of the participants can be affected by the bidders' risk attitudes as well as the premium.

A particularly noteworthy result in this chapter is what we call the “net-premium effect.” This effect says that, given any arbitrary premium rule, as the price rises in an EPA and the bidders update their information about the other bidders' value distributions, the conditional expected utility for the premium from the ongoing auction is always equal to the bidder's utility for the premium should he quit at the current price. The net-premium effect implies that every bidder's conditional expected utility for the premium equals zero until he enters the second stage of EPA. This finding is interesting on its own, as it adds a new insight into the revenue equivalence theorem. Namely, as long as the allocation rule remains the same, adding a premium to the English (or Vickrey) auction always results in a zero-expected utility for the premium for the bidders *ex ante*. While offering a useful tool for comparative welfare analysis of the premium auctions beyond the risk neutrality assumption, this effect predicts revenue equivalence when the bidders are risk neutral.

The main conclusions of this chapter are as follows.

- Under plausible conditions, there exists a unique EPA symmetric equilibrium. In this equilibrium, all bidders bid higher than their

private values—except the one who has the highest possible value and whose bid will be equal to his value.

- Incentive compatibility implies a net-premium effect that is key to the main conclusions of this chapter.
- For any arbitrary premium rule, the expected payment by any bidder decreases as the bidders become more risk averse. Hence, a risk neutral seller is better (worse) off to offer a premium when the bidders are risk loving (averse).
- Under plausible conditions, the bidders prefer the EPA to the English auction if and only if they are risk averse. Therefore, the conventional wisdom that premium auctions tend to attract risk-seeking speculators does not hold in our symmetric auction environment.
- The premium tends to reduce the riskiness of the payment. Hence, a risk averse (loving) seller may like (dislike) the premium policy when the bidders are risk neutral.

Indeed, these results suggest a conflict of interests between the revenue-maximizing seller and the bidders over the use of premiums, and this conflict of interests continues to hold when the seller is risk averse but the bidders are risk loving, or vice versa. However, the seller and the bidders may simultaneously prefer the EPA to the English auction when the bidders are risk neutral or marginally risk averse, and the seller is sufficiently risk averse.

2

Risk Aversion and Optimal Reserve Prices

*This chapter is based on Audrey Hu, Steven A. Matthews, and Liang Zou. 2010. "Risk Aversion and Optimal Reserve Prices in First- and Second-Price Auctions." *Journal of Economic Theory* 145, 1188-1202.*

2.1 Introduction

Most of the studies in the auctions literature begin with the assumption that the bidders are risk neutral for income (or that they have a utility function). A well-known result obtained under this assumption is the revenue equivalence theorem (e.g., Vickrey, 1961; Myerson, 1981; Riley and Samuelson, 1981; Krishna and Maenner, 2001; and Milgrom and Segal, 2002), which predicts that when bidders have independent private information (or signals) the payment rules do not matter under the same allocation rule. In other words, as long as the lowest bidder is indifferent between winning and losing, the allocation rule uniquely determines the expected payoffs of all participants, including the buyers as well as the seller.

None of the above predictions is robust to changes in the bidders' risk preferences, however. Existing studies show (e.g., Holt, 1980; Riley and Samuelson, 1981; Harris and Raviv, 1981; Milgrom and Weber, 1982; Matthews, 1983, 1987; Maskin and Riley, 1984; Cox et al., 1982, 1988) that in symmetric independent private values settings, bidders submit higher bids in the first-price auction when they are risk averse rather than risk neutral. This implies that the expected revenue is higher in the first-price auction than it is in the second-price auction, as the bids in the second-price auction are unaffected by the risk attitudes of the bidders. This unambiguous result has two immediate consequences. First, it suggests that a risk neutral seller would prefer the Dutch or first-price auctions (henceforth, FPA) to the English or second-price auctions (henceforth, SPA) when bidders are risk averse and have symmetric independent private values.¹ Second, even if the bidders' values are interdependent and signals affiliated, by a continuity argument the seller's preference for the

¹We use the term FPA for both the first-price sealed-bid auction and the strategically equivalent Dutch (descending) auction. We use the term SPA for both the second-price sealed-bid (Vickrey) auction and the "button" model of the English ascending-bid auction, as they have the same dominant strategy equilibria in our private values setting (Milgrom and Weber, 1982).

FPA may continue to hold, as long as the interdependence of the bidders' information and valuation is not "too" strong.²

The above-mentioned results have been obtained assuming that the same reserve price is exogenously given in all auctions. However, the reserve price in most real auctions is set by the seller. To the extent that it influences bidding behavior and depends on the type of auction, the endogeneity of the reserve price should be taken into account. In particular, the comparative statics of the optimal reserve price are of direct interest because they bear on ex post efficiency. Lowering the reserve price decreases the probability of the inefficient event in which no sale occurs because the maximum value of the bidders exceeds the seller's value but not the reserve price.

In auction models with private values and independent signals, it is well-known that the seller tends to fix a reserve price strictly higher than his own value for the object for sale.³ This commitment that maximizes the seller's expected revenue (or utility) ex ante entails potential inefficiency ex post, as it excludes some buyers from purchasing the good even though they are willing to pay a price higher than the seller's value. Indeed, exclusion inefficiency of this kind frequently emerge from the studies of various economic problems such as credit rationing (e.g., Stiglitz and Weiss (1981)), monopoly pricing (e.g., Armstrong (1996)), and so on. The general message from these results is that under incomplete information, a monopolistic seller or creditor typically engages in mechanisms that exclude some low value buyers or high risk borrowers – despite the fact that it incurs inefficiency. As such, these results help identify the sources of market failure, offer plausible explanations of otherwise puzzling economic

²This and related results are established, e.g., by Holt (1980), Riley and Samuelson (1981), Harris and Raviv (1981), Milgrom and Weber (1982), Matthews (1983, 1987), Maskin and Riley (1984), Cox et al. (1982, 1988), Smith and Levin (1996), and Eso and White (2004).

³This result was independently found in, e.g., Matthews (1980), Myerson (1981), Riley and Samuelson (1981), and Maskin and Riley (1984). See also Hu, Matthews, and Zou (2010) for a more recent treatment.

phenomena, and suggest possible solutions in terms of relevant regulatory policies.

This chapter focuses on the effects of buyer and seller risk aversion on the seller's optimal reserve price in standard first and second-price auctions. Sharp results are obtained by restricting attention to the otherwise simplest setting, that of symmetric and independent private values. The main results are Theorems 1 – 3.

Theorem 1 establishes that if the seller and/or the buyers are risk averse, then the seller sets a lower reserve price in the FPA than in the SPA. This is in contrast to when all parties are risk neutral, in which case the revenue equivalence theorem implies that the seller's optimal reserve price is the same in both auctions. Risk aversion thus makes the FPA more ex post efficient than the SPA. The result hinges on how the FPA equilibrium bid function is affected by a marginal increase in the reserve price. Risk averse bidders increase their bids less than do risk neutral bidders, and a risk averse seller values the increase in the bids of the high bidders relatively less than does a risk neutral seller because of diminishing marginal utility. Both forces lower the seller's marginal incentive to raise the reserve price.

Theorem 2 establishes that in either auction, a more risk averse seller sets a lower reserve price. Thus, the more risk averse the seller, the more ex post efficient are both auctions. The intuition is straightforward: a more risk averse seller values more (on the margin) a decrease in the risk of not selling the object. The proof, however, is surprisingly intricate.⁴

Theorem 3 establishes that in two fairly general cases, the seller sets a lower reserve price in the FPA if the bidders are more risk averse. (Bidder risk aversion does not affect the SPA equilibrium.) In case (a) the reverse hazard rate function of the bidders' values is decreasing, and either the more risk averse or the less risk averse group of bidders (or both) exhibit nonincreasing absolute risk aversion. In case (b) the more risk averse bidders are strictly more risk averse, in the sense that the minimum of their

⁴Theorem 3 in Waehrer et al. (1998) is our Theorem 2 for the case of risk neutral bidders (and a more general information structure). Our proof takes a different approach.

Arrow-Pratt measure of risk aversion exceeds the maximum of that of the less risk averse buyers. In either case the rate at which the FPA bid function increases in the reserve price is smaller when the bidders are more risk averse. This gives the seller less incentive to raise the reserve price. This effect is stronger if the seller is also risk averse, as then the fact that more risk averse bidders bid higher than less risk averse bidders implies that the seller has a lower marginal utility for the increase in their bids caused by an increase in the reserve price.

The chapter begins with the model in Section 2.2. Useful technical results are in Section 2.3. The FPA equilibrium is studied in Section 2.4. The seller's preferences over auctions with the same reserve price are determined in Section 2.5, and his optimal reserve prices are examined in Section 2.6. Section 2.7 concludes.

2.2 Model

An indivisible object is to be possibly sold to one of $n \geq 2$ potential buyers through either a FPA or a SPA with a reserve price. Each buyer $i \in N = \{1, \dots, n\}$ has a private value for the object, v_i , which is unknown to the others. Ex ante, these values are independently distributed on an interval $[L, H]$ according to the same distribution function F , which has a density function $f = F'$ that is strictly positive and continuously differentiable on $[L, H]$. Some of our results are obtained under the assumption of a decreasing reverse hazard rate function:

$$\text{(DRH)} \quad \frac{f(v)}{F(v)} \text{ strictly decreases on } (L, H).$$

Each participant maximizes expected utility. Each buyer has the same utility function, $u_B : \mathbb{R} \rightarrow \mathbb{R}$. If a buyer with value v wins and pays a price b , his utility is $u_B(v - b)$; his utility is $u_B(0)$ if he loses.⁵ We assume u_B is twice continuously differentiable, with $u'_B > 0$ and $u''_B \leq 0$.

The seller has a value $v_0 \in [L, H]$ for the object, and a twice continuously differentiable utility function, $u_S : \mathbb{R} \rightarrow \mathbb{R}$, satisfying $u'_S > 0$ and $u''_S \leq 0$. The seller's utility is $u_S(b)$ if a sale occurs at price b , and it is $u_S(v_0)$ otherwise.

We consider first and second-price auctions with a reserve price $r \in [L, H]$.⁶ In an equilibrium of either auction, a buyer with a value $v < r$ abstains from bidding. In a SPA, the dominant strategy of a buyer with $v \geq r$ is to submit a bid equal to v . We restrict attention to this equilibrium of the SPA.

Turning to the FPA, it is useful to define $G \equiv F^{n-1}$. If a buyer has value v , then $G(v)$ is the probability that every other buyer has a lower value.

⁵A more general formulation would have $u_B = u(v, -b)$ as the winning bidder's payoff. Under appropriate assumptions, as in Maskin and Riley (1984) or Matthews (1987), our main results extend to this generalization.

⁶This is without loss of generality, since in either auction the equilibrium for $r < L$ is the same as it is for $r = L$ (Maskin and Riley, 1984, Remark 2.1), and for $r > H$ there is zero probability of a sale in any equilibrium.

Let $g \equiv G'$ be the associated density, and let $\ell(v) = g(v)/G(v)$. Lastly, define the function $\gamma = [u_B - u_B(0)]/u'_B$. Then the unique symmetric equilibrium bidding function of the FPA, $b(\cdot, r)$, is equal to the solution on $[r, H]$ of the differential equation,

$$b_1(v, r) = \frac{g(v) [u_B(v - b) - u_B(0)]}{G(v)u'_B(v - b)} = \ell(v)\gamma(v - b), \quad (2.1)$$

that satisfies the initial condition $b(r, r) = r$ (e.g., Maskin and Riley, 1984). We restrict attention to this equilibrium of the FPA.⁷ Observe that for $r \in (L, H)$, $b_1(r, r) = 0$ and $b_2(r, r) = 1 - b_1(r, r) = 1$. For $r = L$ we have $b_1(L, L) = \frac{n-1}{n}$,⁸ and $b_2(L, L)$ is undefined (see footnote 11 below).

Let $R_i = -u''_i/u'_i$ denote the Arrow-Pratt measure of absolute risk aversion for $i = B, S$. The case in which the bidders have constant absolute risk aversion (CARA) provides a benchmark, as then (2.1) can be solved explicitly. When $R_B \equiv a$ for some $a \geq 0$, the FPA equilibrium is

$$b^a(v, r) = \frac{1}{a} \ln \left(e^{av} - a \int_r^v \left(\frac{G(y)}{G(v)} \right) e^{ay} dy \right) \quad \text{for } v \geq r. \quad (2.2)$$

The explicit solution for the CARA bidders allows us to conveniently visualize the behavior of the bid functions. For instance, Figure 2.1 shows how risk aversion causes the bidders to submit higher bids in comparison with the risk neutral bids.

2.3 Technical Preliminaries

It will be repeatedly useful to note that the function γ is related to the risk aversion measure R_B by $\gamma' = 1 + R_B\gamma$. For $t \geq 0$ we have $\gamma(t) \geq 0$,

⁷It is the only equilibrium if $r > L$ and the buyers have nonincreasing absolute risk aversion (Maskin and Riley, 2003).

⁸We obtain $b_1(L, L) = \frac{n-1}{n}$ from (2.1) and L'Hospital's rule:

$$\begin{aligned} b_1(L, L) &= \lim_{v \downarrow L} \frac{(n-1)f(v) [u_B(v - b(v, L)) - u_B(0)]}{F(v)u'_B(v - b(v, L))} \\ &= \lim_{v \downarrow L} (n-1)(1 - b_1(v, L)) = (n-1)(1 - b_1(L, L)). \end{aligned}$$

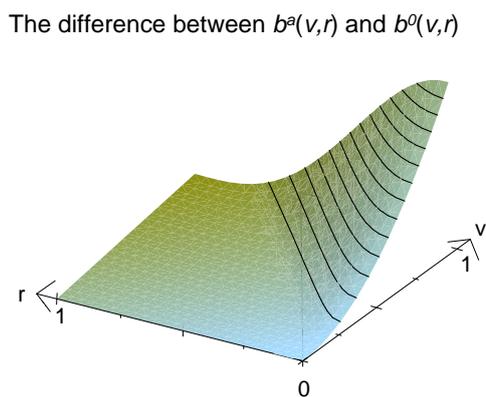


FIGURE 2.1. The FPA bid function increases in the bidders' risk aversion. The figure depicts the difference $b^a(v,r) - b^0(v,r)$ (where $a = 2$ and b^0 is the risk neutral bid). This difference is strictly positive for $a > 0$, and it increases in v and decreases in r .

and so $\gamma'(t) > 0$. If \hat{u}_B is another utility function such that $\hat{R}_B > R_B$, then $\hat{\gamma}(t) > \gamma(t)$ for $t > 0$ (Pratt, 1964, Theorem 1).

We will also use the following two lemmas (their proofs are in the Appendix). The first lemma is a variation of the ‘‘Ranking Lemma’’ of Milgrom (2004).

Lemma 1 *For $c < d \leq \infty$ and $h : [c, d] \rightarrow \mathbb{R}$ differentiable, if $h(c) \geq 0$ then*

- (i) $h > 0$ on $(c, d]$ if $[\forall t \geq c, h(t) = 0 \Rightarrow h'(t) > 0]$,
- (ii) $h > 0$ on $(c, d]$ if $[\forall t > c, h(t) \leq 0 \Rightarrow h'(t) > 0]$.

Lemma 2 For $c < d \leq \infty$ and $i = 1, 2$, let the functions $h_i : [c, d] \rightarrow \mathbb{R}$ be differentiable and satisfy $h'_1 < h'_2$ on (c, d) . Let t_i maximize h_i on $[c, d]$. If $t_i \in (c, d)$ for $i = 1$ or $i = 2$, then $t_1 < t_2$.

2.4 Properties of the FPA Equilibrium

The FPA equilibrium is well known to satisfy $b(v, r) < v$ and $b_1(v, r) > 0$ for any $L \leq r < v$.⁹ Our first proposition provides an expression for b_2 that shows how the equilibrium varies with the reserve price.

Proposition 1 For $r \in (L, H)$ and $v \in [r, H]$,

$$b_2(v, r) = \frac{G(r)}{G(v)} \exp\left(-\int_r^v b_1(y, r) R_B (y - b(y, r)) dy\right), \quad (2.3)$$

and hence

$$0 \underset{(i)}{<} b_2(v, r) \underset{(ii)}{\leq} \frac{G(r)}{G(v)}.$$

Inequality (ii) is an equality if $R_B = 0$; it is a strict inequality if $v > r$ and $R_B > 0$.¹⁰ Lastly, $b_2(v, L) = 0$ for all $v \in (L, H]$.¹¹

Proof. Because the right-hand side of (2.1) is continuously differentiable in b , v , and r (which does not appear explicitly), the solution $b(v, r)$ is continuously differentiable in v and r for $r \in (L, H)$ and $v \in [r, H]$ (e.g., Hale, 2009; Chapter 1, Theorem 3.3). This implies, in turn, that b_{12} exists: by differentiating (2.1) with respect to r and using $\gamma' = 1 + R_B \gamma$, we obtain

$$b_{12}(v, r) = -\ell(v) (1 + \gamma R_B) b_2. \quad (2.4)$$

⁹Fix $r \in [L, H)$, and consider $h(v) = v - b(v, r)$ on $[r, H]$. Note that $h(r) = 0$. Suppose $h(v) \leq 0$ for some $v > r$. Then $h'(v) = 1 - \ell(v)\gamma(h(v)) > 0$. Hence, by Lemma 3 (ii), $v - b(v, r) = h(v) > 0$ for $v \in (r, H]$. This and (2.1) imply $b_1(v, r) > 0$ for $v \in (r, H]$.

¹⁰Here and below, $R_B > 0$ is a functional inequality, meaning that $R_B(y) > 0$ for all y in the relevant interval, which is $[0, H - L]$.

¹¹The derivative $b_2(L, L)$ cannot be defined because $b(L, r)$ is undefined for $r > L$.

Fix $r \in (L, H)$. Since $b_2(r, r) \equiv 1$, the continuity of $b_2(\cdot, r)$ on $[r, H]$ implies the existence of $\bar{v} \in (r, H]$ such that

$$\bar{v} = \max\{v \in [r, H] : b_2(y, r) > 0 \text{ for all } r \leq y < v\}.$$

Note that $b_2(\bar{v}, r) = 0$ if $\bar{v} < H$. Now, $\ln b_2(y, r)$ is well defined for $y \in [r, \bar{v})$, and from (2.4) we have

$$\frac{\partial}{\partial y} \ln b_2(y, r) = -\ell(y)(1 + \gamma R_B).$$

Integrating this on $[r, v]$ for $v \in (r, \bar{v})$ yields, since $\ell(y) = d \ln G$ and $\ln b_2(r, r) = 0$,

$$\begin{aligned} \ln b_2(v, r) &= - \int_r^v d \ln G - \int_r^v \ell(y) \gamma R_B dy \\ &= \ln \frac{G(r)}{G(v)} - \int_r^v b_1 R_B dy, \end{aligned}$$

using (2.1) in the last step. Hence, (2.3) holds at any $v \in (r, \bar{v})$. It also holds at $v = \bar{v}$, by the continuity of $b_2(\cdot, r)$. As this implies $b_2(\bar{v}, r) > 0$, we have $\bar{v} = H$. This proves that (2.3) holds for any $r \in (L, H)$ and $v \in [r, H]$. Inequality (i) is now immediate from $r > L$. Inequality (ii) also follows from (2.3), since $\gamma R_B = 0$ if $R_B = 0$, and $\gamma R_B > 0$ for $v > r$. Lastly, fix $v \in (L, H]$. The right side of (2.3) then converges to 0 as $r \downarrow L$. Hence, $b_2(v, L) = \lim_{r \downarrow L} b_2(v, r) = 0$. ■

Our second proposition establishes the intuitive property that a bidder's profit conditional on winning, $v - b(v, r)$, increases in v , provided that the reverse hazard rate is decreasing.

Proposition 2 *If (DRH) holds, then $b_1(v, r) < 1$ for $L \leq r < v$.*

Proof. We apply Lemma 1(i) to $1 - b_1(\cdot, r)$. Recall $1 - b_1(r, r) > 0$. Suppose $1 - b_1 = 0$ at some $v \geq r$. Then $v > r$. Differentiating (2.1) with respect to v , and evaluating the result at this (v, r) , yields

$$b_{11} = \ell' \gamma + \ell \gamma' \times (1 - b_1) = \ell' \gamma.$$

We have $\gamma(v - b) > 0$ because $b < v$, and $\ell'(v) < 0$ by (DRH). Hence, at this (v, r) , $\partial[1 - b_1]/\partial v = -b_{11} > 0$. Lemma 1(i) now implies $1 - b_1 > 0$ for $v \geq r$. ■

Our third proposition determines the effects of the bidders becoming more risk averse. Part (i) shows that the bid function increases in their risk aversion, generalizing the well-known result that bids are higher when the bidders are risk averse than when they are risk neutral. The remainder of the proposition establishes more surprising results, assuming that (DRH) holds, and the seller and/or the buyers exhibit nonincreasing absolute risk aversion. Parts (ii) and (iii), respectively, show that then, the more risk averse are the bidders, the more rapidly the bid function increases in a bidder's value, but the more slowly it increases in the reserve price. The latter property is largely why the seller's optimal reserve price decreases in the risk aversion of the bidders, as we shall see.

Proposition 3 *Let \hat{u}_B be another utility function satisfying the same assumptions as u_B , with an absolute risk aversion measure satisfying $\hat{R}_B > R_B$ on $[0, H - L]$. Let \hat{b} be the FPA equilibrium when the buyers have utility \hat{u} . Then*

$$(i) \quad \hat{b}(v, r) > b(v, r) \text{ for } v > r.$$

If (DRH) holds, and R_B and/or \hat{R}_B is nonincreasing, then

$$(ii) \quad \hat{b}_1(v, r) > b_1(v, r) \text{ for } v > r, \text{ and}$$

$$(iii) \quad \hat{b}_2(v, r) < b_2(v, r) \text{ for } v > r.$$

Proof. (i) We apply Lemma 1(ii) to $\hat{b}(\cdot, r) - b(\cdot, r)$. We have $\hat{b}(r, r) = b(r, r)$. Suppose $\hat{b} \leq b$ for some $v > r$. Then $\hat{\gamma}(v - \hat{b}) \geq \hat{\gamma}(v - b)$, since $\hat{\gamma}$ is increasing on \mathbb{R}_+ . Since $\hat{R}_B > R_B$ on $[0, H - L]$, we have $\hat{\gamma}(v - b) > \gamma(v - b)$. Hence, $\hat{\gamma}(v - \hat{b}) > \gamma(v - b)$. This and (2.1) yields

$$\hat{b}_1 - b_1 = \left[\hat{\gamma}(v - \hat{b}) - \gamma(v - b) \right] \ell(v) > 0.$$

Lemma 1(ii) now implies $\hat{b} > b$ for all $v > r$.

(ii) We apply Lemma 1(i) to $\hat{b}_1(\cdot, r) - b_1(\cdot, r)$ on intervals of the form $[\xi_k, H]$, where $\xi_k \downarrow r$ as $k \rightarrow \infty$. We will show that $\hat{b}_1(\cdot, r) > b_1(\cdot, r)$ on each interval, and hence on $(r, H]$. To obtain ξ_k , let $\{v_k\}$ be a sequence such that $v_k \downarrow r$. Since $\hat{b}(r, r) = b(r, r)$ and $\hat{b}(v_k, r) > b(v_k, r)$, the mean value theorem implies $\xi_k \in (r, v_k)$ exists such that $\hat{b}_1(\xi_k, r) > b_1(\xi_k, r)$. Note that $\xi_k \downarrow r$. Now, suppose $\hat{b}_1(v, r) = b_1(v, r)$ for some $v \geq \xi_k$. Since R_B or \hat{R}_B is nonincreasing and $\hat{b} > b$ at (v, r) , we have

$$\hat{R}_B(v - \hat{b}) > R_B(v - b). \quad (2.5)$$

Because $\hat{b}_1 = b_1$ at (v, r) , from (2.1) we obtain $\hat{\gamma}(v - \hat{b}) = \gamma(v - b)$. Hence, using (2.1) to differentiate \hat{b}_1 and b_1 yields

$$\begin{aligned} \hat{b}_{11} - b_{11} &= \left[\ell' \hat{\gamma} + \ell \left(1 + \hat{R}_B \hat{\gamma} \right) (1 - \hat{b}_1) \right] \\ &\quad - \left[\ell' \gamma + \ell (1 + R_B \gamma) (1 - b_1) \right] \\ &= \left[\hat{R}_B(v - \hat{b}) - R_B(v - b) \right] \hat{b}_1 (1 - \hat{b}_1) > 0, \end{aligned}$$

where the inequality follows from (2.5), $\hat{b}_1 > 0$, and $\hat{b}_1 < 1$ (by Proposition 2, since we have (DRH) here). Lemma 1(i) now implies $\hat{b}_1(\cdot, r) > b_1(\cdot, r)$ on each $(\xi_k, H]$.

(iii) We apply Lemma 1(ii) to $b_2(\cdot, r) - \hat{b}_2(\cdot, r)$. We have $b_2(r, r) = \hat{b}_2(r, r)$. Suppose $b_2 \leq \hat{b}_2$ for some $v > r$. As (2.1) holds for both b_1 and \hat{b}_1 , differentiating $b_1 - \hat{b}_1$ with respect to r yields

$$\begin{aligned} b_{12} - \hat{b}_{12} &= -(1 + R_B \gamma) \ell b_2 + (1 + \hat{R}_B \hat{\gamma}) \ell \hat{b}_2 \\ &= -(\ell + R_B b_1) b_2 + (\ell + \hat{R}_B \hat{b}_1) \hat{b}_2 \\ &= (\hat{b}_2 - b_2) \ell + \hat{R}_B \hat{b}_1 \hat{b}_2 - R_B b_1 b_2. \end{aligned}$$

Thus, because $\ell > 0$, the hypothesis $b_2 \leq \hat{b}_2$ implies

$$b_{12} - \hat{b}_{12} \geq \left(\hat{R}_B \hat{b}_1 - R_B b_1 \right) \hat{b}_2. \quad (2.6)$$

Since (DRH) holds and $\hat{R}_B > R_B$, Proposition 3(ii) implies $\hat{b}_1 > b_1$. Thus, since $\hat{b}_2 > 0$ by Proposition 1, from (2.6) we obtain $b_{12} - \hat{b}_{12} > 0$. Lemma 1(ii) now implies $b_2 > \hat{b}_2$, for $v > r$. ■

2.5 Seller Preferences over Auctions with the Same Reserve Price

Let $V_I(r)$ and $V_{II}(r)$ denote the seller's equilibrium expected utility in the FPA and SPA auctions, respectively, as a function of the reserve price. The revenue equivalence theorem establishes $V_I(r) = V_{II}(r)$ if all participants are risk neutral.

As shown by Maskin and Riley (1984), risk aversion on the part of the seller and/or the buyers causes the seller to prefer the FPA to the SPA if both have the same reserve price.¹² This is due to two effects. The first is a direct "revenue effect": buyer risk aversion causes them to bid more in the FPA. The second is a "risk effect": the high bid in a FPA is a less risky random variable than it is in a SPA, and so preferred by a risk averse seller.

For future reference we record this result as part (i) of the following proposition. Part (ii) records the result that in a FPA, the seller prefers the buyers to be more risk averse, a consequence of the fact that they then bid more.

Proposition 4 (i) *If R_B and/or R_S is positive, then $V_I(r) > V_{II}(r)$ for $r < H$.*

(ii) *If \hat{u}_B satisfies the same assumptions as u_B , with $\hat{R}_B > R_B$, and $\hat{V}_I(r)$ is the corresponding FPA equilibrium seller payoff, then $\hat{V}_I(r) > V_I(r)$ for $r < H$.*

Proof. Part (i) follows from Theorem 5 in Maskin and Riley (1984). To prove (ii), fix $r < H$ and $(v_1, \dots, v_n) \in [L, H]^n$. Let $v^m = \max_i v_i$. In either case, u_B or \hat{u}_B , a sale occurs if and only if $v^m \geq r$. The price is then $b(v^m, r)$ or $\hat{b}(v^m, r)$, since b_1 and \hat{b}_1 are positive. By Proposition 3(i), $\hat{b}(v, r) > b(v, r)$ for $v > r$. Thus, for almost all value vectors resulting in

¹²Related results are in Vickrey (1961), Matthews (1980), Riley and Samuelson (1981), Waehrer et al. (1998), and Milgrom (2004, Theorem 4.10).

a sale, the sale price is higher given \hat{u}_B than u_B . Since a sale occurs with positive probability because $r < H$, we have $\widehat{V}_I(r) > V_I(r)$. ■

The seller's preferences over auctions with the same fixed reserve price extend immediately to the setting in which the seller sets reserve prices. For example, if R_B and/or R_S is positive, and r_I (r_{II}) is an optimal reserve price for the seller in the FPA (SPA), then Proposition 4(i) immediately implies $V_I(r_I) > V_{II}(r_{II})$.

2.6 Optimal Reserve Prices

We now derive expressions for $V_i(r)$ and $V'_i(r)$ in order to study the seller's optimal reserve prices. The rules of the auctions and the nature of their equilibria imply

$$V_I(r) = n \int_r^H u_S(b(v, r))G(v)dF(v) + F(r)^n u_S(v_0) \quad (2.7)$$

and

$$\begin{aligned} V_{II}(r) &= nG(r)(1 - F(r))u_S(r) \\ &+ n \int_r^H u_S(y)(1 - F(y))dG(y) + F(r)^n u_S(v_0). \end{aligned} \quad (2.8)$$

Differentiating (2.7) yields

$$\begin{aligned} V'_I(r) &= n \int_r^H u'_S(b(v, r))b_2(v, r)G(v)dF(v) \\ &\quad - nG(r)f(r) [u_S(r) - u_S(v_0)]. \end{aligned} \quad (2.9)$$

The first term in (2.9) is the seller's marginal benefit from raising the reserve price in the FPA, due to the resulting increase in the bid function on $[r, H]$. The second term is the marginal cost, due to the lost sales at price $b(r, r) = r$ caused by a marginal increase in the reserve price.

Differentiating (2.8) yields

$$\begin{aligned} V'_{II}(r) &= nG(r)(1 - F(r))u'_S(r) \\ &\quad - nG(r)f(r)[u_S(r) - u_S(v_0)]. \end{aligned} \quad (2.10)$$

Again, the first and second terms are the seller's marginal benefit and marginal cost of raising the reserve price. Comparing (2.9) to (2.10) shows that the marginal cost is the same in the SPA as in the FPA. The marginal benefit of raising the reserve price in the SPA differs, as it is due to the resulting increase in the price received in the event that precisely one bidder has a value greater than r .

The next proposition establishes that in both auctions, optimal reserve prices exist, and they are all strictly between v_0 and H . Furthermore, the optimal reserve price in the SPA is unique and invariant to the number of bidders under the regularity assumption that a bidder's virtual valuation increases in his value (Myerson, 1981).

For $i = I, II$, the proposition refers to the set of reserve prices in $[L, H]$ that maximize V_i , denoted as \mathcal{R}_i . It also refers to a function defined by

$$\Phi(r) \equiv \frac{u_S(v_0) - u_S(r)}{u'_S(r)} + \frac{1 - F(r)}{f(r)}, \quad (2.11)$$

which is of relevance because (2.10) implies

$$V'_{II}(r) = nG(r)f(r)u'_S(r)\Phi(r). \quad (2.12)$$

Proposition 5 *Both \mathcal{R}_I and \mathcal{R}_{II} are nonempty subsets of (v_0, H) . Any $r_{II} \in \mathcal{R}_{II}$ satisfies $\Phi(r_{II}) = 0$, and \mathcal{R}_{II} is a singleton and independent of the number of bidders if $v - \frac{1-F(v)}{f(v)}$ is strictly increasing on (L, H) .*

Proof. Because $[L, H]$ is compact and V_i is continuous, $\mathcal{R}_i \neq \emptyset$. For any $r \in (L, v_0)$, the second term in (2.9) is nonpositive, and the first term is positive by Proposition 1. Hence, $V'_I > 0$ on (L, v_0) . Expression (2.10) directly implies $V'_{II} > 0$ on (L, v_0) . We thus have $\mathcal{R}_i \subseteq [v_0, H]$. From (2.9) and (2.10) we see that

$$V'_i(r) \rightarrow nf(H)[u_S(H) - u_S(v_0)] < 0$$

as $r \uparrow H$. Hence, $\mathcal{R}_i \subseteq [v_0, H)$.

Assume for now that $\mathcal{R}_{II} \subseteq (v_0, H)$. Then, any $r_{II} \in \mathcal{R}_{II}$ satisfies $V'_{II}(r_{II}) = 0$ and $r_{II} > L$, and hence $\Phi(r_{II}) = 0$. Differentiating (2.11) yields

$$\Phi'(r) = \left(\frac{u_S(v_0) - u_S(r)}{u'_S(r)} \right) R_S(r) - \left(r - \frac{1 - F(r)}{f(r)} \right)'$$

The first term is nonpositive for $r \geq v_0$. Hence, if $v - \frac{1-F(v)}{f(v)}$ is strictly increasing, then $\Phi' < 0$ on $[v_0, H]$. This interval then contains a unique r_{II} satisfying $\Phi(r_{II}) = 0$, and so $\mathcal{R}_{II} = \{r_{II}\}$. Since Φ does not depend on n , neither does r_{II} .

It remains only to show $v_0 \notin \mathcal{R}_i$, and so $\mathcal{R}_i \subseteq (v_0, H)$. From (2.11) we have $\Phi(v_0) > 0$, since $v_0 \in [L, H)$ and $f(v_0) < \infty$. Hence, (2.12) implies that $V'_{II}(v_0) \geq 0$, and that $\bar{r} > v_0$ exists such that $V'_{II}(r) > 0$ for $r \in (v_0, \bar{r})$. This proves $v_0 \notin \mathcal{R}_{II}$.

To prove $v_0 \notin \mathcal{R}_I$, note first that for $v_0 > L$, we have $V'_I(v_0) > 0$ from (2.9) and Proposition 1, and hence $v_0 \notin \mathcal{R}_I$. So assume $v_0 = L$. Then, since $b_2(\cdot, L) = 0$ on (L, H) (Proposition 1), $V'_I(v_0) = V'_I(L) = 0$. Define

$$m \equiv \exp \left(- \int_L^H b_1(y, L) R_B(y - b(y, L)) dy \right).$$

The function $b_1(\cdot, L)$ is bounded on $[L, H]$, as it is continuous on $(L, H]$ and $b_1(v, L) \rightarrow \frac{n-1}{n}$ as $v \downarrow L$ (footnote. 8). The integral in the definition of m is thus finite, and so $m > 0$. Note now that from Proposition 1, for any $v \in (L, H]$ we have

$$\lim_{r \rightarrow L} b_2(v, r) \frac{G(v)}{G(r)} = \exp \left(- \int_L^v b_1(y, L) R_B(y - b(y, L)) dy \right) \geq m.$$

Consequently, since $f(L) < \infty$, there exists $\bar{r} \in (L, H)$ such that for $r \in (L, \bar{r})$,

$$\begin{aligned} V_I'(r) &= nG(r) \int_r^H u_S'(b(v, r)) \left[b_2(v, r) \frac{G(v)}{G(r)} \right] dF(v) \\ &\quad - nG(r)f(r) [u_S(r) - u_S(L)] \\ &\geq nG(r) \left(\int_r^H u_S'(b(v, r)) \left[\frac{1}{2}m \right] dF(v) - f(r) [u_S(r) - u_S(L)] \right) \\ &> 0. \end{aligned}$$

This proves $v_0 \notin \mathcal{R}_I$. ■

We now show that the seller sets a lower reserve price in the FPA than in the SPA if he and/or the bidders are risk averse. The proof is based on the observation that because the seller's marginal cost of raising the reserve price is the same in both auctions, the difference in his incentives is the difference in the marginal benefits: (2.9) and (2.10) yield

$$\begin{aligned} V_I'(r) - V_{II}'(r) &= \underbrace{n \int_r^H u_S'(b(v, r)) b_2(v, r) G(v) dF(v)}_{MB_I} \\ &\quad - \underbrace{nG(r) (1 - F(r)) u_S'(r)}_{MB_{II}}. \end{aligned} \quad (2.13)$$

It is easy to see that this difference is negative if the bidders and/or the seller is risk averse. By the revenue equivalence theorem, $V_I'(r) = V_{II}'(r)$ if they are all risk neutral, and so then $MB_I = MB_{II}$. As the seller becomes risk averse, the ratio $u_S'(b(v, r))/u_S'(r)$ falls because $b(v, r) > r$, and hence MB_I falls relative to MB_{II} . As the bidders become risk averse, b_2 falls by Proposition 1, which lowers MB_I and leaves MB_{II} unchanged. The proof of our first theorem makes this logic precise.

Theorem 1 *Suppose R_B and/or R_S is positive. Then, for any $r_I \in \mathcal{R}_I$ and $r_{II} \in \mathcal{R}_{II}$, we have $r_I < r_{II}$.*

Proof. Write (2.13) as

$$\begin{aligned} & V'_I(r) - V'_{II}(r) \\ &= nG(r)u'_S(r) \int_r^H \left[\left(\frac{u'_S(b(v,r))}{u'_S(r)} \right) \left(\frac{G(v)}{G(r)} b_2(v,r) \right) - 1 \right] dF(v). \end{aligned}$$

Since $r > v_0 \geq L$, this expression is positive if and only if the integral is positive. Fix $v > r$. Since $b(v,r) > r$ and u_S is concave we have $u'_S(b(v,r))/u'_S(r) \leq 1$, and this inequality is strict if $R_S > 0$. From Proposition 1 we have $G(v)b_2(v,r)/G(r) \leq 1$, and this inequality is strict if $R_B > 0$. Hence, as at least one of R_S and R_B is positive, the integrand in the above expression is negative at all $v \in (r, H]$. This proves $V'_I < V'_{II}$ on (L, H) . Since $r_{II} \in (v_0, H)$ by Proposition 5, Lemma 2 now implies $r_I < r_{II}$. ■

Our second theorem shows that in either auction, a more risk averse seller sets a lower reserve price. The intuition is that the more risk averse the seller is, the more he wishes to avoid the risk of not selling the object for a profitable price.

Theorem 2 *Let \hat{u}_S satisfy the same assumptions as u_S , with $\hat{R}_S > R_S$. Let $\hat{\mathcal{R}}_i$ and \mathcal{R}_i be the sets of optimal reserve prices given \hat{u}_S and u_S , for $i = I, II$. Then, for any $\hat{r}_i \in \hat{\mathcal{R}}_i$ and $r_i \in \mathcal{R}_i$, we have $\hat{r}_I < r_I$ and $\hat{r}_{II} < r_{II}$.*

Proof. We first prove $\hat{r}_{II} < r_{II}$. Let $\hat{\Phi}(r)$ be defined by replacing u_S by \hat{u}_S in (2.11). By Pratt (1964, Theorem 1), $\hat{R}_S > R_S$ implies that for $r > v_0$,

$$\frac{\hat{u}_S(v_0) - \hat{u}_S(r)}{\hat{u}'_S(r)} < \frac{u_S(v_0) - u_S(r)}{u'_S(r)}, \quad (2.14)$$

and hence $\hat{\Phi}(r) < \Phi(r)$. Since $\Phi(r)$ is invariant to any linear transformation of u_S , and since $V'_{II}(r) = nG(r)f(r)u'_S(r)\Phi(r)$, a linear transformation of u_S will only lead to a positive ratio transformation of $V'_{II}(r)$ and hence will not affect the optimal reserve prices. Now let $\hat{r}_{II} = \max \hat{\mathcal{R}}_{II}$. By

Proposition 5, $\widehat{\mathcal{R}}_{II}$ is a nonempty subset of (v_0, H) and therefore $\widehat{\Phi}(\widehat{r}_{II}) = 0 < \Phi(\widehat{r}_{II})$. W.l.o.g., we now normalize u_S such that $u_S(v_0) = \widehat{u}_S(v_0)$ and $u_S(\widehat{r}_{II}) = \widehat{u}_S(\widehat{r}_{II})$. Since $\widehat{R}_S > R_S$, by Pratt (1964, Theorem 1) $u_S(r) < \widehat{u}_S(r)$ for all $r \in (v_0, \widehat{r}_{II})$. Consequently, for all $r \in (v_0, \widehat{r}_{II})$,

$$\begin{aligned} & V_{II}(r) - \widehat{V}_{II}(r) \\ &= n \int_r^H \left[(u_S(r) - \widehat{u}_S(r)) G(r) + \int_r^v (u_S(y) - \widehat{u}_S(y)) dG(y) \right] dF(v) \\ &< n \int_{\widehat{r}_{II}}^H \int_{\widehat{r}_{II}}^v (u_S(y) - \widehat{u}_S(y)) dG(y) dF(v) \\ &= V_{II}(\widehat{r}_{II}) - \widehat{V}_{II}(\widehat{r}_{II}), \end{aligned}$$

which implies $V_{II}(\widehat{r}_{II}) - V_{II}(r) > \widehat{V}_{II}(\widehat{r}_{II}) - \widehat{V}_{II}(r) \geq 0$ since \widehat{r}_{II} maximizes \widehat{V}_{II} . This establishes $\widehat{r}_{II} \leq r_{II}$. The strict inequality now follows from $V'_{II}(\widehat{r}_{II}) > 0$ since $\Phi(\widehat{r}_{II}) > 0$.

We now use a similar approach to prove $\widehat{r}_I < r_I$. W.l.o.g., we may assume $\widehat{r}_I = \max \widehat{\mathcal{R}}_I$. Define function Ψ by

$$\Psi(r) \equiv \frac{u_S(v_0) - u_S(r)}{u'_S(r)} + \int_r^H \frac{u'_S(b(v, r))}{u'_S(r)} \frac{b_2(v, r)G(v)}{G(r)f(r)} dF(v), \quad (2.15)$$

which allows (2.9) to be written as $V'_I(r) = nG(r)f(r)u'_S(r)\Psi(r)$. The role of Ψ is analogous to that of Φ in that $\Psi(r) \geq 0$ iff $V'_I(r) \geq 0$, and that a linear transformation of u_S does not affect Ψ and only leads to a positive ratio transformation of $V'_I(r)$. Consequently, without affecting the optimal reserve prices we can now normalize u_S such that $u_S(v_0) = \widehat{u}_S(v_0)$ and $u_S(\widehat{r}_I) = \widehat{u}_S(\widehat{r}_I)$. It follows from $\widehat{R}_S > R_S$ that $u_S(r) < \widehat{u}_S(r)$ for all $r \in (v_0, \widehat{r}_I)$, and that

$$\frac{u'_S(r)}{\widehat{u}'_S(r)} > \frac{u_S(r) - u_S(v_0)}{\widehat{u}_S(r) - u_S(v_0)} > 0 \quad \forall r \in (\widehat{r}_I, H).$$

For all r and v such that $r \leq \widehat{r}_I < v$, since $b_2 \geq 0$ and $b(v, \widehat{r}_I) > \widehat{r}_I$, we then have

$$\max\{u_S(b(v, r)) - \widehat{u}_S(b(v, r)), 0\} \leq u_S(b(v, \widehat{r}_I)) - \widehat{u}_S(b(v, \widehat{r}_I))$$

The above conditions imply that for $r \in (v_0, \hat{r}_I)$,

$$\begin{aligned}
V_I(r) - \hat{V}_I(r) &= n \int_r^H [u_S(b(v, r)) - \hat{u}_S(b(v, r))] G(v) dF(v) \\
&= n \int_r^{\hat{r}_I} [u_S(b(v, r)) - \hat{u}_S(b(v, r))] G(v) dF(v) \\
&\quad + n \int_{\hat{r}_I}^H [u_S(b(v, r)) - \hat{u}_S(b(v, r))] G(v) dF(v) \\
&< n \int_{\hat{r}_I}^H [u_S(b(v, r)) - \hat{u}_S(b(v, r))] G(v) dF(v) \\
&\leq n \int_{\hat{r}_I}^H \max\{u_S(b(v, r)) - \hat{u}_S(b(v, r)), 0\} G(v) dF(v) \\
&\leq n \int_{\hat{r}_I}^H [u_S(b(v, \hat{r}_I)) - \hat{u}_S(b(v, \hat{r}_I))] G(v) dF(v) \\
&= V_I(\hat{r}_I) - \hat{V}_I(\hat{r}_I),
\end{aligned}$$

or that $V_I(\hat{r}_I) - V_I(r) > \hat{V}_I(\hat{r}_I) - \hat{V}_I(r) \geq 0$ since \hat{r}_I maximizes \hat{V}_I . This implies $\hat{r}_I \leq r_I$. Now define $\hat{\Psi}(r)$ by replacing u_S by \hat{u}_S in (2.15). Since $b(v, r) > r$ and $b_2 \geq 0$, the second term in (2.15) decreases in the seller's risk aversion. Thus, using (2.14) we obtain $\Psi(r) > \hat{\Psi}(r)$ for all $r \in (v_0, H]$ and, in particular, $\Psi(\hat{r}_I) > \hat{\Psi}(\hat{r}_I) = 0$. Therefore we must have $\hat{r}_I < r_I$. ■

Our third and final theorem establishes that under two fairly general conditions, the seller sets a lower reserve price in the FPA if the bidders are more risk averse. The logic of the result is twofold. First, under the assumed conditions the FPA bid function increases in the reserve price at a slower rate if the bidders are more risk averse. This lowers the incentive of the seller to raise the reserve price. Second, because more risk averse bidders bid more, the increase in their bids in response to an increase in the reserve price generates a lower marginal utility increase for the (weakly) risk averse seller. The proof reflects these two forces.

Theorem 3 *Let \hat{u}_B satisfy the same assumptions as u_B , with $\hat{R}_B > R_B$. Let \mathcal{R}_I ($\hat{\mathcal{R}}_I$) be the set of optimal reserve prices for the seller given u_B*

- (\hat{u}_B). Then, for any $r_I \in \mathcal{R}_I$ and $\hat{r}_I \in \widehat{\mathcal{R}}_I$, we have $\hat{r}_I < r_I$ if either
- (a) (DRH) holds and R_B and/or \widehat{R}_B is nonincreasing, or
 - (b) $\min_{t \in D} \widehat{R}_B(t) > \max_{t \in D} R_B(t)$, where $D = [0, H - L]$.

Proof. Let $\widehat{V}(r)$ be the seller's payoff given \hat{u}_B and reserve r . We show that (a) and (b) each imply $\widehat{V}'_I(r) < V'_I(r)$ for $r > L$. This and Lemma 2 then yield the result, $\hat{r}_I < r_I$, since these reserve prices are in the interval (v_0, H) .

From (2.9) we obtain

$$\begin{aligned} & \widehat{V}'_I(r) - V'_I(r) \\ &= n \int_r^H \left[u'_S(\hat{b}(v, r)) \hat{b}_2(v, r) - u'_S(b(v, r)) b_2(v, r) \right] G(v) dF(v). \end{aligned}$$

The concavity of u_S , together with $\hat{b}(v, r) > b(v, r)$ (Proposition 3(i)), yields $u'_S(\hat{b}) \leq u'_S(b)$ for $v > r$. Hence, to show that (a) and (b) each imply $\widehat{V}'_I(r) < V'_I(r)$ for $r > L$, it suffices to show that they each imply

$$\hat{b}_2(v, r) < b_2(v, r) \text{ for } L < r < v. \quad (2.16)$$

By Proposition 3(iii), (a) implies (2.16). Now assume (b) holds. Then a constant a exists such that $\widehat{R}_B > a > R_B$ on $[0, H - L]$. Fix $L < r < v$. Letting b^a be the CARA equilibrium given by (2.2), by Proposition 3(i) we have $\hat{b} > b^a > b$. Hence, by Proposition 1,

$$\begin{aligned} G(v)b_2(v, r) &= G(r) \exp \left(- \int_r^v b_1(y, r) R_B(y - b(y, r)) dy \right) \\ &> G(r) \exp \left(-a \int_r^v b_1(y, r) dy \right) \quad (\text{since } R_B < a) \\ &= G(r) \exp(-a(b(v, r) - r)) \\ &> G(r) \exp(-a(b^a(v, r) - r)) \quad (\text{since } b < b^a) \\ &= G(v)b_2^a(v, r). \end{aligned}$$

Thus, $b_2^a < b_2$. Similarly, $\widehat{R}_B > a$ yields $\hat{b}_2 < b_2^a$. So (b) indeed implies (2.16). ■

2.7 Concluding Discussion

We have shown that when the seller sets the reserve price, he sets it lower the more risk averse he is and, in a first-price auction, the more risk averse the buyers are. The seller's optimal reserve price is lower in the first-price auction than in the second-price auction, unless all parties are risk neutral. Risk aversion thus reduces the probability of not selling the object when a buyer's value for it exceeds that of the seller, especially in the first-price auction.

The buyers may agree, *ex ante*, with the seller's preference for the first-price auction. Indeed, if they exhibit constant (or increasing) absolute risk aversion, every type of buyer weakly (strictly) prefers at the interim stage the first-price to a second-price auction that has the same reserve price (Matthews, 1987). *Ipso facto*, in these cases the buyers prefer the first-price auction if it has the lower reserve price, as it does when the seller sets the reserve price and he or the buyers are risk averse. By continuity, the buyers must also prefer the first-price auction if their absolute risk aversion measure is approximately constant, so long as they and/or the seller are risk averse.¹³ More generally, buyers with values in the interval $(r_I, r_{II}]$ strictly prefer the FPA, and hence so must the buyers with values in some interval (r_I, \hat{v}) , where $\hat{v} > r_{II}$.

We have focused tightly in this chapter on the effects of risk aversion on optimal reserve prices in two standard auctions, holding fixed their other features. Endogenizing these other features and determining the effects of risk aversion on their levels is a topic for future research. For example, if the seller is able to charge bidders an entry fee, he may wish to do so if the bidders are risk averse (Maskin and Riley, 1983), but not if he is risk averse and can also set the reserve price (Waehrer et al., 1998). The nature of optimal combinations of entry fees and reserves when the seller

¹³Formally, if $|R_B(y) - a| < \varepsilon$ for all y , and if $a > 0$ and/or $R_S > 0$, then $\bar{\varepsilon} > 0$ exists such that when $\varepsilon < \bar{\varepsilon}$, every type of buyer interim prefers the FPA to the SPA when the seller sets the reserve prices.

or buyers are risk averse is unknown. Another example is entry: if each of a large number of potential bidders must pay a cost to learn his value, the number of bidders becomes endogenous. In this case the seller may want to lower the reserve price in order to increase entry.¹⁴ Our results suggest that risk aversion on the part of the seller or buyers should strengthen this effect, especially in the first-price auction.¹⁵

Future work may also generalize the setting of our results. It may be fruitful, for example, to consider asymmetric bidders with different value distributions, which give rise to a different ex post inefficiency (sale to the wrong bidder) than the one (no sale) that we have considered. Settings with ex post risk or interdependent values are naturally of interest as well.

2.8 Appendix

Proof of Lemma 1. (i) Assume $h(t) \leq 0$ for some $t \in (c, d]$. The hypothesis and the continuity of h imply the existence of $\hat{t} \in [c, t)$ such that $h(\hat{t}) < 0$. Let $\bar{s} = \sup\{s \in [c, \hat{t}) : h(s) \geq 0\}$. As h is continuous, $\bar{s} < \hat{t}$ and $h(\bar{s}) = 0$. The hypothesis now implies the existence of $s \in (\bar{s}, \hat{t})$ such that $h(s) > 0$. This contradicts the definition of \bar{s} .

(ii) Assume $h(t) \leq 0$ for some $t \in (c, d]$. Let m be the largest minimizer of h on $[c, t]$. Since $h(c) \geq h(t)$, $m > c$. We thus have $h'(m) \leq 0$, as well as $h(m) \leq h(t) \leq 0$. This contradicts the hypothesis. ■

Proof of Lemma 2. Let $i \in \{1, 2\}$ be such that $t_i \in (c, d)$, and let $j \neq i$ be the other index. Then $h'_j(t_i) \neq h'_i(t_i) = 0$. This proves $h_j(t_j) > h_j(t_i)$, and hence $t_1 \neq t_2$. Defining $h = h_2 - h_1$, we now have $h(t_2) > h(t_1)$. By

¹⁴The effects of endogenous entry on optimal reserve prices are studied, in risk neutral settings, by McAfee and McMillan (1987), Engelbrecht-Wiggans (1993), and Levin and Smith (1994).

¹⁵Endogenous entry can reverse the seller's preference for the FPA, since the SPA may induce more entry if the buyers have DARA risk preferences, as is shown in Smith and Levin (1996). This reversal should occur less often, however, when the seller sets the reserve price, since he sets it lower in the FPA.

the mean value theorem, there exists t strictly between t_1 and t_2 such that

$$(t_2 - t_1)h'(t) = h(t_2) - h(t_1) > 0.$$

This proves $t_1 < t_2$, since $h'(t) > 0$. ■

3

Deterring Collusion using Premium Auctions

This chapter is based on Audrey Hu, Theo Offerman and Sander Onderstal. "Fighting collusion in auctions: An experimental investigation." Forthcoming in International Journal of Industrial Economics.

3.1 Introduction

Fighting collusion is a primary concern for auctioneers because bidders who manage to form a cartel can seriously harm the seller’s revenue. Klemperer (2002) argues that collusion and other competition policy related issues like predation and entry deterrence are more relevant for practical auction design than risk-aversion, affiliation, and budget-constraints that play a prominent role in mainstream auction theory. Case law shows that collusion in auctions is not just a theoretical possibility: Krishna (2004) reports that in the 1980s, 75% of the US cartel cases were related to auctions.¹ Apparently, competition law enforcement does not sufficiently deter bidders to collude. In fact, Motta (2004) argues that “[i]t is better to try to create an environment that discourages collusion in the first place than trying to prove unlawful behavior afterwards.”

The literature provides several ways for auctioneers to implement auction rules that discourage bidders to collude. It is well known that the auctioneer may impose a reserve price to do so (Graham and Marshall, 1987). Recent papers show that collusion-proof mechanisms exist under fairly general circumstances. These mechanisms raise as much revenue as a revenue-maximizing mechanism in the absence of collusion (Laffont and Martimort, 1997, 2000, Jeon and Menicucci, 2005, and Che and Kim, 2006, 2008).

These theoretical solutions have several practical limitations. The optimal reserve price and the collusion-proof mechanism require the auctioneer to know the distribution functions from which bidders draw their values. In addition, the auctioneer needs to know which bidders belong to which cartel. In practice, such information is difficult, if not impossible, to acquire.² For practical mechanism design, Wilson (1987) strongly advocates

¹However, this high percentage does not necessarily suggest that the auctions cartels can be detected easily, as we do not know the number of those (tacit) collusions that have escaped public attention.

²Other limitations of proposed collusion-proof mechanisms are the following. Both Laffont and Martimort (1997, 2000) and Jeon and Menicucci (2005) require a risk-neutral, “benevolent” third-party to coordinate the side-payments for the coalition to function. Che and Kim’s (2006) collusion-proof

the implementation of “detail-free” auctions, i.e., auctions of which the rules do not depend on the above mentioned peculiarities of the environment.

Therefore, we will focus on a more practical solution and search for an existing “detail-free” auction format that prevents collusion as much as possible. Among the existing auctions, the literature suggests using the first-price sealed-bid auction (FP) instead of the English auction (EN) (Robinson, 1985, and Marshall and Marx, 2007). The reason is that a cartel agreement is stable in EN, where no bidder has an incentive to deviate from the cartel agreement because the cartel will continue bidding up to the highest value of its members. In contrast, a cartel in FP has to shade its bid below the highest value in the group to make a profit, which gives individual cartel-members an incentive to cheat on the agreement and submit a higher bid than the cartel.

Still, there have been many FP auctions where bidders colluded, for instance by submitting identical bids (Scherer, 1980; McAfee and McMillan, 1992). Recent examples of collusion in FP include infrastructure procurement (Porter and Zona, 1993, and Boone et al., 2009) and school milk tenders (Porter and Zona, 1999, and Pesendorfer, 2000). Apparently, many cartels have been able to overcome the free-rider incentives in FP, possibly because repeated interaction renders collusion stable in FP (Blume and Heidhues, 2008, Abdulkadiroğlu and Chung, 2003, Aoyagi, 2003, 2007, and Skrzypacz and Hopenhayn, 2004). Motivated by these examples, we focus on the toughest possible case for auctioneers, the one where cartel members do not have to fear that there will be defection from within the cartel and where side-payments are possible between cartel members (a “strong cartel” in McAfee and McMillan’s (1992) terminology, and a “bid submission mechanism” in Marshall and Marx’s (2007)). Our choice to focus on strong cartels is also supported by experimental evidence. Phillips, Menkhaus and Coatney (2003) show that even groups of 6 bidders who

model does not rely on the special structures or bidder-types like the previous work. However, selling to the coalition is often not feasible in practice.

interact repeatedly are able to form stable coalitions when communication is allowed. In their communication treatment, Hamaguchi, Ishikawa, Ishimoto, Kimura and Tanno (2007) find that in procurement auctions subjects do not cheat on the agreement reached in the communication phase.

In this chapter, we compare how effective FP, EN, and a lesser known format based on a premium auction are in deterring collusion. In a premium auction, the auctioneer pays the runner-up a premium for driving up the price paid by the winner. In situations where the auctioneer fears collusion, a premium auction may make collusion less attractive because it encourages bidders outside of the cartel to compete for the premium.³ In Europe, premium auctions are used to sell houses, land, boats, machinery and equipment. There are many variants of premium auctions, that differ in institutional details. In fact, in the Netherlands and Belgium, many of the larger cities have their own variant that they claim to be unique in the world.

Here, we consider a premium auction investigated in Goeree and Offerman (2004), the Amsterdam second-price auction (AMSA). This auction is one of the simpler formats and it has the advantage that its equilibrium is analytically tractable. AMSA consists of two phases. In the first phase, the auctioneer raises the price successively while bidders decide whether or not to drop from the auction. This process continues until two bidders remain. The price at which the last bidder dropped out defines the endogenous reserve price or bottom price for the second phase. In this phase, both remaining bidders independently submit sealed bids, which must be at least as high as the bottom price. The highest bidder wins and pays a price equal to the second highest bid. Both bidders of the second

³The literature identifies other situations where premium auctions may perform well relative to standard auction formats such as FP and EN. Goeree and Offerman (2004) show, both theoretically and in an experiment, that premium auctions may generate more revenue than standard auctions when bidders are asymmetric. Milgrom (2004) argues that the prospect of receiving a premium may attract “weak” bidders to a premium auction who would not have entered in a standard auction where they have no hope to beat the strong bidder.

phase receive a premium, which is a fraction $0 < \alpha < 0.5$ of the difference between the second highest bid and the bottom price.

Notice that there are some similarities between the use of premium auctions and shill bidding. With shill bidding, the seller invents fake bids or asks a confederate to submit fake bids to stir up the bidding. In contrast to the use of a premium in auctions, shill bidding is usually explicitly forbidden. For instance, eBay unambiguously prohibits shill bidding. The rationale provided by eBay is that family members, roommates and employees of the seller have a level of access to information on the good for sale that is not available to other bidders. This is an important difference with a premium auction, where the bidders who pursue the premium are not better informed than the bidders who are genuinely interested in the good. In addition, an important difference is that all the bidders who participate in a premium auction are exactly informed about the rules of the game, while in shill auctions the genuine bidders are not informed of the presence of a shill bidder. For such reasons premium auctions are legally more acceptable than shill bidding, even though they both intend to stir up the bidding.

In most countries collusion is forbidden, and, if it is detected, cartel members receive a fine. In addition, players incur costs when they decide to set up a cartel. Instead of closely modeling such processes, we simply introduce a cost that bidders have to pay when they decide to collude. If the eligible bidders agree to form a cartel, they determine in a pre-auction knockout who will proceed to the auction and how much he has to pay to compensate the other members for not participating.

We examine two settings in which bidders can collude. In the symmetric setting, all bidders can collude, and in the asymmetric setting, only a subset can do so. In the symmetric environment, all bidders draw their values from the same distribution function. In the asymmetric one, we distinguish between “weak” and “strong” bidders. A strong bidder always has a higher value than a weak bidder. This form of asymmetry characterizes many situations in practice, where serious, genuinely interested

bidders compete with fortune-hunters out for a bargain. Maskin and Riley (2000) motivated this setup with a reference to the “Getty effect”, after the wealthy museum known for consistently outbidding the competition. Only strong bidders have the opportunity to collude. The rationale for this choice is that in practice there is basically an infinite supply of bidders with a weak preference for the good, so it is prohibitively costly to try and include all of them in a cartel. On the other hand, there is usually only a limited number of seriously interested bidders, and for them it may be very interesting to prevent competition from each other.

The theoretical properties of this model are the following. In the symmetric case, collusion is equally likely in the three auctions, i.e., it is equally likely that bidders form a cartel. In the asymmetric case, collusion occurs more often in EN than in FP despite the assumption that the cartel, if formed, is also stable in FP. In the stage game where the designated bidder of a cartel faces weak bidders, AMSA turns out to have multiple equilibria, which mainly depend on how aggressively weak bidders bid. If they remain “passive” and bid up to value, AMSA and EN are equally conducive to collusion, and both mechanisms are dominated by FP. However, in an “aggressive equilibrium”, AMSA outperforms both FP and EN in terms of fighting collusion.

Which equilibrium of AMSA is the most likely to be played remains an open question, which we address using a laboratory experiment. Another reason for using a laboratory experiment to empirically test our theoretical findings is that field data on cartels are difficult to obtain by its illegal nature. In the experiment, we compare AMSA with FP and EN. We observe the following results. In the symmetric setting, EN and AMSA are equally successful in fighting collusion. Both mechanisms outperform FP. In the asymmetric setting, AMSA triggers less collusion than the other two auctions, which perform equally poorly. Overall, our experiments suggest that AMSA is the superior choice to fight collusion. To the extent that the experimental results deviate from the theoretical predictions, we provide a coherent explanation for why they differ.

In single-unit auctions, collusion does not arise under standard experimental procedures.⁴ The exception is provided in Lind and Plott (1991), who report attempts at collusion in one of their five common value auction sessions. There is surprisingly little experimental work that allows subjects to explicitly collude in single-unit auctions. The main exception is Isaac and Walker (1985) who gave bidders the opportunity to talk before they submitted their sealed bids in a first-price private value auction. In four out of their six series where a single unit was put up for sale, the four bidders managed to collude. Kagel (1995) discusses two unpublished studies that also study collusion in single-unit auctions. In one study, Dyer investigated tacit collusion in first-price private value auctions by comparing bidding in fixed groups and known identities with bidding in groups that were randomly rematched between auctions. His results were inconclusive. In the other study, Kagel, Van Winkle, Rondelez and Zander let subjects communicate prior to bidding in a first-price common value auction. When the reserve price was announced, subjects used a rotation rule and almost always submitted bids at the reserve price. With a secret reserve price, bidders were less successful in colluding and earned somewhat less than half the amount they made when the reserve price was announced and the amount that they made when there was no communication. More recently, Hamaguchi et al. (2007) study collusion in procurement auctions and the effectiveness of leniency programs. As in Isaac and Walker (1985), bidders could talk before submitting bids. They observe that virtually all bids are at the monopoly price, so that bidders clearly manage to collude. Our experiment goes a step further than the previous literature by examining how successful bidders are in forming cartels under different auction formats, and by studying the role of bidders outside the cartel who may render a cartel unattractive if they bid aggressively.⁵

⁴See Kagel (1995) for an overview of experimental single unit auctions.

⁵In contrast, there is a vast experimental literature on collusion in multi-unit auctions. This literature includes Goswami, Noe, and Rebello (1996), Shertsyuk (1999, 2002), Kwasnica (2000), Kagel and Levin (2001, 2005), Phillips, Menkhaus and Coatney (2003), Sade, Schnitzlein and Zender (2006), Kwasnica and Sherstyuk (2007).

The remainder of this chapter is organized as follows. In Section 3.2, we describe the theoretical background of our experiments. Section 3.3 includes our experimental design. In Section 3.4, we present our experimental findings and section 3.5 concludes. The experimental instructions can be found in the Appendix of this chapter.

3.2 Theory

A seller offers one indivisible object in FP, EN, or AMSA to $n \geq 2$ risk neutral bidders, $s \geq 2$ strong ones and $w \equiv n - s \geq 0$ weak ones. Bidders who are active in the second phase of AMSA obtain a premium equal to a fraction $\alpha \in (0, 1/2)$ of the difference between the second highest bid and the reserve price. Weak bidders draw their value from the uniform distribution on the interval $[0, 1]$, while strong bidders' values are uniformly distributed on $[L, H]$, $H > L \geq 0$.⁶ All values are drawn independently. We let $v^{[2]}$ denote the second order statistic of s draws from the uniform distribution on $[L, H]$.

Bidders interact in a three-stage game. In the first stage, strong bidders vote for or against forming a cartel. A cartel forms if and only if all strong bidders vote "yes". All bidders in the cartel incur a commonly known exogenous cost $c > 0$ if and only if the cartel is actually formed. If a cartel forms, in stage two, the strong bidders interact in a pre-auction knockout mechanism like the one described in McAfee and McMillan (1992). In this knock-out auction, all bidders independently submit a sealed bid. The highest bidder wins, he pays a fraction $1/(s - 1)$ of his bid to each of the other $s - 1$ strong bidders, and proceeds to stage three. In the third stage, in the case of a cartel, the designated strong bidder interacts in the auction (AMSA, EN, or FP) with the weak bidders.⁷ The designated bidder can submit shill bids on behalf of the other cartel-members. This

⁶We shall assume $L \geq 1$ for the asymmetric case later on.

⁷Boone et al. (2009) describe how members of a Dutch construction cartel used a similar mechanism to determine the designated winner and his side-payments to the other cartel members.

realistic feature helps to conceal the fact that the strong bidders collude. When bidders do not form a cartel, stage two is skipped and all bidders compete in the auction in stage three. As solution concept, we use the perfect Bayesian equilibrium.⁸

In the first stage, a strong bidder will vote for collusion if and only if the (expected) benefits of collusion outweigh its costs c . Let us assume that the strong bidder with the highest value always wins, with or without collusion. Let P denote the price the designated strong bidder expects to pay in the actual auction. The following proposition characterizes when strong bidders will vote for collusion in stage one.

Proposition 6 *Suppose that the subgame after the voting stage has an equilibrium in which the auction always allocates the object to the strong bidder with the highest value (in both the collusive and the non-collusive case) and that the strong bidders' lowest type expects zero profit in the non-collusive case. Then, in the equilibrium of the entire game, a strong bidder, regardless of his value, votes in favor of the cartel if and only if*

$$c \leq \frac{1}{s} [E \{v^{[2]}\} - P]. \quad (3.1)$$

Proof. We let $F^{[1]}$ [$F^{[2]}$] denote the distribution function of the first [second] order statistic of s draws from the uniform distribution on the interval $[L, H]$. Myerson (1981) shows that a strong bidder's expected pay-off from an auction can be expressed as

$$\pi(v) = \underline{\pi} + \int_L^v Q(x) dx$$

where $\underline{\pi}$ denotes the expected pay-off for the strong bidder's lowest type and $Q(x)$ the probability that a strong bidder with value x wins. Let $\underline{\Pi}$ be the expected pay-off for the strong bidder's lowest type in the case of collusion. Because the auction always allocates the object to the bidder

⁸When we speak about the equilibrium of an auction, we refer to the Bayesian equilibrium of the last subgame in which $w + 1$ [$w + s$] bidders participate if a cartel is [not] formed.

with the highest value (both in the collusive and the non-collusive case) and the strong bidders' lowest type expects zero profit in the non-collusive case, a strong bidder with value v is willing to join the cartel if and only if $c \leq \underline{\Pi}$. McAfee and McMillan (1992) show that in the knock-out auction, the following bidding function constitutes a symmetric Bayesian Nash equilibrium:

$$B(v) = \frac{s-1}{s} F^{[1]}(v)^{-1} \int_L^v (x-P) dF^{[1]}(x).$$

Given this equilibrium, $\underline{\Pi}$ can be expressed as

$$\begin{aligned} \underline{\Pi} &= \frac{1}{s-1} \int_L^H B(v) dF(v)^{s-1} \\ &= \frac{1}{s} \int_L^H F(v)^{-s} \int_L^v (x-P) dF(x)^s dF(v)^{s-1} \\ &= \frac{1}{s} \int_L^H (x-P) \int_x^H F(v)^{-s} dF(v)^{s-1} dF(x)^s \\ &= \frac{1}{s} \int_L^H (x-P) s(s-1) [1-F(x)] F(x)^{s-2} dF(x) \\ &= \frac{1}{s} \int_L^H (x-P) dF^{[2]}(x) \\ &= \frac{1}{s} (E\{v^{[2]}\} - P), \end{aligned}$$

where F denotes the value distribution function of a strong bidders. The third equality follows by changing the order of integration. The other steps are straightforward. ■

Note the expected benefits of collusion for the strong bidders is the difference between the expected (net) payments without collusion (which is the expectation of the second highest value) and the price the designated winner expects to pay with collusion (which is P). This additional pie will be divided equally among the s strong bidders, which explains the expected benefits on the right-hand side of (3.1). The result that the

willingness-to-pay for forming a cartel does not depend on a bidder's value follows from Myerson's (1981) revenue equivalence theorem. Given that two auctions always assign the object to the bidder with the highest value, the difference in expected utility for a bidder is determined only by the difference in expected utility for the bidder with the lowest value L . The following corollary follows from Proposition 6.

Corollary 1 *If two auctions always allocate the object to the bidder with the highest value in equilibrium (both in the collusive and the non-collusive case) and the lowest type expects zero profit in the non-collusive case, then the auction with the lower P is more conducive to collusion.*

Corollary 1 shows that we only have to compare the expected price the designated winner has to pay in FP, EN, and AMSA to predict which of the three auction formats is less prone to collusion.

We consider a symmetric and an asymmetric setting. In the symmetric one, $w = 0$ (there are no weak bidders), $H = 1$, and $L = 0$. The following proposition immediately follows from Corollary 1, because the three auctions are efficient (with and without collusion), the lowest type expects zero profit in the absence of collusion, and in all three auctions, the designated winner pays zero for the object in the case of a cartel.⁹

Proposition 7 *If $w = 0$, $H = 1$, and $L = 0$, collusion is equally likely in equilibrium in FP, EN, and AMSA.*

In the asymmetric case, $w \geq 1$ (there is at least one weak bidder) and $H > L \geq 1$ (a strong bidder's value is always higher than a weak bidder's). We will establish how the auctions rank in terms of incentives to collude on the basis of the equilibria of the subgame played in stage three. For FP, let $B^{FP}(v)$ [$b^{FP}(v)$] be a strong [weak] bidder's bid if his value is v .

⁹In the unique equilibrium of FP, a bidder with value v bids $B^{FP}(v) = v - v/n$, while EN has an equilibrium in weakly dominant strategies in which each bidder bids value. Goeree and Offerman (2004) establish that AMSA has an equilibrium in which a bidder with value v bids $\frac{v+\alpha}{1+\alpha}$ in both stages.

Using Maskin and Riley's (2003) uniqueness result, it follows that all non-collusive equilibria of FP in non-dominated strategies are characterized by

$$\begin{aligned} B^{FP}(v) &= v - \frac{v - L}{s}; \\ b^{FP}(v) &\in [0, v]. \end{aligned}$$

The following proposition describes collusive equilibria for FP for sufficiently high L .

Proposition 8 *Suppose that strong bidders form a cartel. If $w \geq 1$ and $L \geq \frac{w+1}{w}$, then in any equilibrium of FP in non-dominated strategies, the designated strong bidder bids $B^{FP}(v) = 1$ and always wins the auction.*

Proof. For weak bidders, bids above their value are weakly dominated. Therefore, none of the weak bidders bids more than 1 in equilibrium so neither does the designated strong bidder. Therefore, the proof is established if we show that the designated strong bidder will never bid less than 1 in equilibrium. Now, suppose his lowest equilibrium bid equals $b < 1$. Then a weak bidder with value $v_w \in (b, 1]$ best responds by submitting a bid in the interval (b, v_w) , while those with a value below b bid less than b . Therefore, the designated winner's expected profit given his value v equals $U(b, v) = (v - b)b^w$ if he bids b and $v - 1$ if he bids 1. Note that, for all $v \in [L, H]$, $\frac{\partial U(b, v)}{\partial b} = b^{w-1}(wv - (w + 1)b) > 0$ if $L \geq \frac{w+1}{w}$ so that $(v - b)b^w < v - 1$. A contradiction is established because bidding $b < 1$ is not a best response for the designated winner. An equilibrium in which $B^{FP}(v) = 1$ can be readily constructed by letting all weak bidders bid value. ■

So, if $L \geq \frac{w+1}{w}$, the expected payment by the designated winner equals

$$P^{FP} = 1.$$

For EN it is always a weakly dominant strategy to bid value. So, in the case of collusion, the designated winner expects to pay the highest value

among the weak bidders:

$$P^{EN} = \frac{w}{w+1}.$$

The following proposition establishes an equilibrium for AMSA in the absence of collusion.¹⁰

Proposition 9 *Let $w \geq 1$ and $L \geq 1$. Suppose that the strong bidders do not form a cartel. The following bidding strategies constitute an equilibrium outcome of AMSA in non-dominated strategies. In the first phase, a strong bidder with value v remains in the auction up to $\frac{v+\alpha H}{1+\alpha}$. The weak bidders all drop out at any price between their value and L . In the second phase, a strong bidder with value v bids $\frac{v+\alpha H}{1+\alpha}$.*

Proof. We begin by solving the second phase given the strategies in the first phase. If $s > 2$, let v_3 denote the value of a strong bidder whose bid in phase one equals the bottom price $X = B_1(v_3)$. Otherwise, $v_3 = L$. Moreover, v_2 denotes the other strong bidder's value. The second phase expected payoff of a strong bidder with value v who bids $B_2(\hat{v}) \geq X$ can be expressed as:

$$\pi(\hat{v}|v) = \frac{1}{H - v_3} \times \left(\int_{v_3}^{\hat{v}} (v - B_2(v_2)) dv_2 + \alpha \int_{v_3}^{\hat{v}} (B_2(v_2) - X) dv_2 + \alpha (B_2(\hat{v}) - X)(H - \hat{v}) \right).$$

The first (second) [third] term on the RHS refers to the bidder's value minus his payment if he wins (the premium if he wins) [the premium if he

¹⁰For weak bidders the strategy to bid value weakly dominates bidding below value.

loses]. The FOC is:

$$\left. \frac{\partial \pi(\hat{v}|v)}{\partial \hat{v}} \right|_{\hat{v}=v} = \frac{1}{H - v_3} \times \left(\begin{array}{c} v - B_2^a(v) + \alpha(B_2^a(v) - X) \\ + \alpha B_2^{a'}(v)(H - v) - \alpha(B_2^a(v) - X) \end{array} \right) = 0$$

from which we obtain the optimal bidding strategy for strong bidders. It is readily verified that the SOC $\text{sign}\left(\frac{\partial \pi(\hat{v}|v)}{\partial \hat{v}}\right) = \text{sign}(v - \hat{v})$ is satisfied. Because a strong bidder with value L has not reason to bid more than $\frac{L+\alpha H}{1+\alpha}$, a weak bidder surely has no reason to do so. Therefore, he has no reason to deviate from the above bids. ■

As we discussed before, the designated bidder can submit bids on behalf of the other strong bidders. In FP and EN, these shill bids do not affect the equilibrium outcome. In AMSA, however, the designated bidder may discourage weak bidders from pursuing the premium by keeping at least one of the shill bidders in the auction as long as weak bidders continue bidding. The possibility of shill bids makes it harder for AMSA to outperform the standard auctions. The following proposition shows that the AMSA may have several equilibria, which depend on how “aggressively” weak bidders bid.

Proposition 10 *Let $w \geq 1$ and $L \geq 1$. Suppose that the strong bidders form a cartel. The following bidding strategies constitute an equilibrium outcome of AMSA in non-dominated strategies. In the first phase, the strong bidder and his shill bidder remain in the auction up to his value. The weak bidders all drop out at any price between their value and L . In the second phase, the strong bidder bids value and the shill bidder bids the bottom price.*

The above proposition holds true because a weak bidder has no incentive to bid more than L , the lowest bid submitted by a strong bidder. If he does, he will outbid some types of strong bidders and end up paying more

for the object than his value. It is easy to see that the strong bidder has no reason to deviate either. We say that weak bidders bid aggressively [passively] if they bid up to L [value] in the first phase, which is the highest [lowest] bid which is consistent with Propositions 9 and 10. Let P_{agr}^{AMSA} [P_{pas}^{AMSA}] denote the designated winner's expected payment in the aggressive [passive] equilibrium in the case of a cartel. Then:

$$P_{agr}^{AMSA} = L$$

and

$$P_{pas}^{AMSA} = \frac{w}{w+1}.$$

Table 1 ranks the three auctions in terms of likelihood of collusion for the two equilibrium extremes of AMSA. Corollary 1 and the expected equilibrium payments in FP, EN, and AMSA imply that in equilibrium, collusion is (weakly) less likely in AMSA than in EN. If $L \geq \frac{w+1}{w}$, collusion is more likely in EN than in FP, while the ranking of AMSA relative to FP depends on which of the equilibria in AMSA is played in the case of collusion. If the passive [aggressive] equilibrium is played, collusion is more [less] likely in AMSA than in FP.

Table 1
Ranking of auctions w.r.t. likelihood of collusion: the asymmetric case

Ranking	Auction	Expected (net) payment	
		No collusion	collusion
1	AMSA: passive equilibrium	$E\{v^{[2]}\}$	$w/(w+1)$
1	EN	$E\{v^{[2]}\}$	$w/(w+1)$
2	FP	$E\{v^{[2]}\}$	1
3	AMSA: aggressive equilibrium	$E\{v^{[2]}\}$	L

Notes: This is the ranking for $w \geq 1$ and $L \geq (w+1)/w$. A higher ranking (=lower number) refers to a higher likelihood of collusion in the sense that there is a larger range of cartel costs c for which collusion is profitable in equilibrium.

3.3 Experimental Design and Procedure

The computerized experiment was conducted at the Center for Experimental Economics and political Decision making (CREED) of the University of Amsterdam. A total of 180 students from the undergraduate population of the University were recruited by public announcement and participated in 9 sessions. Subjects earned points in the experiment, that were exchanged in euros at a rate of 5 points for €1. On average subjects made €25.70 with a standard deviation of €7.45 in sessions that lasted between 100 and 140 minutes.¹¹ Subjects read the computerized instructions at their own pace. Before they could proceed to the experiment, they had to correctly answer some questions testing their understanding of the rules. Before the experiment started, subjects received a handout with a summary of the instructions (see Appendix).

We employed a between-subjects design, in which subjects participated in one of three treatments only, FP, EN, or AMSA. The treatments only differed in the auction rules. In FP, subjects simultaneously submitted sealed bids for the good for sale. The highest bidder bought the good for sale and paid a price equal to the own bid (in all auctions, tied bids were randomly resolved by the computer). In EN, a thermometer showing the current price started rising from 0. Bidders decided whether or not to quit at the current price. When all but one bidder had pushed the quit button, the thermometer stopped rising and the remaining bidder bought the good at the current price. In AMSA, the auction process consisted of two phases. In the first phase, a thermometer started rising from 0. The thermometer stopped rising when all but two bidders had pushed the quit button. This price was called the bottom price for phase two. The two remaining bidders proceeded to the second phase where they simultaneously submitted sealed bids at least as high as the bottom price.

¹¹At the end of the first session, we found out that subjects earned less than we had expected. Therefore, we provided them with an unannounced gift of €5 that was added to the total that they had made in the experiment. We kept the same procedure in the other sessions.

The highest bidder bought the good for sale at a price equal to the second highest sealed bid.

In all auctions, the winner earned a payoff equal to the own value minus the price paid. In addition, in AMSA the two bidders of the second phase each earned a premium equal to 30% of the difference between the lowest bid in the second phase and the bottom price. We now describe the features that were the same in each treatment.

The experiment consisted of three subsequent parts: a symmetric environment without collusion, a symmetric environment with collusion, and an asymmetric environment with collusion. The three parts consisted of 6 periods, 8 periods and 10 periods, respectively. Subjects received the instructions of a subsequent part only after the previous part had been completed. In each period, subjects were assigned to groups of 6. We randomly rematched subjects between periods within a matching-group of 12 subjects. In each session, we ran two independent matching-groups simultaneously, unless we did not have sufficient subjects in which case we ran one group. In each treatment, we obtained data on 5 independent matching-groups of 12 subjects each.

We started part one without collusion because we wanted the subjects to gain experience with the auction rules before they proceeded to the more complicated game where they were allowed to collude. At the outset of part one, subjects received a starting capital of 50 points. In addition, they earned and sometimes lost points with their decisions. In each period, a good was sold in each group of subjects. We communicated to the subjects that each subject received a private value for the good for sale, which was a draw from a $U[0,50]$ distribution. Draws were independent across subjects and periods. Subjects were only informed of their own value. We kept draws constant across treatments for the sake of comparability of the results.

In part two, subjects were allowed to collude. In each period, subjects were first informed of the costs of cooperating, which were the same for

all subjects.¹² Then subjects simultaneously voted whether or not to cooperate. Only if all 6 players voted for cooperation, the group actually cooperated. When a group cooperated, each bidder paid a cost of cooperation. This cost varied across periods, but it was constant across treatments to make results comparable. Group-members were informed of the total number of votes for cooperation in their group. If the group cooperated, all 6 bidders simultaneously submitted sealed bids in a knock-out auction for the right to be the designated bidder. The highest bidder became the designated bidder and automatically bought the good for zero in the auction. The designated bidder paid his bid in the knock-out auction, which was equally shared by the 5 other bidders. If the group did not cooperate, subjects did not incur the costs of collusion and the good was sold with the same auction rules as in part one.

Part three introduced asymmetry between bidders. In each period, three out of six bidders in a group were assigned the role of weak bidder and the three others the role of strong bidder. Weak bidders received a value from $U[0,50]$, while strong bidders received a value from $U[70,120]$. Roles and values were assigned privately and independently across subjects and periods. In part three, only strong bidders had the possibility to collude. At the outset of the period, all bidders were informed of the costs that strong bidders would incur if they actually cooperated. A period started with strong bidders voting to cooperate or not. If all three strong bidders voted for cooperation, strong bidders did cooperate. Only strong bidders were informed of the outcome of the voting process. Therefore, weak bidders were not sure whether or not they faced a cartel. When strong bidders cooperated, they paid the cost of cooperating and proceeded to a knock-out auction, where they submitted sealed bids for the right to be designated bidder. The highest bidder won and paid a price equal to the own bid. This price was equally shared by the other two strong bidders. Then the designated bidder proceeded with the weak bidders to the main auction

¹²In the instructions we avoided the word collusion, because many subjects are unfamiliar with its meaning.

to bid for the good for sale. In the main auction, the designated bidder submitted shill bids on behalf of the other strong bidders and serious bids on the own behalf. Designated bidders did not share the profits (and premiums) that they made in the main auction. In case the strong bidders did not collude, all bidders immediately proceeded to the main auction with the same auction rules as in the previous parts.

During EN and the first phase of AMSA, other bidders in the group were immediately informed when one of their rivals had dropped out and, in part three, whether this bidder was weak or strong. At the end of a period, all bidders were informed of all bids in the group, and, when applicable, the strength of the bidder making the bid. Table 2 summarizes our experimental design.

Table 2
Experimental design

characteristics treatments					
treatment	price paid by winner	premium?	group-size	matching-group size	# matching-groups
FP	b_1	no	6	12	5
EN	b_2	no	6	12	5
AMSA	b_2	$0.3*(b_2-b_3)$	6	12	5
other features that were the same in all treatments					
part	collusion? who?	bidders' types	valuations	# periods	cost collusion
1	no	6 symmetric	U[0,50]	6	no
2	yes, all	6 symmetric	U[0,50]	8	0,5,2,3,7,6,4,1
3	yes, only strong	3 weak and 3 strong	weak: U[0,50] strong: U[70,120]	10	2,20,12,16,14, 6,10,8,18,4

Notes: b_i refers to the i -th highest bid. The column cost collusion reports the costs per period, from the first period to the last one.

We deliberately chose to build up the strategic complexity throughout the experiment. In the first part, subjects became familiar with the auction rules. In the second part, they were introduced to the possibility of collusion. By varying the costs of collusion, we encouraged them to vote for collusion when costs were low and to vote against collusion

when costs were high. This way they rapidly gained experience with how profitable collusive bidding is compared to competitive bidding. Finally, in part three we introduced asymmetry between the bidders after they had become familiar with the auction rules and the possibility of collusion. To some extent our design mimics a natural process where bidders are engaged in a new series of auctions and then start spotting opportunities for collusion after time progresses. The main difference between our design and a natural process outside the lab is that we force our subjects to think about the possibility of collusion. However, we do not think that our designs triggers too much or too little collusion compared to a more natural process. Because subjects experience collusive auctions as well as competitive auctions, they can make well informed choices after a limited amount of time. Our design choices make it easier for subjects to learn. The enhanced possibilities for learning may compensate for the lack of experience that our subjects have in participating in auctions.

In any case, the most important goal of our experiment is to compare behavior between treatments. Since the sequencing is the same for any treatment, there is no reason to expect a bias in the comparison of the auction formats.

3.4 Results

We present the results in three parts. Before we start we want to make the caveat that all our results depend on the particular parameters that we employ in our experiments. However, there is no reason to expect that our parameter choices bias the qualitative comparison between the auctions. In Subsection 3.4.1, we compare the three auction formats at the aggregate level. In Subsection 3.4.2, we take a closer look at individual bidding behavior and in Subsection 3.4.3 we provide a coherent explanation of the main results.

Table 3
Collusion

		part 2		part 3	
		group colludes	votes collusion	group colludes	strong bidders' votes collusion
FP	realized	42.5%	81.9%	56.0%	79.3%
	Nash	75.0%	75.0%	70.0%	70.0%
EN	realized	26.3%	75.8%	53.0%	81.0%
	Nash	75.0%	75.0%	90.0%	90.0%
AMSA	realized	30.0%	75.0%	40.0%	68.7%
	Nash	75.0%	75.0%	--	--
	Nash passive	--	--	90.0%	90.0%
	Nash aggressive	--	--	40.0%	40.0%

3.4.1 *Between auction comparisons*

We evaluated the three auction formats on the basis of how they scored with respect to deterring collusion, raising revenue, and pursuing efficient outcomes. Table 3 presents the percentages of bidders who voted in favor of collusion together with the theoretical predictions that depend on the costs of collusion. In part two, where all bidders were symmetric, theory predicts that the auctions are equally vulnerable to collusion. The data show that the votes on collusion were very close to Nash in EN and AMSA. In FP, we observed moderately more votes for collusion than predicted. Notice that the proportions of cases where groups actually colluded were substantially smaller than the theoretically predicted ones. This is due to the fact that subjects did not exactly follow the theoretical threshold rule. Combined with the unanimity rule for collusion, this led to much fewer occasions where the groups actually colluded.

In part three with asymmetric bidders, AMSA triggered considerably fewer votes for collusion than FP and EN did. The proportion of votes for collusion in AMSA was about halfway the level predicted by the aggressive and the one predicted by the passive equilibrium. Remarkably, EN and FP performed about equally poorly in preventing collusion, while theory

predicted that FP should trigger less collusion. We will come back to these results in Subsection 3.4.3 after we have dealt with individual behavior.

In part three, theoretical predictions on when bidders collude vary with the treatments. The EN auction and the passive equilibrium of AMSA predict that collusion is only prevented for a cost of 20. In FP, players should not collude for costs higher than or equal to 16, and in the aggressive equilibrium of AMSA, players should not collude for costs of 10 and higher. Therefore, for a cost level of 20 and cost levels below 10 the predictions were the same for all treatments. Table 3 pools across all levels of costs of collusion, also the ones for which the theoretical predictions are the same. Figure 3.1 provides an view on the relationship between costs of collusion and votes for collusion. It is striking that in part three votes for collusion were very similar across treatments for cost levels below 10 and at 20, as theory predicts. The difference in votes was indeed produced in the theoretically relevant cost set $\{10, 12, \dots, 18\}$.

We now investigate to what extent these qualitative results were statistically significant. To take account of the panel data structure of our experiment, we estimated the following logit model with random effects. Let $y_{i,t}$ represent the vote of individual i in period t ; $y_{i,t} = 1$ if i voted for collusion in period t and $y_{i,t} = 0$ if i voted against. We introduce the underlying latent variable $y_{i,t}^*$:

$$y_{i,t}^* = \gamma + \beta_{cost} * cost_t + \beta_{value} * value_{i,t} + \beta_{dumam} * dumam_i + \beta_{dumfp} * dumfp_i + \alpha_i + \varepsilon_{i,t}$$

$$y_{i,t} = 1 \quad \text{if } y_{i,t}^* > 0$$

$$y_{i,t} = 0 \quad \text{if } y_{i,t}^* \leq 0$$

Here, γ represents the constant; $cost_t$ refers to the costs of collusion in period t ; $value_{i,t}$ to the value of i in period t ; $dumam_i$ is a dummy that

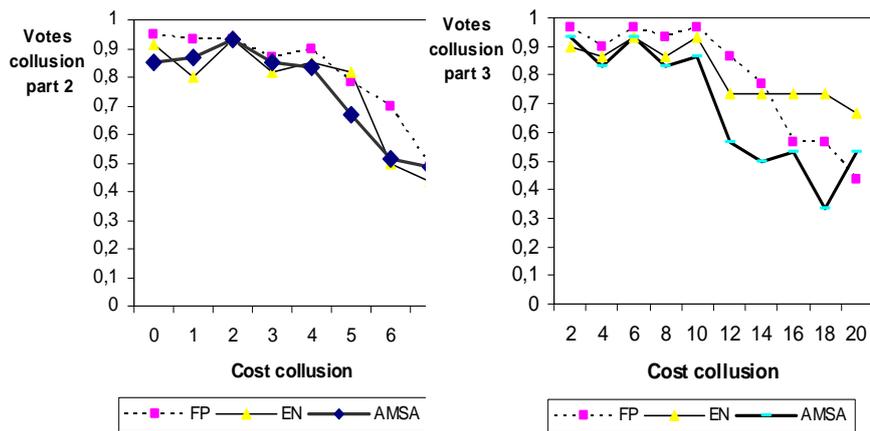


FIGURE 3.1. **Votes for collusion.** *Notes:* for each cost of collusion level the proportion of strong bidders' votes for collusion is displayed (vote collusion=1 if individual voted for collusion). Left panel: part2; right panel: part 3.

equals 1 if i participated in AMSA and 0 elsewhere, and $dumfp_i$ is the corresponding dummy for FP. In addition, we included “group dummies” in the regressions to correct for matching-group specific effects (not reported) and “period dummies” to correct for timing effects. Table 4 reports the treatment effects compared to the omitted treatment EN.

It turns out that in part two (the symmetric case), FP attracted significantly more votes for collusion than EN ($p = 0.01$) and AMSA ($p = 0.00$, Wald test). EN raised slightly more votes for collusion than AMSA did, and the difference is significant at $p = 0.05$. In part three (the asymmetric case), we observe less collusive votes in AMSA than in FP ($p = 0.01$, Wald test) and EN ($p = 0.00$), whereas there is no statistical difference between FP and EN ($p = 0.30$). As expected, there was a clear significant negative effect of the cost of collusion on the inclination to vote for collusion in both regressions reported.

Table 4
Random effects logit model on vote for collusion

	part 2	part 3
	estimate (s.e.)	estimate (s.e.)
dumam	-1.12 (0.57)**	-2.61 (0.71)**
dumfp	2.00 (0.73)**	-0.76 (0.73)
cost	-0.41 (0.05)**	-0.17 (0.03)**
value	-0.03 (0.01)**	-0.01 (0.01)
dump1	0.64 (0.25)**	-0.21 (0.38)
dump2	0.98 (0.38)**	-0.20 (0.34)
dump3	0.53 (0.29)*	-0.15 (0.36)
dump4	0.04 (0.23)	0.89 (0.62)
dump5	0.89 (0.29)**	1.06 (0.54)**
dump6	0.16 (0.34)	0.28 (0.48)
dump7		-0.25 (0.34)
dump8		-0.38 (0.54)
constant	3.71 (0.54)**	5.45 (1.08)**
Wald-test		
dumam-dumfp=0	-3.12 (1.01)**	-1.85 (0.67)**
-logL	602.21	393.45

Notes: standard errors in parentheses. ** (*) denotes significance at 5% (10%) level; the omitted treatment is EN; dumam equals 1 for AMSA and 0 otherwise; dumfp equals 1 for FP and 0 otherwise; for part 2, dump1, ..., dump6 equal 1 for periods 8, ..., 14, respectively, and 0 otherwise; for part 3, dump1, ..., dump8 equal 1 for periods 17, ..., 24, respectively, and 0 otherwise.

A remarkable result is that in part two the period dummies are always positive and significantly so in 4 of 6 cases. This suggests that subjects learned in the sense that they more easily voted for collusion after they obtained some experience with the game. In contrast, there does not seem to be a systematic pattern in the period dummies in part three. Interestingly, the coefficient for value is significantly negative in part two. There, subjects were more inclined to vote for collusion when they received lower values. This is in contrast to theory, which indicates that the decision to vote for collusion does not depend on value. Possibly, subjects with low values realized that they would not have a fair chance in a competitive auction, which made them more inclined to vote for collusion. The coef-

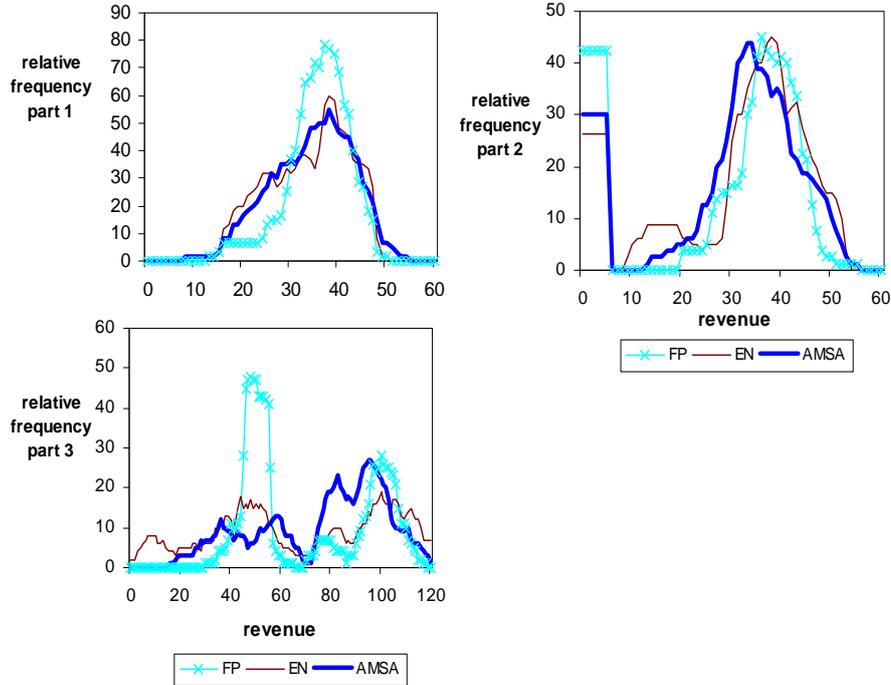


FIGURE 3.2. **Revenue.** *Notes:* for each revenue level the percentage of outcomes that fall in the interval $[\text{revenue}-5, \text{revenue}+5]$ is displayed. Upper-left panel: part 1; upper-right panel: part 2; lower panel: part 3.

ficient for value ceases to be significant in part three, though, which may be a sign that subjects learned across the two parts.

We now turn to the comparison of revenues between the treatments. Figure 3.2 shows revenue histograms for parts one, two and three. In part one, revenue was on average somewhat higher and less dispersed in FP than in EN and AMSA. The histogram of revenues in AMSA was almost identical to the one in EN. In agreement with the finding that subjects colluded more often in part two of FP, the upper-right panel shows a larger spike at 0 in FP than the other two formats. Thus in the symmetric setup, the possibility to collude counteracted the usual revenue dominance

Table 5
Revenue

		part 1	part 2	part 3
FP	realized	35.7 (5.7)	21.3 (18.8)	70.9 (25.2)
	Nash	34.9 (6.1)	9.5 (16.6)	63.7 (21.4)
EN	realized	33.5 (8.1)	26.4 (17.7)	68.0 (33.8)
	Nash	35.8 (7.5)	9.8 (17.3)	45.2 (20.5)
AMSA	realized	34.3 (8.0)	24.9 (17.5)	76.8 (25.3)
	Nash	35.7 (5.5)	9.6 (16.8)	--
	Nash passive	--	--	45.0 (19.4)
	Nash aggressive	--	--	86.1 (14.5)

Notes: standard errors in parentheses.

of FP over EN found in previous experimental auctions.¹³ The lower-left panel shows that the largest differences in revenues were observed for asymmetric bidders. Here, a bimodal distribution resulted in FP, with the largest number of outcome close to 50 and most of the other outcomes close to 100. In contrast, the revenue histograms for EN and AMSA were much more spread out. EN was the most vulnerable auction in terms of raising very low revenues.

Table 5 reports the average revenues in the experiment in comparison with the theoretical revenues given the values and collusion costs employed in the experiment. In part one, FP generated the highest revenue, followed by AMSA and EN. The differences were small, though, and all quite close to the theoretically expected levels. The second part reveals that in the case of symmetric bidders and potential cartel formation, EN and AMSA both raised higher revenues than FP. In all treatments, revenues were much higher than the theoretical predictions. Because unanimity was required for a cartel to form, the number of actual cartels was much lower than predicted by theory, and, as a consequence, actual revenue was higher. In the third part, AMSA performed best while EN and FP raised similar revenues. Here, EN performed much better than theoretically predicted.

¹³See Kagel (1995).

The revenue of AMSA was closer to the revenue expected in the aggressive equilibrium than the revenue in the passive equilibrium.

To investigate the significance of the revenue comparisons, we estimated a random effects model that took the interaction in the experiment into account. Let $r_{i,j,t}$ represent the revenue of group i ($i = 1$ or $i = 2$) in matching-group j in period t :

$$r_{i,j,t} = \gamma + \beta_{cost} * cost_t + \beta_{dumam} * dumam_j + \beta_{dumfp} * dumfp_j + \alpha_j + \varepsilon_{i,j,t}$$

Here, γ denotes the constant; $cost_t$ represents the costs of collusion in period t ; $dumam_j$ ($dumfp_j$) is a dummy that equals 1 if the matching-group j was run in AMSA (FP) and 0 elsewhere.

Table 6 reports the results compared to the omitted treatment EN. In part one, only the difference in revenue between FP and EN is significant at $p=0.08$. In part two there are no significant differences between the treatments. In the asymmetric situation of part three, it becomes attractive for sellers to employ the AMSA format, as it raised roughly 10% more revenue than FP and EN. The difference in revenue between AMSA and FP is significant at $p=0.08$ (Wald test) and the difference in revenue between AMSA and EN is significant at $p=0.04$. The difference between FP and EN is not significant ($p=0.50$). In both regressions there is a significant effect of costs of collusion. With higher costs of collusion, groups colluded less and more revenue was raised.

Table 6
Random effects model on revenue

	part 1	part 2	part 3
	estimate (s.e.)	estimate (s.e.)	estimate (s.e.)
dumam	0.80 (1.34)	-1.49 (4.34)	8.76 (4.35)**
dumfp	2.12 (1.20)*	-5.15 (4.36)	2.90 (4.30)
cost		2.31 (0.79)**	1.39 (0.35)**
dump1	-5.88 (2.07)**	-10.07 (3.20)**	0.09 (6.08)
dump2	2.92 (0.97)**	3.66 (4.11)	6.98 (5.46)
dump3	-1.17 (1.34)	3.72 (4.19)	4.68 (6.42)
dump4	-4.75 (1.20)**	6.15 (3.36)*	-9.25 (5.40)*
dump5	-3.93 (1.73)**	-9.73 (3.53)**	-17.23 (4.56)**
dump6		9.44 (4.30)**	-4.37 (5.97)
dump7			6.53 (5.69)
dump8			3.54 (7.44)
constant	35.67 (1.16)**	17.95 (4.34)**	53.65 (6.22)**
Wald-test			
dumfp-dumam=0	1.32 (1.16)	-3.66 (4.33)	-5.86 (3.37)*
R ² within	0.18	0.28	0.20
R ² between	0.44	0.11	0.29
R ² overall	0.18	0.26	0.21

Notes: (robust) standard errors in parentheses. ** (*) denotes significance at 5% (10%) level; the omitted treatment is EN; dumam equals 1 for AMSA and 0 otherwise; dumfp equals 1 for FP and 0 otherwise; for part 1, dump1, ..., dump5 equal 1 for periods 2, ..., 6, respectively, and 0 otherwise; for part 2, dump1, ..., dump6 equal 1 for periods 8, ..., 14, respectively, and 0 otherwise; for part 3, dump1, ..., dump8 equal 1 for periods 17, ..., 24, respectively, and 0 otherwise.

Table 7 presents revenue in parts two and three conditional on whether a cartel was established. In part two, conditional on a cartel not being formed, the results were very similar as the ones for part one. Thus, the different revenue results in parts one and two are mainly attributed to the differences in votes for collusion between the treatments. In part three, both in the cases where collusion occurred and the cases where collusion did not occur, actual revenues were very close to the theoretical predicted outcomes in EN and FP. The result that, pooled across all cases, revenue in EN was much higher than theoretically expected must thus be attributed to the fact that this auction was much less conducive to collusion than predicted. Overall, revenue in AMSA was higher than the other two formats

Table 7
Revenue conditional on (no) collusion

		part 2		part 3	
		no collusion		no collusion	collusion
FP	realized	37.0 (5.2)	97.8 (9.6)	49.8 (5.8)	
	Nash	36.1 (5.3)	96.7 (5.6)	50.0 (0.0)	
EN	realized	35.8 (9.2)	99.0 (12.7)	40.5 (19.9)	
	Nash	36.7 (9.1)	101.3 (11.9)	39.2 (8.6)	
AMSA	realized	35.6 (7.2)	93.8 (10.5)	51.3 (18.6)	
	Nash	37.3 (6.7)	98.6 (7.8)	--	
	Nash passive	--	--	38.8 (7.5)	
	Nash aggressive	--	--	70.0 (0.0)	

Notes: in part 2 realized and theoretical revenues in case of collusion equal 0 by definition; the Nash predictions are computed on the basis of the actual voting behavior. Standard errors in parentheses.

despite the fact that AMSA realized less revenue in the absence of collusion. Conditional on collusion, AMSA dominated EN, but raised similar revenues as FP did. Therefore, the results that AMSA revenue dominated FP and EN must be attributed to bidders being less inclined to vote for collusion. Note that the observations are closer to the Nash predictions than in Table 5, with AMSA in part three being closer to the “passive” equilibrium than the “aggressive” one.

Finally, we point the spotlight on efficiency. Table 8 includes the average efficiency of the auctions in each part.¹⁴ Theory predicts that all auctions are 100% efficient because in equilibrium, the bidder with the highest value always wins the object. In all three parts, EN was more efficient than FP, and AMSA was less efficient than FP and EN. The efficiency differences were substantial in parts one and three. Running similar regressions as the ones reported for revenue, we find that the differences in efficiency in part one between FP and AMSA and EN and AMSA are both significant at the 5% level. In part three, the differences between EN and AMSA and FP and EN are significant at the 10% level. All other differences in efficiency are not significant at conventional levels. So while AMSA tends

¹⁴We define efficiency as $(v_{\text{winner}} - v_{\text{min}})/(v_{\text{max}} - v_{\text{min}})$, where v_{winner} , v_{min} , and v_{max} represent the value of the winner, the lowest value among the bidders and the highest value among the bidders, respectively.

to outperform FP and EN in terms of cartel formation and revenue, the auctioneer may still prefer EN if efficiency is considered the important criterion.

Table 8
Efficiency in %

	part 1	part 2	part 3
FP	95.4 (13.9)	96.0 (12.6)	92.7 (15.3)
EN	95.4 (16.0)	97.0 (13.1)	96.1 (11.9)
AMSA	86.7 (27.1)	94.4 (18.3)	90.9 (21.5)

Notes: standard errors in parentheses.

3.4.2 Individual bidding behavior

In this section, we take a close look at subjects' bidding behavior before we turn to an explanation of the main results. First, we deal with how subjects behaved in the knock-out auctions once they had decided to collude. Figure 3.3 shows average bids conditional on value in parts two and three. In part 2, the Nash predictions trace average bids in FP very well, as can be observed in the upper-left panel. In EN, submitted bids fall below the Nash prediction while in AMSA bidders tended to overshoot compared to Nash. Nevertheless, deviations from Nash were rather small when symmetric bidders bid in the knock-out auction.

The other panels of Figure 3.3 display how strong bidders bid after they voted to collude in part three, where the theoretical predictions depended on the employed auction format. Like in part two, actual average bids in FP were very close to the theoretical prediction. In contrast, in EN strong bidders submitted substantially lower bids than predicted. It was as if bidders preferred to leave the task to exploit the right to be designated bidder to others in this treatment. In Subsection 3.4.3, we will provide an explanation of this remarkable result. In AMSA, average knock-out bids were above the amounts that were predicted by the aggressive equilibrium but below the amounts of the passive equilibrium.

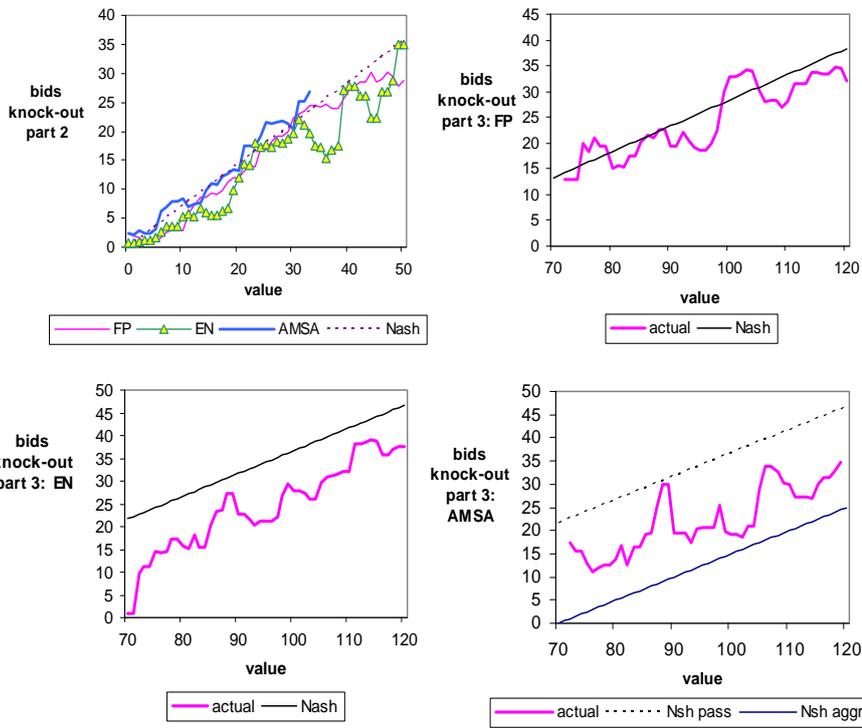


FIGURE 3.3. **Bids knock-out auction.** *Notes:* for each value the average of knock-out bids that fall in the interval $[value-2, value+2]$ is displayed. Upper-left panel: part 2 all auctions; upper-right panel: part 3 FP; lower-left panel: part 3 EN; lower-right panel: part 3 AMSA.

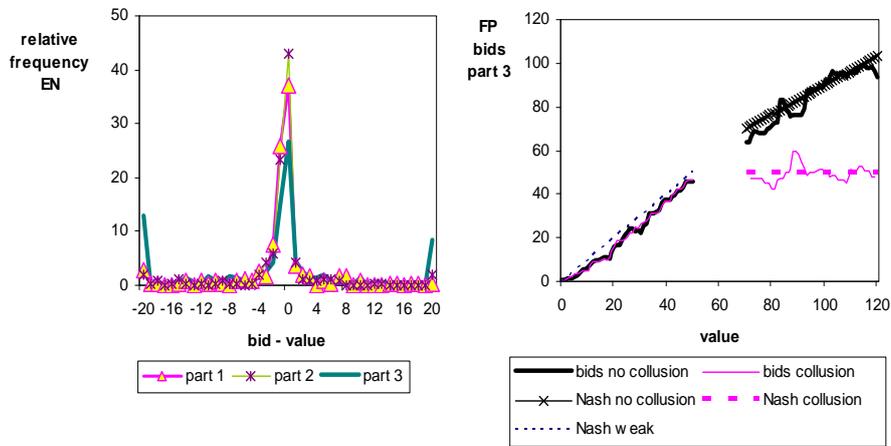


FIGURE 3.4. **Bids in FP (part 3) and EN.** *Notes:* the left-panel shows the histogram of deviations from value (bid-value) in main-auction EN; the right-panel presents for each value the average of main-auction FP bids that fall in the interval $[\text{value}-2, \text{value}+2]$.

We now turn to the bids submitted in the main auction. For EN, the left-panel of Figure 3.4 provides the histograms for the deviations of bids from value in all three parts. Most submitted bids were equal to value, and only few deviated more than two points from value. Only in part three a small minority of bids deviated substantially from value. Most of these deviating bids were submitted by weak bidders, who either gave up from the start or who chose to drive up the price for the strong bidders.

In the first two parts of FP, bidders' behavior agreed with the general picture coming from symmetric private value auctions. Bidders submitted bids that were on average slightly higher than Nash. The right-panel of

Figure 3.4 shows average bids together with theoretical predictions for the much less investigated asymmetric case. Average bids were remarkably close to the theoretically predicted ones, both for weak and for strong bidders.

Figure 3.5 tells a somewhat different story for the first two parts of AMSA. In the first phase of these auctions, bidders on average exited a bit sooner than predicted by Nash, while the subjects that went on to the second phase with low values submitted higher sealed bids than expected (upper-left panel). A similar pattern was observed in Goeree and Offerman (2004). One possible explanation is that subjects differ in their risk-attitudes. The AMSA format automatically selects the risk-averse types to drop in the first phase while the risk seeking ones tend to continue to the second phase. Alternatively, low-valued subjects who proceeded to the second phase may have decided to submit high bids to rationalize their risky bidding in the first phase. Occasionally, low-valued bidders thus became the winner of the auction, which agrees with the poor efficiency performance of this format. A similar picture emerged in part three of AMSA. Again, in the first phase weak bidders behaved rather cautiously, on average exiting only somewhat higher than their value (lower-left panel). Those weak bidders that continued to the second phase tended to take high risks (lower-right panel).

Conditional on collusion, bidders faced a coordination problem in part three of the AMSA auction. In the experiment, the passive equilibrium, predicting that weak bidders bid up to value, and the aggressive equilibrium, predicting that weak bidders submitted bids equal to 70, attracted bidders' attention. To classify the collusive cases, we divided the interval between the prediction of the passive equilibrium (i.e., the highest value of the weak bidders) and the prediction of the aggressive equilibrium (i.e., 70) in three equal parts. If the realized revenue was to the left of the middle interval, it was classified as being close to the passive equilibrium and if it was to the right of the middle interval, it was classified as being close to the aggressive equilibrium. A substantial part of 50% of the col-

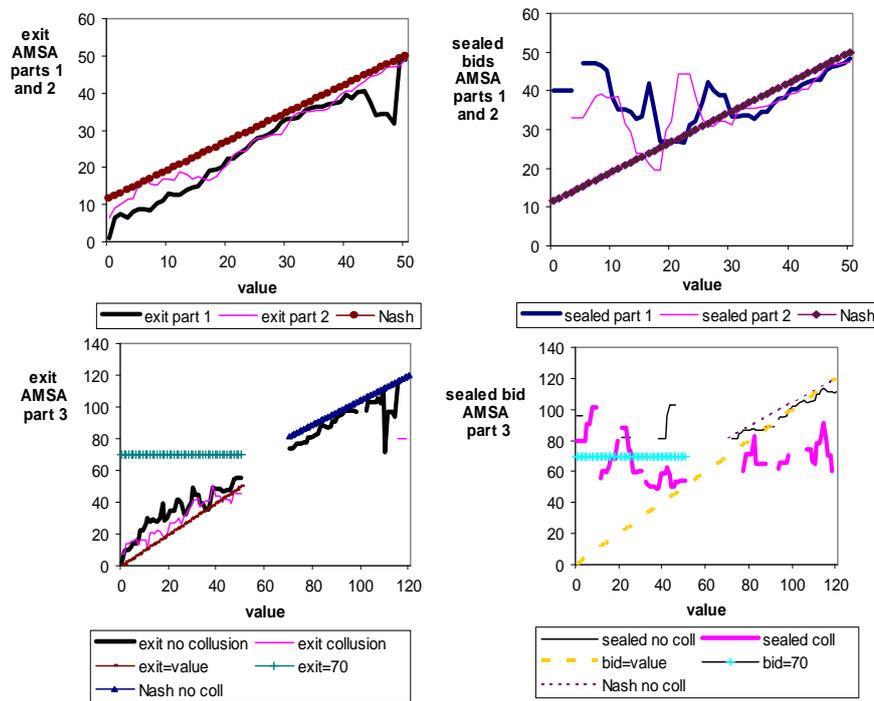


FIGURE 3.5. **Bids in AMSA.** *Notes:* for each value the average of main-auction sealed bids in AMSA that fall in the interval $[value-2, value+2]$ is displayed. Upper-left panel: AMSA exit first phase parts 1 and 2; upper-right panel: AMSA sealed bids second phase parts 1 and 2; lower-left panel: AMSA exit first phase part 3; lower-right panel: AMSA sealed bids second phase part 3.

cluding groups ended up being close to the passive equilibrium, while 25% finished close to the aggressive equilibrium. The remaining 25% of the collusive cases was in between the passive and the aggressive equilibrium.

According to both equilibria, the designated bidder should be tough and keep the shill bidders in the auction as long as the weak bidders had not yet exited. Only if a designated bidder plays tough, the bottom price is not determined by weak bidders. In agreement with this feature of the equilibria, designated bidders played tough in 72.5% of the cases and received higher profits if they did so. That is, the designated bidder's profit on the transaction equalled 48.9 (at an s.e. of 25.3) for tough play and it equalled 35.7 (at an s.e. of 28.3) when they let their shill bidders drop before the weak bidders did.¹⁵

3.4.3 Explanation of the main results

In this section, we provide an explanation for the main results on collusion. In part two, theory predicted that the auctions were revenue equivalent and that, as a consequence, the auctions would be equally conducive for collusion. Instead, we observed that FP triggered significantly more votes than the other two formats.¹⁶ We think that the key to explaining these differences is given by the revenues actually raised in part one. There, bidding was most competitive in FP, while AMSA and EN raised similar profits. FP-bidders experienced that the main auction was not so profitable, which made collusion more attractive compared to the other two formats. In fact, when revenue equivalence breaks down in the way it did in part one, the theoretical predictions on collusion change in the direction that we actually observed.

¹⁵In AMSA, the profit on the transaction equals the own value minus price paid plus premium in case the bidder bought the good and it equalled the premium or 0 in case the bidder did not buy the good.

¹⁶In addition, EN was significantly more conducive to collusion than AMSA, but the difference in collusive votes between these two auctions was rather small.

Table 9
Prospects for designated bidder part 3

Part 3	profit transaction winner collusion	price paid by designated bidder	% cases designated bidder buys product
FP	49.6 (19.3) n=56	50.3 (5.6) n=52	92.9%; n=56
EN	61.8 (26.5) n=53	40.9 (19.6) n=33	96.2%; n=53
AMSA	42.3 (26.1) n=40	51.6 (17.9) n=33	82.5%; n=40

Notes: standard errors in parentheses.

The main results in part three were that AMSA proved less conducive to collusion than the other auctions, and that, rather unexpectedly, FP performed equally unsuccessful in fighting collusion as EN did. Table 9 presents some statistics that provide an explanation for these results. The table lists for each auction how much profit the designated bidder actually made on the transaction in the main auction, at which price the designated bidder bought the good for sale, and what the probability was that the designated bidder actually bought the good in the main auction.¹⁷ On all these criteria, AMSA offered the worst prospects for the designated bidder. Given that collusion was most unattractive in AMSA, it makes sense that bidders voted more often against collusion in this format.

What remains puzzling though was that FP did not attract less collusive votes than EN did, even though the statistics in Table 9 show that the prospects for the designated bidder were worse in FP. We think that two features may have contributed to this result. The first one is that, even though the designated bidder made on average a higher profit in EN than in FP, this occurred at a higher variance. Subjects who disliked risk may have been more reluctant to become designated bidder in EN. This was also reflected in the knock-out bids shown in Figure 3.3. Bids in the knock-out auction were close to the theoretically predicted ones in FP, while they were considerably below the theoretical bids in EN. We think

¹⁷For AMSA, the profit on the transaction was defined in footnote 15. In EN, it was equal to the own value minus the price paid in the main auction if the designated bidder won the auction and 0 otherwise; in FP, it was equal to value minus own bid in case of winning and 0 otherwise.

that perhaps an even stronger force behind this result may have been designated bidders' lack of control in the EN auction. In the FP auction, it was easy for a bidder to predict how much profit was available in the main auction. Bidders knew that a bid of 50 or 51 would win the main auction almost surely. Therefore, a colluding strong bidder could easily anticipate the profit to be made in FP, which may have led to more confident bidding in the knock-out auction and more confident voting for collusion in the voting stage. In contrast, in EN the price that the designated bidder was going to pay completely depended on the behavior of weak bidders. As Figure 3.6 shows, this price was much more volatile in EN than in FP. The extra ambiguity in EN that designated bidders faced in the main auction may have discouraged voting for collusion.¹⁸

It is important to remember that in our experiment the cartel was stable by design in all auctions. This feature of our experiment diminishes the relevance of our results for one-shot auctions. When some bidder cheats on the cartel agreement, bidders may retaliate within a one-shot EN auction but not within a one-shot FP auction. Therefore, when bidders do not have the possibility to retaliate in the future, EN auctions may be more prone to collusion than FP auctions (Robinson, 1985; Marshall and Marx, 2007). Instead, our results are relevant to situations where bidders interact repeatedly as in bidding for projects in the construction industry. In such situations, there is ample evidence that even in FP auctions bidders refrain from cheating on the cartel, presumably out of fear for future retaliation (Scherer, 1980; McAfee and McMillan, 1992; Porter and Zona, 1993; Porter and Zona, 1999; Pesendorfer, 2000; Boone et al., 2009).

3.5 Concluding Discussion

In this chapter, we studied the collusive properties of EN, FP and AMSA using a laboratory experiment. We did so in two settings. In the first one,

¹⁸Notice that in AMSA designated bidders were faced with a similar lack of control as in EN, so this factor also worked against collusion in AMSA.

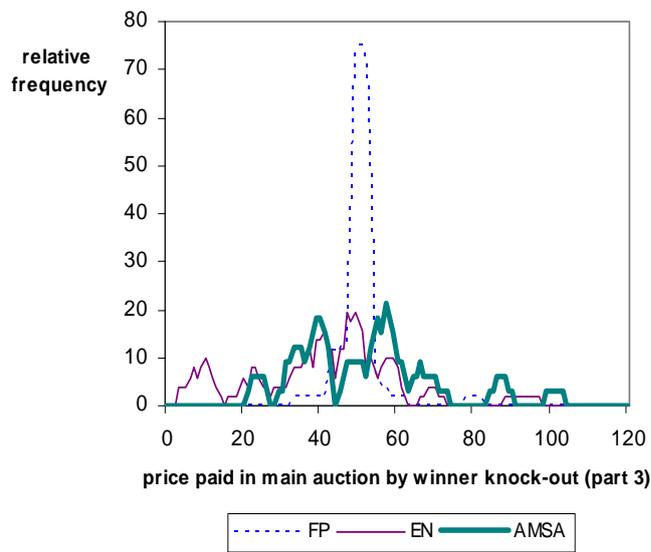


FIGURE 3.6. **Prices paid in main auction by designated bidder part 3.**
Notes: for each price paid in the main auction the percentage of outcomes that fall in the interval $[\text{price}-2, \text{price}+2]$ is displayed.

bidders were symmetric and all could participate in the cartel. Here we observed that FP triggers more collusive votes than the other formats. This result is consistent with the finding that without collusion, the FP auction was the most competitive one. Therefore, the incentive to collude was highest in this format. Interestingly, with the possibility to collude, the revenue dominance of FP over EN usually reported in experimental private value auctions completely disappears.

In the second setting, both strong and weak bidders competed for the good for sale. Only strong bidders were eligible for collusion. In theory, FP should outperform EN in preventing collusion, because in the former a designated bidder could not afford to bid below the higher end of the support of the weak bidders, which makes collusion relatively less attractive. In contrast to this prediction, we observed that EN triggered about as much collusion as FP did. We think that there are two reasons behind this result. First, the designated bidder ran a higher risk in EN when he had to beat the weak bidders in the auction. Second, the designated bidder faced less ambiguity in FP than in EN. That is, in FP the designated bidder could easily anticipate the amount of profit that he would almost surely make in the main auction, whereas in EN the actual price paid in the main auction varied substantially. Consistent with these explanations is our finding that in EN strong bidders tended to submit low bids in the knock-out auction, as if they preferred to leave the right to be designated bidder to others.

According to theory, AMSA is less conducive to collusion than the other formats only if weak bidders bid sufficiently aggressively in the case of collusion. In the experiment, bidders focussed sufficiently on the aggressive equilibrium to make collusion unattractive. AMSA triggered less collusion than the other auctions did.

3.6 Appendix

INSTRUCTIONS EXPERIMENT¹⁹

Welcome to this experiment on decision-making! You can make money in this experiment. Your choices and the choices of the other participants will determine how much money you will make. Read the instructions carefully. There is paper and a pen on your table. You can use these during the experiment. Before the experiment starts, we will hand out a summary of the instructions.

THE EXPERIMENT

You will earn points in the experiment. At the end of the experiment, your points will be exchanged in euros. Each point will yield 20 cent. You will start with a starting capital of 50 points.

The experiment consists of three parts. The three parts are linked to one another, so it is important that you understand the instructions of current part before you proceed with the experiment. Only when a part is completely finished you will receive instructions for the next part. The first part lasts for 6 periods. Your earnings in the first part will be the sum of your earnings in these 6 periods plus the starting capital. Your earnings in the experiment will equal the sum of your earnings in all three parts.

Each period you will be allocated to a group of 6 persons to whom a product is to be sold. The composition of the group varies from period to period. This means that you will be involved in an auction with different players in each period.

VALUE OF THE PRODUCT

For each participant, the product has a different value lying between 0 points and 50 points. Every number between 0 and 50 is equally likely. The value assigned to one participant does not depend on the values of the

¹⁹These are the instructions for treatment AMSA. Other instructions are available upon request.

other participants. This means that your value is probably different from those of others. At the start of a period, you will be informed about your own value, which will not be revealed to the other participants. Likewise, the other participants' values are not revealed to you.

SALE OF THE PRODUCT: PHASE 1

Each period consists of two phases. In the first phase, the "temperature of a thermometer" will rise point by point. The level of the temperature indicates the price of the product. In the first phase, all six participants have the possibility to push the button "QUIT". By pushing the QUIT button, a participant indicates that he or she will not buy the product in this period. When four participants have pushed the QUIT button, the first phase is finished. The level of the temperature where the fourth bidder pushed QUIT is called the "BOTTOM PRICE". The four participants that have pushed the QUIT button in the first phase do not receive any payoff in this period.

If accidentally two (or more) participants push the QUIT button at the same time then chance will determine which one of these participants will continue. The temperature in the thermometer will never rise higher than the upper bound of the value interval: 50 points. If the thermometer reaches the upper bound, the computer will automatically push the QUIT button for you.

When a participant pushes the QUIT button in the first phase, the other participants (no matter whether they have pushed the QUIT button or not) can observe the price at which he or she chooses to quit in the current period.

SALE OF THE PRODUCT: PHASE 2

In the second phase, the two participants that have not quit will submit their "ultimate bid". The participant with the highest ultimate bid buys the product. The price that this participant pays is NOT equal to the own ultimate bid, but to the ultimate bid of the OTHER PARTICIPANT!

The ultimate bid has to be larger than or equal to the bottom price determined in phase 1. If accidentally both bidders submit the same ultimate bid, then chance will determine which of these two bidders buys the product. It is not allowed to submit an ultimate bid higher than 50.

The buyer will not literally receive a product. He or she will receive an amount equal to the value of the product minus the price of the product (in points).

SALE OF THE PRODUCT: PREMIUM

Both bidders of the second phase will receive a premium that depends on the extent to which the bottom price from the first phase is actually increased. To calculate this premium, the difference between the lowest ultimate bid in the second phase and the bottom price is determined. Each bidder of the second phase receives a premium of 30% of this difference (in points).

EXAMPLE

The procedure to sell the product will now be illustrated with an example. THE NUMBERS IN THE EXAMPLES ARE ARBITRARILY CHOSEN.

The temperature of the thermometer starts rising from 0. At a price of 22, 25, 30, the first, second, and the third bidder pushes the QUIT button respectively. The temperature keeps rising until the fourth bidder pushes the QUIT button at a price of 32. This is the end of the first phase. The two remaining bidders submit an ultimate bid higher than or equal to 32 in the second phase. Assume that the fifth bidder bids 42 and the sixth bidder bids 45, and then the results are as follows:

The sixth bidder buys the product at a price of 42. Both the fifth and the sixth bidder receive a premium of 30% of $42 - 32$ (=the lowest ultimate bid minus the bottom price): this yields an amount of 3 points to either of them.

The sixth bidder also obtains the gains from trade. The gains from the trade equal her or his value for the product minus the price paid for the product.

GAIN AND LOSS

Notice that the highest bidder in a period can make a loss. If the highest bidder pays a price higher than her or his value for the product, and if this is not compensated by the premium, he or she makes a loss. Just as any gain is automatically added to the amount earned up to that period, any loss will automatically be subtracted.

RESULTS OF THE PERIOD

At the end of a period, the results of the period will be communicated. You will be told whether you had the highest bid and how much payoff you have earned. All bidders in a group will also be informed about the two ultimate bids submitted in the second phase.

Then a new period will be started. In the new period, again a product will be sold. Each participant receives a new value for the product. Your value for the product in the one period will not depend on your value for the product in any other period

If you are positive that you have understood the instructions, please click 'Next' below for the test questions.

THE NUMBERS IN THE QUESTIONS ARE ARBITRARILY CHOSEN.

(1) Assume that in the second phase your ultimate bid equals 48 while the other bidder has an ultimate bid of 46. What is the price that you will have to pay for the product?

(2) Assume that in the second phase your ultimate bid equals 46 while the other bidder's ultimate bid equals 36. The bottom price from the first phase equals 16. What is the premium for each of the bidders in the second phase?

(3) Is the following statement correct: in each period you will be matched with the same other players

END

You have reached the end of the instructions. If you wish to read some parts of the instructions again, please click 'Finished', then you will return to the beginning of the instructions. When you are ready to start the experiment, please push the button READY. If by then not all the participants have pushed READY, you will return to the beginning of the instructions. When all participants have pushed READY, the experiment will start. When the experiment has started, you will NOT be able to return to these instructions. Before the experiment is started, a summary of the instructions will be handed out.

If you still have questions, please raise your hand!

INSTRUCTIONS: PART 2

The second part of the experiment lasts for 8 periods. Your earnings of part 1 will be transferred to the first period of part 2. As the general rules in the second part are quite similar to those of the first part, we will focus on the differences between the two parts in the following.

At the beginning of every period in the second part, you are given AN OPTION TO VOTE FOR COOPERATION with the other players in your group at a given cost. You will know the cost of cooperation before each period starts; your value for the product will be communicated to you at the same time. The cost of cooperation differs in each period. Within a period, each player faces the same cost of cooperation. The gain or loss a participant makes in each period is immediately subtracted from or added to his or her total earnings.

DECISION TO VOTE FOR COOPERATION

ALL BIDDERS are asked if they would like to vote for the cooperation. A bidder can either click the YES button on the screen if he or she would like to cooperate with the other bidders; or click the NO button if he or she thinks otherwise. Only in the situation where all six bidders vote for the cooperation, will the group of bidders cooperate.

Also, note the bidders are obliged to pay the costs of cooperation only when the cooperation ACTUALLY takes place. That is, if one or more bidders vote against cooperation, then the group will not cooperate and consequently no bidder has to pay the costs for cooperation.

When all bidders have made their choice about cooperation, you will be informed about how many bidders have voted against cooperation and whether the cooperation will take place in your group.

COOPERATION

If a group cooperates, ONLY ONE BIDDER among the six obtains the right to enter the main auction. This bidder is called the DESIGNATED BIDDER.

The following procedure will determine who will be the designated bidder in a group: each bidder has to submit a bid for the right to be the only bidder in the main auction. For each participant, the bid submitted cannot be higher than 60 points. If the bid you submit for the right is the highest among the six, you will then become the designated bidder of your group at a price equal to your own bid. To compensate all the other five bidders who will not be present at the main auction, your payment will be EQUALLY shared by them. Likewise, if your bid is not the highest among the six, you will receive a fifth of the bid of the highest bidder. In exchange, you will not compete for the product in the main auction. The bids submitted have to be an integer between 0 and 50 points. If two or more bidders submit the same bid, then chance will determine which of these bidders will receive the product.

As the designated bidder will effectively be the only ACTIVE bidder in the main auction, we will skip the course of the main auction and directly reward the product to the designated bidder at price 0. The results will be communicated at the end of the cooperation phase. You will be told whether you have had the highest bid, how much profit you have made and how high the other five bids are. To summarize, the payoff of a designated bidder equals his or her own value minus the bid he or she submitted for the right to be the designated bidder and minus the costs of cooperation.

The payoffs of the other bidders equal the designated bidder's bid by 5 minus the costs of cooperation.

NO COOPERATION

If one or more bidders vote against cooperation, then the group will not cooperate. In this case, the rules will be the SAME as in part 1. That is, all bidders will compete in the same way for the product as described in the instructions of the previous part.

EXAMPLE

The procedure to sell the product will now be illustrated with an example. THE NUMBERS IN THE EXAMPLES ARE ARBITRARILY CHOSEN.

Bidders A, B, C, D, E and F are allocated in one group for the current period. The cost of cooperation is 2; the bidders' values for the product are 0, 10, 20, 30, 40 and 50 points respectively. All six bidders choose to vote for the cooperation, so the group will cooperate.

The next step is to determine the designated bidder: bidder A, B, C, D, E and F each submits one bid for the right to be the designated bidder. Their bids are 0, 5, 15, 20, 25, and 30 respectively. Hence, bidder F gets the right to be the designated bidder at a price equal to his or her own bid, namely 30 points. Bidders A, B, C, D and E will not participate in the main auction and will equally share bidder F's payment (each bidder 6 points). After the costs of cooperation are subtracted, their pay-off is 4 points.

As the designated bidder (bidder F) will effectively be the only ACTIVE bidder in the main auction, we will skip the course of the main auction and directly reward the product to bidder F at price 0. Therefore, bidder F's payoff is equal to 50 (his value) minus 30 (his payment to the other bidders) minus 2 (the cost of cooperation), which equals 18.

OTHER ASPECTS

All other aspects of part 2 will be the same as they were in part 1. In particular, the same procedure will be used to determine the values. That

is, each participant will receive in each period a different value between 0 and 50 points, and every number between 0 and 50 is equally likely.

If you are positive that you have understood the instructions, please click 'Next' below for the test questions.

THE NUMBERS IN THE QUESTIONS ARE ARBITRARILY CHOSEN.

(1) Assume that four out of six bidders in your group push the YES button when they are asked to vote for cooperation. Will your group cooperate?

(2) Assume that four out of six bidders in your group push the YES button when they are asked to vote for cooperation. You were among the players who voted for cooperation. Will you pay the costs for cooperation?

(3) Assume that your group cooperates; your bid for the right to buy the product at a price of 0 equal to 28 while the other bids submitted are 12, 20, 24, 25, and 30. The cost for cooperating equals 2 in this period. What is your payoff in this period?

(4) Assume that your group cooperates; your bid for the right to buy the product at a price of 0 equals 30 while the other bids submitted are 12, 20, 24, 26, and 28. The cost for cooperating equals 2 in this period. Your value for the product equals 40. What is your payoff in this period?

INSTRUCTIONS: PART 3

The third part of the experiment lasts for 10 periods. Your earnings of parts 1 and 2 will be transferred to the first period of part 3. As the rules in the third part of the experiment are quite similar to the rules in the second part, we will focus on the differences between part 2 and 3 in the following instructions. An important aspect where part 3 differs from part two is that TWO TYPES of roles are assigned to the participants: the role of STRONG BIDDER and the role of WEAK BIDDER.

STRONG AND WEAK BIDDERS

The role of each type of bidder is determined by chance at the start of each period. Hence, a participant who receives the role as weak bidder in

one period may be asked to play as a strong bidder in the next period and vice versa. In part 3, each group consists of three players who receive the role of the strong bidders and three players who receive the role of weak bidders. Like in parts 1 and 2, players will be re-matched into a different group when a new period starts.

VALUE OF THE PRODUCT

In the third part of the experiment, the strong bidders and the weak bidders have different intervals from which the values are drawn. For the strong bidders, the value of the product lies between 70 points and 120 points, and every number between 70 and 120 is equally likely. For the weak bidders, the value of the product lies between 0 and 50 points, and every number between 0 and 50 is equally likely.

Again, the value assigned to one participant does not depend on the values of the other participants. Hence, your value is probably different from those of the others. At the start of a period, you will be informed about your own role and your own value, which will not be revealed to the other participants. Likewise, the other participants' values and roles are not revealed to you.

DECISION TO VOTE FOR COOPERATION IN PART 3

Different from part 2, the option of cooperation is offered to the **STRONG BIDDERS ONLY**. The costs of cooperation are the same for each strong bidder but they differ from period to period; the weak bidders never have to bear such costs because they are excluded from cooperation. Both the strong bidders and the weak bidders will be informed about the strong bidders' costs of cooperation before each period begins.

In the third part of the experiment, all participants who receive the roles as strong bidders are asked to vote for the cooperation with the other two strong bidders in the same group. Each strong bidder can either click the **YES** button on the screen if he or she would like to cooperate with the other strong bidders; or click the **No** button if he or she thinks otherwise.

Only in the situation where all three strong bidders vote for cooperation, will the strong bidders of a group cooperate. Strong bidders pay for the costs of cooperation only when cooperation ACTUALLY takes place. In other words, if one or more strong bidders vote against cooperation, cooperation will not take place and consequently no bidder has to pay the costs for cooperation. The results of voting are communicated after all three strong bidders in the group have made their choice (Yes or No). The strong bidders will be informed about how many of them have voted against cooperation and whether the cooperation will take place. The weak bidders will not receive this information.

COOPERATION AMONG STRONG BIDDERS

If the strong bidders cooperate, ONLY ONE STRONG BIDDER among the three obtains the right to enter the main auction, bidding against the three weak bidders for the product. This strong bidder is called the DESIGNATED BIDDER.

Apart from bidding for his or her own, the designated bidder also bid on behalf of his or her fellow strong bidders in the same group, who according to the rules, are not allowed to participate in the main auction.

The following procedure will determine who will be the designated bidder in a group: each strong bidder submits a bid (number of points) for the right to be the designated bidder. The strong bidder with the highest bid will be the designated bidder (in case of ties among the highest bidders, the winner is selected by chance from them). The designated bidder will pay an amount equal to the own bid to compensate the other two strong bidders in the group. This amount will be EQUALLY shared by the other two strong bidders. Bids to become the designated bidder have to be at least 0 points and they cannot exceed 120 points. When all strong bidders submitted their bids, the results will be communicated. All strong bidders will be informed about if they have submitted the highest bid, and how high all three bids submitted are. (Weak bidders will not receive this information.)

SALE OF THE PRODUCT WITH COOPERATING STRONG BIDDERS: PHASE 1

Unlike part 2, there are altogether four active bidders involved in the main auction: the designated bidder and the three weak bidders, who will bid for the product for sale in a similar way as introduced in part 1. The difference is that the designated bidder has to bid on all three strong bidders' behalf in the third part. The designated bidder is free to set the bids for the other two strong bidders in any way that he or she likes, as long as these bids do not exceed his or her own bid (so the strong bidders who are not present will not become the winner of the auction). That is, when bidding in the first phase, a designated bidder must push the QUIT button for the other two strong bidders before he or she pushes his or her own QUIT button. Notice that the designated bidder may push the QUIT button for the other two strong bidders at a price of 0, but he or she may also decide to wait before letting one or two of the other strong bidders quit.

In the third part, the temperature of the thermometer may rise until 120 points before the computer will automatically push the QUIT button for any bidder who has not pushed the button then. When a participant pushes the QUIT button in the first phase, the other participants in the same group (including the strong bidders who are NOT ACTIVE in the main auction) will not only observe the QUIT price of this participant like in the first two parts, but also the role of this bidder (i.e. a weak bidder or a strong bidder). If a strong bidder pushes the QUIT button, all the other bidders will observe a blue colored bid together with the letter "b" next to the thermometer. If a weak bidder pushes the QUIT button, all the other bidders will observe the yellow colored bid together with a letter "w" next to the thermometer. From this, both of the remaining bidders in phase 2 can infer the other bidder's role.

SALE OF THE PRODUCT WITH COOPERATING STRONG BIDDERS: PHASE 2

In the second phase, the two participants that have not quit in the first phase will submit their "ultimate bid" for the product. The participant with the higher ultimate bid between the two buys the product at a price equal to the lower ultimate bid. An ultimate bid in phase 2 has to be an integer between the bottom price determined in phase 1 and 120 points. This restriction holds for both weak bidders and strong bidders, which means that for the weak bidders whose value is between 0 and 50, the upper limit of bids is higher in part 3 (120 points) than in part 1 and 2 (50 points).

Both bidders who enter the second phase earn a premium that is determined in the same way as in the previous parts. It is possible that a designated bidder enters the second phase with one of the other strong bidders on whose behalf the designated bidder is bidding for (when all the weak bidders quit before the designated bidder lets the second strong bidder quit). In such a case, we will SKIP the course of the second phase (so the designated bidder DOES NOT have to submit any ultimate bid) and directly reward the designated bidder with the product at the bottom price. In other words, we equate the lower ultimate bid (which is the product price for the winner in the second phase) to the bottom price. As a result, no premium is rewarded to both bidders (i.e. the designated bidder and the strong bidder the designated bidder bids for). Therefore, the designated bidder's gains from trade in this period equals his or her value minus the bottom price determined in the first phase minus the price paid for the right to be the designated winner minus the cost of cooperation of this period.

NO COOPERATION

If one or more strong bidders vote against cooperation, then the strong bidders will not cooperate. In this case, all bidders will compete in the same way for the product for sale as described in the instructions of part 1. The only difference is that from now there are weak and strong bidders, and that both types of bidders may bid higher than they were allowed in parts 1 and 2 (up to a maximum of 120 points). When a participant

pushes the QUIT button, the other participants in the same group can observe the QUIT price of the participant and the role of this bidder (i.e. a weak bidder or a strong bidder). Note that you get the same information as in the case that strong bidders vote for cooperation.

EXAMPLE

The following example illustrates the procedure. THE NUMBERS IN THE EXAMPLE ARE ARBITRARILY CHOSEN.

Strong bidders A, B and C and weak bidders D, E, and F are allocated in one group for the current period. The cost of cooperation is 10. The strong bidders A, B, C's values are 80, 90 and 110, respectively. The weak bidders D, E, F's values are 10, 30 and 50, respectively. When asked to vote for cooperation, all three strong bidders (bidders A, B and C) vote 'yes'. Hence, the strong bidders cooperate.

Next, the three strong bidders submit a bid for the right to be the designated bidder. Bidders A, B and C bid 10, 20 and 30, respectively. As a result, bidder C receives the right and pays a price equal to 30 (the own bid). This amount is shared by bidders A and B: each receives an amount of 15 points. In total, bidders A and B make: 15 (share) - 10 (cost cooperation) = 5 each.

Weak bidders do not observe the results of cooperation.

In the main auction, bidder C has to bid for him or herself as well as for bidders A and B. Assume that in the first phase the weak bidders D, E and F push the QUIT button at prices of 8, 31 and 49, respectively. The strong bidder pushes the QUIT button at 50 for one of the other strong bidders. This terminates the first phase. Because all weak bidders quit in the first phase, the second phase is skipped. The result is that bidder C buys the product at a price of 50. In total, her or his payoff equals 110 (value) - 50 (price product) - 30 (own bid for designated bidder) - 10 (cost cooperation) = 20. The weak bidders do not earn any payoff in this example.

If you are positive that you have understood the instructions, please click 'Next' below for the test questions.

(1) Assume that you are a weak bidder in the current period. Are you going to have the option to cooperate with the other weak bidders?

(2) Assume that the cost of cooperating equals 5 points and that all strong bidders vote for cooperation. You are a strong bidder who bids 16 to become the designated bidder, while the other two bids submitted are 8 and 18. What is your payoff in this period?

(3) Assume that the cost of cooperating equals 5 points and that all strong bidders vote for cooperation. You are a strong bidder who bids 20 to become the designated bidder, while the other two bids submitted are 8 and 16. You become the designated bidder who competes against the weak bidders for the product. You push the QUIT button for one other strong bidder at a price of 10. Then the three weak bidders push the QUIT button at prices of 40, 60 and 65. Thus the bottom price is 65. You are told the second phase is to be skipped. What is your payoff in this period if your value equals 105?

4

Non-Quasilinear Preferences in Premium Auctions

This chapter is based on Audrey Hu, Theo Offerman, and Liang Zou. 2010. "A Theory of Premium Auctions." Working Paper, University of Amsterdam.

4.1 Introduction

The preceding chapter has followed the existing literature on premium auctions and focused on situations where risk neutral bidders exhibit strong asymmetries prior to the auction. In particular, the attention was drawn to a situation where one strong bidder (or cartel) competed with several weak bidders. This literature remains inconclusive as to how premium auctions perform in circumstances beyond these special cases. The objective of this chapter is to provide a theory of premium auctions that broadens the scope of the existing studies. We study a class of English premium auctions (henceforth, EPA), in which risk averse or risk loving bidders compete in a standard symmetric private values setting. By comparing this type of auctions with the standard English auction (henceforth, EA), we develop new insights into the circumstances where it does, or does not, make sense for the seller to employ a premium auction.

The EPA proceeds in two stages. In the first stage, the seller raises the price until all but two bidders (finalists) have withdrawn. In the second stage, the price is raised further and stops as soon as one finalist withdraws. The remaining finalist wins the object and pays the price at which the auction stops. In addition, both finalists receive a premium determined by a pre-specified function of the difference between the ending prices of the two stages. Our model is a generalization of the “Amsterdam Second-Price Auction” model studied in Goeree and Offerman (2004). Instead of limiting to the linear premium rules and uniform value distributions, we allow for a general class of premium functions and value distribution functions, as well as for bidders to have a general utility function.

The few existing theoretical investigations of such premium tactics were mainly guided by the researchers’ intuition about the circumstances where premium auctions would perform well. Goeree and Offerman (2004) consider a setting in which there is a strong bidder and a population of weak

bidders. In their model, it is common knowledge that the strong bidder will have the highest value for the object for sale. Hence, the weak bidders have little incentive to participate in a standard first or second-price auction. The lack of competition could then allow the strong bidder to win the object at a very low price. In a premium auction, the same weak bidders can be attracted to the auction and bid competitively for the premium, thereby enhancing the seller's expected revenue. Milgrom (2004, p. 239-241) analyzes an example with endogenous entry that is similar in spirit. In his model, there is a small positive entry cost. In equilibrium, the weak bidders enter the premium auction with positive probability even though they know that a strong bidder will have the largest value for the good for sale. Other tactics of a similar sort, such as using bidding credits or set-asides, have also been shown to enhance competition when bidders are asymmetric (e.g., Ayres and Cramton, 1996).

Another strand of literature studies how risk aversion affects the expected utilities of the bidders, apart from that of the seller, under different auction policies. For instance, Matthews (1987) shows that in symmetric independent private values settings, the buyers who exhibit constant Arrow-Pratt absolute risk aversion has the same expected utilities in any of the standard auctions: English, Dutch, first-price, or second-price. If the buyers exhibit decreasing (increasing) risk aversion, however, then they have strictly higher (lower) expected utilities in the English or second-price auctions. Understanding the buyers' preferences over auction formats can be important because even though an individual bidder may not have much power to influence the auction design, he has nevertheless the choice to "vote with his feet." This possibility is relevant to the seller especially where the potential bidders are few and they face certain costs of participating in the auction (e.g., Smith and Levin, 1996).

In this chapter, we first characterize an EPA symmetric equilibrium and derive the basic properties of the bid function in such an equilibrium (Theorem 4). We then show that an EPA symmetric equilibrium exists and, moreover, has to be unique for a certain class of premium and utility

functions (Theorem 5). In Theorem 6, we derive a “net-premium effect” of EPA that is key to the welfare conclusions of this paper. This effect predicts that a finalist’s conditional expected utility for the premium from the ongoing auction, when calculated in isolation of other random payoffs, is always the same as his utility for the premium if he drops out at the current price level. An important implication of this effect is that at the start of the second stage of EPA, both finalists would have a conditional expected utility for the premium that is equal to zero. This finding is interesting on its own, as it adds a new insight into the types of auctions à la Vickrey (1961)—for the more general utility functions of the bidders. In particular, the net-premium effect implies revenue equivalence when the bidders are risk neutral.

In Theorem 7, we show that for any arbitrary premium function, the expected revenue decreases as the bidders become more risk averse. Since the expected revenue will be invariant with the premium when the bidders are risk neutral (Myerson, 1981), Theorem 7 implies that a risk neutral seller is better (worse) off to offer a premium only when the bidders are risk loving (averse). At first sight, this result may seem to be counter intuitive, especially given the result of Lemma 5 that the premium has the effect of reducing the riskiness of the payment in an English auction. But then what causes risk lovers to bid more aggressively than risk averters in an EPA? The answer is given by the net-premium effect. Risk lovers stay longer in an EPA because they derive a higher expected utility from the uncertain premium.

The result that risk seekers, or speculators, bid aggressively in a premium auction might suggest that they “love” the premiums and will therefore be more willing to participate in an EPA rather than an EA. This intuition turns out to be incorrect. From the bidders’ perspective, we show in Theorem 8 that under certain conditions the bidders prefer an EPA to an EA if and only if they are risk averse. Therefore, the conventional wisdom that premium auctions tend to attract risk-seeking speculators does not apply in our symmetric auction environment. Indeed, our results sug-

gest a conflict of interests between the revenue-maximizing seller and the bidders over the choice between EA and EPA. This conflict of interests continues to hold when the seller is risk averse but the bidders are risk loving, or vice versa (Theorem 9). However, the seller and the bidders may simultaneously prefer the EPA to the EA when the bidders are risk neutral or marginally risk averse, and the seller is sufficiently risk averse.

In general, our results suggest that there are circumstances under which a premium auction performs better than a standard English auction, and circumstances under which it performs worse. As Klemperer (2002) already emphasized, auction design is not a matter of one size fits all. Instead, it calls for “different horses for different courses.” The seller should judge the field of bidders and choose the auction format accordingly. In this respect, it is interesting to note that some auction houses do switch repeatedly between using the premium auctions and the standard Ebay-like auction procedures.¹

The rest of the chapter is organized as follows. Section 4.2 presents the EPA model and the basic assumptions. Section 4.3 characterizes, and establishes the existence and uniqueness, of an EPA equilibrium. A closed-form equilibrium solution is derived in this section for the case where the bidders exhibit constant absolute risk aversion (CARA). Section 4.4 derives the net-premium effect and analyzes its consequences for the expected revenue and the players’ expected utilities. Section 4.5 concludes.

¹For instance, Troostwijk adopted a premium auction for its recent sale of a Boeing 737-400 in November 2009, although the auction house stays with the standard procedures more often. See <http://www.troostwijkauctions.com/nl/> for more examples.

4.2 The Model

A single object is to be sold to one of n (> 2) bidders via an English premium auction (EPA).² Each bidder has a private value (v) for the object that is independently distributed ex ante according to cumulative distribution F , which has a continuously differentiable density function $f = F'$ that is strictly positive on its support $[L, H]$.

Although the auction can be conducted incessantly until the object is sold, it is equivalent and analytically convenient to perceive it, as we do, as a two-stage auction. In the first stage, a price for the object rises continuously from a sufficiently low level and each bidder stays in the auction until he chooses to quit (e.g., by pressing an electronic button). This stage rounds up as soon as only two bidders, called *finalists*, remain and the price level (X) at which the last bidder quits, called the *bottom price*, will serve as a reserve price onwards. In the second stage, the price level rises from the bottom price X until one of the finalists quits. The last one who stays wins the object and pays the price (b) at which the other finalist quits. In addition, both finalists receive a cash premium from the seller that is equal to $\varphi(b - X)$,³ where $\varphi : [0, H - L] \rightarrow \mathbb{R}_+$ is a twice continuously differentiable function such that $\varphi(0) = 0$, and $0 < \varphi' \leq 1/2$. We call such φ a *premium function*. As usual, ties are assumed to be resolved randomly in both stages. If two or more bidders simultaneously withdraw at price X in the first stage, with only one bidder left, then a random device will choose one of these bidders to be a finalist. If both finalists withdraw at the same price b , then both will receive a premium equal to $\varphi(b - X)$, and one of them will be randomly chosen to receive the object and pays the price b . Clearly, if $\varphi \equiv 0$ then the model reduces

²We focus on the English-type of premium auctions in this study mainly because of their popularity in practice.

³A virtually equivalent model is to award the premium only to the highest losing bidder. This will lead to a different equilibrium bid function as derived in Theorem 4, but the qualitative conclusions of Theorems 5-9 will remain the same. See footnote 8.

to an English auction (EA)—with a possible (coffee) break when only two bidders remain.

The bidders have the same utility function, $u : \mathbb{R} \rightarrow \mathbb{R}$, that is strictly increasing and twice continuously differentiable. If a bidder quits before entering the second stage, his utility is normalized to be $u(0) = 0$. We also normalize the seller's reservation value for the object to be zero, and assume that $L \geq 0$ so that the sale will take place with certainty.

Some of our results are obtained under the following assumptions:

$$(A1) \quad \varphi'' \leq 0.$$

$$(A2) \quad \forall z \in \mathbb{R}, \ln[u(w) - u(z)] \text{ is weakly concave in } w \text{ on } (z, \infty).$$

$$(A3) \quad \left(\frac{f(v)}{1 - F(v)} \right)' > 0 \text{ on } (L, H).$$

All these assumptions are quite standard. (A1) requires that the premium function be weakly concave, which includes the linear premium rule that is frequently observed in practice. (A2) assumes that the utility surplus $u(w) - u(z)$ is (weakly) log-concave in w for all $w > z$. This condition is commonly invoked for the existence of equilibria in the first-price sealed-bid auctions (e.g., Athey (2001)), which allows the bidders to be risk neutral, risk averse, and to some extent risk loving.⁴ Clearly, (A2) is satisfied automatically for any degree of convexity of u that is exponential, a special case that will be studied in the Example in Section 4.3. The assumption (A3) says that the hazard rate of F is increasing, which is a well-known property that many common distributions possess.

4.3 The EPA Symmetric Equilibrium

We say that $b : [L, H]^2 \rightarrow [L, H]$ represents a bid function if $b(v, p)$ strictly increases in v , and $b(v, p) - p$ strictly decreases in p . The bid function b is an *EPA symmetric equilibrium (EPA-SE)* if the following strategy maximizes

⁴We use the terms risk loving, risk preferring, or risk seeking interchangeably—all refer to the case where the utility function is (weakly) convex.

each bidder's expected utility in each stage of the auction, conditional on updated information and the common belief that the strategy will be adopted by everyone.⁵ In the first stage, as price p increases continuously, each bidder with value v remains in the auction as long as $b(v, p) > p$, and the bidder quits the auction as soon as $b(v, p) = p$. Given the bottom price X where the first stage ends, each of the two finalists adopts the bid function $b(\cdot, X)$ in the second stage such that with value v , the bidder remains in the auction until he wins the object or quits when the price level reaches $p = b(v, X)$.

Clearly, an EPA-SE implies that the low value bidders will drop out first. It can also be shown (see the proof of Theorem 4) that if $b(v, p)$ is an EPA-SE, then it is differentiable in v so that $b_1(v, p) > 0$.⁶ This implies that at any price $p \geq X$ in the second stage, there is an r solving $b(r, X) = p$ such that if both finalists remain in the auction they must have values higher than r . For $v \geq r$, $[F(v) - F(r)] / [1 - F(r)]$ is thus each finalist's updated probability that the other finalist has a lower type than v . We call such r the *current screening level* (or screening level for short), which is implicitly defined through its one-to-one relation with the ongoing price $p (\geq X)$ in the second stage of EPA, given any bottom price X .

Now fix a bottom price $X \in [L, H)$ and consider the second-stage EPA that is going on at price $p \geq X$. For ease of notation, w.l.o.g. we may directly refer to the screening level r ($b(r, X) = p$) rather than the price level p in describing an ongoing second-stage EPA. If both finalists adopt the bid function $b(\cdot, X)$, then each with value $v \geq r$ will have a conditional

⁵It is known that even in ex ante symmetric settings there may exist multiple asymmetric equilibria (e.g., Maskin and Riley, 2003). We focus on symmetric equilibria in this study.

⁶For functions with two variables, we use subscripts to denote their partial derivatives with respect to the corresponding variable.

expected utility equal to

$$\begin{aligned}
 & U(v|r, b(\cdot, X)) \\
 \equiv & \frac{1}{1 - F(r)} \int_r^v u(v - b(y, X) + \varphi(b(y, X) - X)) dF(y) \\
 & + \frac{1 - F(v)}{1 - F(r)} u(\varphi(b(v, X) - X)), \tag{4.1}
 \end{aligned}$$

where the first term comes from the event that the bidder wins, and the last term from the event that the bidder loses.

An interesting aspect of the premium auction is that even though the bidders have private values, the premium in effect introduces a strong “affiliation” of the values for the two finalists. A higher value of the opponent can now be “good news” as it increases the expected premium for each bidder. However, there are some important (strategic) differences between EPA and the standard English auction (EA) with affiliated signals and interdependent values (e.g., Milgrom and Weber, 1982; Eso and White, 2004). For instance, when a bidder decides to drop out at a price in an EA “he would be just indifferent between winning and losing at that price.” (Milgrom and Weber, 1982, p.1105). This is true for the first-stage EPA but is not true for the second-stage EPA. It will be shown that the equilibrium bid in the EPA is strictly higher than a bidder’s true value. Consequently, should both finalists simultaneously drop out, they would both prefer losing rather than winning. Moreover, the EA equilibrium strategy calls for each bidder to stay in the auction until his expected utility conditional on winning in a tie is zero (or equal to the status-quo utility level that is commonly known beforehand). This property implies a straightforward solution for the symmetric bid function of the EA. In the second-stage EPA, the bidder will drop out at a price level at which his expected utility is positive due to the premium collected, but this expected utility level is private information and cannot be determined unless the bid function is already given.

Nevertheless, the difficulty of applying the standard EA analysis to the EPA can be circumvented by appealing to an argument of Milgrom and

Weber (1982, p. 1105), which implies that despite the interdependency of the utility payoffs, the second-stage EPA can also be seen as a strategically equivalent second-price sealed bid (or Vickrey) premium auction. This result derives from the fact that when only two bidders remain, the bid function cannot be made dependent on the revealed value of the opponent because the auction ends as soon as the opponent drops out—it will then be too late for the winner to adjust his bid. By the same logic, this argument can be applied to any price level p of the second-stage EPA as long as the auction continues. In other words, fix any $p \geq X$, the remaining EPA can be analyzed equivalently as though it is a Vickrey premium auction with reserve price p , in which the highest bidder wins the object and pays the second highest price b , while both finalists receive a premium $\varphi(b - X)$. We shall follow this approach from now on.

The limiting case with $X = H$ is trivial, for then both finalists would have $v = H$ and both would bid $b(H, H) = H$. Now fix $X \in [L, H)$. As long as the auction continues, given the updated screening level $r \in [r(X), H)$, where $b(r(X), X) = X$, the expected utility of a finalist who has value $v \in [r, H]$ and who bids as though his value is $z \in [r, H]$ equals

$$\begin{aligned} & \bar{U}(v, z|r, b(\cdot, X)) \\ \equiv & \frac{1}{1 - F(r)} \int_r^z u(v - b(y, X) + \varphi(b(y, X) - X)) dF(y) \\ & + \frac{1 - F(z)}{1 - F(r)} u(\varphi(b(z, X) - X)). \end{aligned} \quad (4.2)$$

Hence, given that each finalist adopts the bid function b , equilibrium (or incentive compatibility) holds in the second stage if and only if for all $X \in [L, H)$, $r \in [r(X), H)$, and $v, z \in [r, H]$,

$$U(v|r, b(\cdot, X)) \geq \bar{U}(v, z|r, b(\cdot, X)). \quad (4.3)$$

For some of the results we will use the following lemma. It is a variation of the “Ranking Lemma” of Milgrom (2004; p.124). See also Lemma 1 in Chapter 2, and Milgrom and Weber (1982, Lemma 2).

Lemma 3 For $-\infty < c < d < \infty$ and $h : [c, d] \rightarrow \mathbb{R}$ continuous with $h(d) \geq 0$,

- (i) if $\left\{ \begin{array}{l} h \text{ is differentiable on } [c, d] \text{ and} \\ [\forall t \in [c, d], h(t) = 0 \Rightarrow h'(t) < 0] \end{array} \right.$, then $h > 0$ on $[c, d]$;
- (ii) if $\left\{ \begin{array}{l} h \text{ is differentiable on } [c, d] \text{ and} \\ [\forall t \in [c, d), h(t) \leq 0 \Rightarrow h'(t) \leq 0] \end{array} \right.$, then $h \geq 0$ on $[c, d]$.

Proof. The proof of part (i) is analogous to Lemma 1(i), hence is omitted. To show part (ii), assume that h is differentiable on $[c, d]$. Suppose $h(t) < 0$ for some $t \in [c, d)$. Then the continuity of h and the assumption $h(d) \geq 0$ imply the existence of $\hat{t} \in (t, d]$ such that $h(\hat{t}) = 0$ and $h(s) < 0$ for all $s \in [t, \hat{t})$. By the mean value theorem, this implies that there exists $s \in (t, \hat{t})$ such that $h'(s) = [h(\hat{t}) - h(t)] / (\hat{t} - t) > 0$. Therefore, the hypothesis in the square brackets of part (ii) does not hold. ■

The next theorem provides the characterization of an EPA symmetric equilibrium.

Theorem 4 (Necessary and sufficient condition for EPA-SE) For any utility function u and premium function φ , the function $b : [L, H]^2 \rightarrow [L, H]$ is an EPA-SE if and only if for all $X \in [L, H)$, $b(\cdot, X)$ is the solution of the following differential equation and boundary condition: for $v \in [L, H)$,

$$b_1(v, X) = \frac{u(\varphi(b - X)) - u(v - b + \varphi(b - X))}{u'(\varphi(b - X)) \varphi'(b - X)} \frac{f(v)}{1 - F(v)} \quad (4.4)$$

$$b(H, X) = \lim_{v \uparrow H} b(v, X) = H \quad (4.5)$$

on the domain

$$\mathcal{D}(X) = \{(v, b) \in [L, H]^2 : b(v, X) \in [X, H)\}.$$

Before proving this theorem, we first present a lemma and show that (4.4)-(4.5) imply that any solution b necessarily satisfies the required properties of an EPA-SE, i.e., $b(v, p)$ strictly increases in v and $b(v, p) - p$ strictly decreases in p . These two properties imply that as $p \uparrow H$, only the bidder(s) with $v = H$ will remain in the auction; and hence $b(H, H) = H$.

Lemma 4 *If $b : [L, H]^2 \rightarrow [L, H]$ is a solution of (4.4)-(4.5), then (i) $b_1(v, p) > 0$ on $\mathcal{D}(p)$ for all $p \in [L, H]$; and (ii) $b(v, p) - p > b(v, \hat{p}) - \hat{p}$ whenever $\hat{p} > p$, for all $v \in [L, H]$ such that $b(v, \hat{p}) \geq \hat{p}$.*

Proof. To show property (i), fix an arbitrary $p \in [L, H]$. Because the right-hand side of (4.4) is continuously differentiable in v and b (except at $v = H$), by the mean value theorem we can write, for $v < H$,

$$b_1(v, p) = \frac{u'(\xi)(b(v, p) - v)}{u'(\varphi(b(v, p) - p))\varphi'(b(v, p) - p)} \frac{f(v)}{1 - F(v)},$$

where $\xi \rightarrow \varphi(H - p)$ as $v \uparrow H$ (hence $b(v, p) \rightarrow H$). Since the above expression has a 0/0 form at $v = H$, by L'Hospital's rule and taking limit as $v \uparrow H$ yields

$$\lim_{v \uparrow H} b_1(v, p) = \frac{f(H)}{\varphi'(H - p)} \lim_{v \uparrow H} \frac{b(v, p) - v}{1 - F(v)} = \frac{1 - \lim_{v \uparrow H} b_1(v, p)}{\varphi'(H - p)}.$$

This allows us to denote by $b_1(H, p)$ the value of the continuous extension of $b_1(v, p)$ at $v = H$:

$$b_1(H, p) = \lim_{v \uparrow H} b_1(v, p) = \frac{1}{1 + \varphi'(H - p)}. \quad (4.6)$$

From (4.4) it can be seen that for all $v \in [L, H]$, $b_1(v, p) > 0$ is equivalent to $b(v, p) > v$. We now apply Lemma 3(i) to $h(v) \equiv b(v, p) - v$. Since $h(H) = 0$ and $h'(H) = b_1(H, p) - 1 < 0$ (see (4.6)), we have $h(v) > 0$ on (\bar{v}, H) for some $\bar{v} < H$. If $h(v) = 0$ for some $v \leq \bar{v}$, then (4.4) implies $b_1(v, p) = 0 < 1$ so that $h'(v) < 0$. Lemma 3(i) now implies $h(v) > 0$ and therefore $b_1(v, p) > 0$ on $\mathcal{D}(p)$. Clearly, by the arbitrariness of p this property holds for all $p \in [L, H]$.

To show property (ii), fix any $\hat{p} > p$. We apply Lemma 3(i) to $h(v) \equiv b(v, p) - p - (b(v, \hat{p}) - \hat{p})$. Because $b(H, p) = b(H, \hat{p})$, by continuity $b(v, p) - b(v, \hat{p})$ is arbitrarily close to zero for v sufficiently close to H . Thus, there exists $\bar{v} < H$ such that $h > 0$ on $(\bar{v}, H]$. Now suppose $h(v) = 0$ for some $v \leq \bar{v}$. Then $\varphi(b(v, p) - p) = \varphi(b(v, \hat{p}) - \hat{p})$ and $b(v, \hat{p}) > b(v, p)$. Hence,

$$\begin{aligned} & b_1(v, \hat{p}) \\ = & \frac{u(\varphi(b(v, p) - p)) - u(v - b(v, \hat{p}) + \varphi(b(v, p) - p))}{u'(\varphi(b(v, p) - p))\varphi'(b(v, p) - p)} \frac{f(v)}{1 - F(v)} \\ > & \frac{u(\varphi(b(v, p) - p)) - u(v - b(v, p) + \varphi(b(v, p) - p))}{u'(\varphi(b(v, p) - p))\varphi'(b(v, p) - p)} \frac{f(v)}{1 - F(v)} \\ = & b_1(v, p) \end{aligned}$$

or $h'(v) < 0$. Since $h(\bar{v}) \geq 0$, Lemma 3(i) now implies $h(v) > 0$ or $b(v, \hat{p}) - \hat{p} < b(v, p) - p$ for all $v \in [L, H]$. We conclude that $b(v, p) - p$ is a strictly decreasing function of p for all $v \in [L, H]$. ■

In addition to the stated results, from its proof we can see that Lemma 4 has also established an intuitive property that $b(v, p) > v$ for all $v < H$, i.e., the premium induces all bidders to bid higher than their true values except the one who has the highest possible value H .

Proof of Theorem 4.

We show by backward induction that b is an EPA-SE if and only if for all $X \in [L, H]$, $b(v, X)$ satisfies (4.4)-(4.5) on $\mathcal{D}(X)$.

Suppose b is an EPA-SE. We start with the second stage, assuming that the first stage ends with a bottom price $X \in [L, H]$ and that the current second-stage price level implies a screening level $r \geq r(X)$. Because $b(r, X) = p$ and $b(v, X)$ strictly increases in v , both finalists must have values higher than or equal to r if they have followed the equilibrium strategy until now. Clearly, $(v, b) \in \mathcal{D}(X)$ for $v \geq r$. Using the same arguments of Maskin and Riley (1984; p. 1485-1486), it can be readily shown that the equilibrium condition (4.3) implies that $b(\cdot, X)$ is continuous on $[r, H]$ and differentiable on $[r, H)$.

Now for all $v, z \in [r, H)$, differentiating $\bar{U}(v, z|r, b(\cdot, X))$ (see (4.2)) with respect to z gives

$$\begin{aligned} \bar{U}_2(v, z|r, b(\cdot, X)) = & \\ & \frac{f(z)}{1 - F(r)} [u(v - b(z, X) + \varphi(b(z, X) - X)) - u(\varphi(b(z, X) - X))] \\ & + \frac{1 - F(z)}{1 - F(r)} u'(\varphi(b(z, X) - X)) \varphi'(b(z, X) - X) b_1(z, X). \end{aligned} \quad (4.7)$$

Because the right-hand side of the above expression strictly increases in v ,

$$\bar{U}_2(v, v|r, b(\cdot, X)) = 0 \text{ implies } \bar{U}_2(v, z|r, b(\cdot, X)) \begin{cases} > 0 \text{ if } z < v \\ = 0 \text{ if } z = v \\ < 0 \text{ if } z > v \end{cases} .$$

As $z \uparrow H$, the term in (4.7) vanishes (because $\bar{U}_2(v, v|r, b(\cdot, X)) = 0$ implies $\lim_{v \uparrow H} b_1(v, X) < \infty$, as shown in (4.6)). We then have

$$\begin{aligned} \bar{U}_2(v, H|r, b(\cdot, X)) &= \frac{f(H)}{1 - F(r)} \times \\ & [u(H - b(H, X) + \varphi(b(H, X) - X)) - u(\varphi(b(H, X) - X))] \\ & \lesseqgtr 0 \text{ for } b(H, X) \gtrless H. \end{aligned}$$

Since $X \in [L, H)$ is arbitrary, we conclude that (4.3) holds if and only if for all $X \in [L, H)$, $b(v, X)$ satisfies (4.5) and (4.4) for all $v \in [r(X), H)$ (which is equivalent to $(v, b) \in \mathcal{D}(X)$). This completes the necessity part of the proof (and the sufficiency part of the proof for stage two).

Now consider the decision of a bidder with value v in the first stage. Suppose that b satisfies (4.4)-(4.5), and that the bidder computes his expected second-stage utility using b in (4.1). The question is: "If the bidder becomes a finalist at the current price p , does he expect a positive utility in the second stage?" Clearly, as long as $b(v, p) > p$ so that $v > r(p)$, where $r(p)$ derives from $b(r(p), p) = p$, it is a dominant strategy to stay because $U(v|r(p), b(\cdot, p)) > U(r(p)|r(p), b(\cdot, p)) = 0$. Once the price reaches

the level where $b(v, p) = p$, then staying becomes a (weakly) dominated strategy, because it leads to a higher bottom price than p . It makes no difference in expected utility if the bidder can quit later in the first stage. But if the bidder quits too late and becomes a finalist, with bottom price $X > p$, he must then bid as if his value is higher than v in the second stage. This implies that his expected utility will be non-positive. (The best he might then do is to bid the bottom price X . But this is weakly dominated by quitting earlier: in case the other finalist also bids the bottom price, the random resolution of the tie could allocate the object to the bidder for too high a price, without any compensating premium.) We conclude that b constitutes an EPA-SE, having now completed the sufficiency part of the proof for both stages. ■

It is worth noting that n does not appear in (4.4)-(4.5). This suggests that, like standard private values English (or Vickrey) auctions, in an EPA the bidders in fact need not know the exact number of bidders – it suffices that each bidder only knows that there are some (more than one) other bidders competing in the first stage. Another observation is that the bid function is independent of any screening level r , which suggests that the possibility of updating the screening level will not affect bidding behavior. This observation lies at the bottom of the argument that the English (premium) auction and the Vickrey (premium) auction are strategically equivalent when only two bidders remain.

The next theorem concerns the existence and uniqueness of an EPA-SE. Because the right-hand side of differential equation (4.4) is undefined at $v = H$ (let alone Lipschitzian at this boundary point), we cannot directly apply the fundamental theorem of ordinary differential equations for a (unique) solution. The existence of a solution of (4.4)-(4.5) can be readily established, however, by employing standard techniques from ordinary differential equations theory. On the other hand, we can also look for sufficient conditions that directly guarantee the existence of an EPA-SE. As shown in Athey (2001), the set of sufficient conditions for the existence of equilibria in a large class of games of incomplete information includes a

single crossing condition as proposed in Milgrom and Shannon (1994). In our context, the Milgrom-Shannon single crossing condition holds as long as $\bar{U}(v, z|r, b(\cdot, X))$ is supermodular in (v, z) (e.g., Athey (2001)).⁷ We take this (shorter) approach in Theorem 5, verify that $\bar{U}(v, z|r, b(\cdot, X))$ is supermodular, and further show that the EPA-SE is unique under the assumptions (A1)-(A2).

Theorem 5 (Existence and uniqueness of EPA-SE) *For any utility function u and premium function φ , (i) there exists an EPA-SE $b : [L, H]^2 \rightarrow [L, H]$ and (ii) if the assumptions (A1)-(A2) hold, then b is unique.*

Proof. (Existence) By Theorem 4, it suffices to establish the existence of a second-stage equilibrium $b(\cdot, X)$ in an EPA for all $X \in [L, H]$. Theorem 4 also implies that there is no loss of generality to restrict attention to differentiable and strictly increasing bid functions in search of an EPA-SE. The case with $X = H$ is trivial, since it is then common knowledge that both finalists have values equal to H and $b(H, H) = H$. Now suppose $X \in [L, H)$ and that the opponent of a finalist adopts an increasing and differentiable bid function $b(\cdot, X)$. Then this finalist's expected utility at any screening level $r \geq r(X)$ is given by $\bar{U}(v, z|r, b(\cdot, X))$, where v is the bidder's true value and z determines his bid $b(z, X)$. Since $b(z, X)$ is continuous and strictly increasing in z , without ambiguity we can treat z as the bidder's "action." It follows from (4.2) that

$$\begin{aligned} & \bar{U}_{12}(v, z|r, b(\cdot, X)) \\ &= \frac{f(z)}{1 - F(r)} u'(v - b(z, X) + \varphi(b(z, X) - X)) > 0. \end{aligned}$$

This inequality implies that \bar{U} is supermodular in (v, z) (Topkis (1978)). Thus, by Athey (2001; Corollary 2.1), there exists an increasing second-stage equilibrium $b(\cdot, X)$ of EPA. (It is easy to verify that other assumptions of Athey's Corollary 2.1 are satisfied in our context.) Since X is ar-

⁷See also Milgrom and Weber (1982) for the general definition of supermodularity.

bitrary, this proves that the bid function $b(\cdot, X)$ exists for all $X \in [L, H]$. By Theorem 4, $b(\cdot, X)$ is necessarily a solution of (4.4)-(4.5) whenever $b(v, X) \in [X, H)$, and, moreover, $b_1 > 0$ and $b(v, X) - X$ strictly decreases in X . These properties further imply that b meets the criteria of the first-stage equilibrium of the EPA (see the proof of Theorem 4). The existence of an EPA-SE is thus established.

(Uniqueness) Now assume that (A1)-(A2) hold. Let b be a solution of (4.4)-(4.5). By Theorem 4, $b(v, X) \in (v, H]$. Fix any $X \in [L, H)$ and $v \in [r(X), H)$. Then, the right-hand side of (4.4) strictly increases in b for $v < b \leq H$ as can be seen from

$$\begin{aligned} & \frac{\partial}{\partial b} \frac{u(\varphi(b-X)) - u(v-b+\varphi(b-X))}{u'(\varphi(b-X))\varphi'(b-X)} \\ = & 1 + \frac{u'(v-b+\varphi)\frac{1-\varphi'}{\varphi'}}{u'(\varphi)} \\ & - \frac{u(\varphi) - u(v-b+\varphi)}{u'(\varphi)\varphi'} \left(\frac{u''(\varphi)\varphi'}{u'(\varphi)} + \frac{\varphi''}{\varphi'} \right) \end{aligned} \tag{4.8}$$

$$\begin{aligned} & > 1 - \frac{u(\varphi) - u(v-b+\varphi)}{u'(\varphi)} \frac{u''(\varphi)}{u'(\varphi)} \quad (\text{by } 0 < \varphi' \leq \frac{1}{2} \text{ and (A1)}) \\ & \geq 0 \quad (\text{by (A2)}) \end{aligned} \tag{4.9}$$

Suppose there exists another solution of (4.4)-(4.5) for some $X \in [L, H)$, say, $\hat{b}(v, X)$. Then we fix this X and apply Lemma 3(ii) to $h(v) \equiv b(v, X) - \hat{b}(v, X)$. We have $h(H) = 0$. From (4.4) and (4.9), it is easy to see that $h(v) \leq 0 \Rightarrow h'(v) \leq 0$ for all $v \in [L, H)$. Thus $h \geq 0$ on $[L, H]$. However, this logic applies also to $-h$, which implies $h \leq 0$ on $[L, H]$. We therefore conclude that $b(v, X) = \hat{b}(v, X)$ so that the solution of (4.4)-(4.5) is necessarily unique on $\mathcal{D}(X)$, which is equivalent to that the EPA-SE is unique.

■

Let us consider an example in which the bidders have constant absolute risk aversion (CARA). As we do not restrict attention to risk averse bidders, the Arrow-Pratt measure $-u''/u'$ can be positive (risk averse) or negative (risk loving).

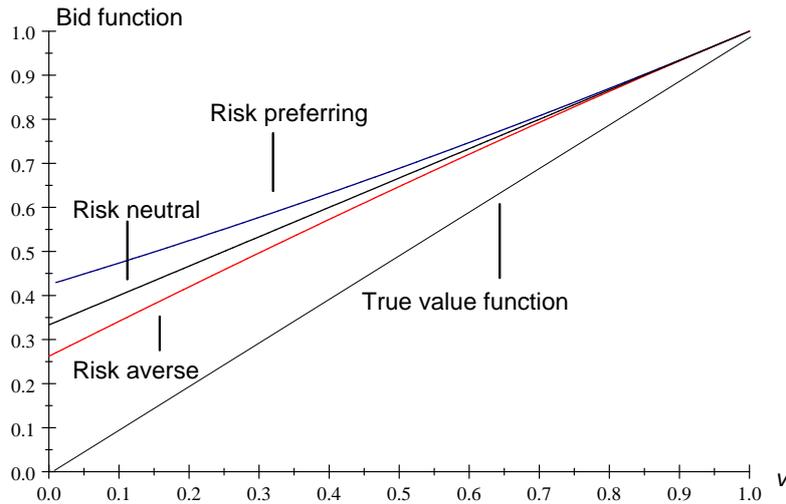


FIGURE 4.1. The premium induces the buyers to bid higher than their true values. The more risk tolerant the buyers are, the higher will be their bids. Here, F is assumed to be uniform on $[0, 1]$ and $\alpha = 0.5$. The risk averse bid function assumes $\lambda = 3$, and the risk seeking bid function assumes $\lambda = -3$.

Example. Suppose that the bidders have utility function

$$u(x) = \frac{1 - \exp(-\lambda x)}{\lambda}, \quad \lambda \in \mathbb{R}.$$

Suppose further that φ is linear, with $\varphi(x) = \alpha x$ for some constant $\alpha \in (0, 1/2]$. The differential equation (4.4) then reduces to

$$b'_\lambda(v) = \frac{\exp(\lambda(b_\lambda(v) - v)) - 1}{\alpha\lambda} \frac{f(v)}{1 - F(v)} \tag{4.10}$$

where we observe that the bid function $b(v, X) \equiv b_\lambda(v)$ is independent of X .

The differential equation (4.10) can be solved explicitly to yield the EPA-SE. To see this, rearranging terms in (4.10), and multiplying both sides by $\exp(-\lambda b_\lambda(v))$, we obtain

$$\begin{aligned} & \alpha \lambda b'_\lambda(v) (1 - F(v)) \exp(-\lambda b_\lambda(v)) + f(v) \exp(-\lambda b_\lambda(v)) \\ &= f(v) \exp(-\lambda v) \end{aligned}$$

Now multiply both sides of the above equation by $(1 - F(v))^{\frac{1}{\alpha}-1}$ to get

$$-\frac{\partial}{\partial v} \left(\alpha (1 - F(v))^{\frac{1}{\alpha}} \exp(-\lambda b_\lambda(v)) \right) = (1 - F(v))^{\frac{1}{\alpha}-1} f(v) \exp(-\lambda v)$$

Then, integrating and rearranging terms yields the desired closed-form solution:

$$b_\lambda(v) = -\frac{1}{\lambda} \ln \left(\frac{1}{\alpha} \int_v^H \frac{e^{-\lambda x}}{1 - F(x)} \left(\frac{1 - F(x)}{1 - F(v)} \right)^{\frac{1}{\alpha}} dF(x) \right)$$

Figure 4.1 depicts the bid functions of the risk averse, risk neutral, and risk preferring bidders. The figure confirms that the premium, in general, induces the bidders to bid higher than their true values. It also shows that the bids are uniformly higher (lower) if the bidders are more risk tolerant (averse).

4.4 The Premium Effects

We now investigate how the premium will affect the expected payment and the players' expected utilities. Let us start with a standard English auction (EA) without any premium, in which it is a (weakly) dominant strategy for each bidder to remain in the auction until the price reaches his true value v . When there are only two bidders left in an EA, who will also be the two finalists in an EPA, the current screening level r is equal to the current price p . The conditional expected utility at the screening

level r of each remaining bidder in the EA then equals

$$\begin{aligned}
 W(v|r) &\equiv \frac{1}{1-F(r)} \int_r^v u(v-y) dF(y) \\
 &= \frac{1}{1-F(r)} \int_r^H u(\max(v-y, 0)) dF(y) \\
 &= E[u(\max(v-y, 0))|r],
 \end{aligned} \tag{4.11}$$

where y denotes the opponent's possible value, and the expectation $E(\cdot|r)$ is taken with respect to the conditional cumulative distribution of y , i.e., $[F(y) - F(r)] / [1 - F(r)]$.

Consider next an EPA with a premium function φ . Suppose the first stage is over and the bottom price is X . At the start of the second-stage EPA, both finalists know that their values are higher or equal to $r(X)$ from $b(r(X), X) = X$. In order to highlight the effects of the premium, let us define a "gamble" $\Phi(\cdot|v, X)$ conditional on v and X . The payoff of Φ depends on the realization of $y \in [r(X), H]$ as follows:⁸

$$\Phi(y|v, X) = \begin{cases} y - b(y, X) + \varphi(b(y, X) - X) & \text{if } y \in [r(X), v] \\ \varphi(b(v, X) - X) & \text{if } y \in (v, H] \end{cases} . \tag{4.12}$$

It can be the case that Φ is discontinuous at $y = v$, although it only occurs with a zero probability in the present model.

⁸If only the highest losing bidder receives the premium, then replacing the gamble Φ by Φ^0 , defined as follows, will give the same predictions as in the subsequent theorems:

$$\Phi^0(y|v, X) = \begin{cases} y - b(y, X) & \text{if } y \in [r, v] \\ \varphi(b(v, X) - X) & \text{if } y \in (v, H] \end{cases}$$

From (4.1), at any screening level $r \geq r(X)$ the conditional expected utility of a finalist can now be written as

$$\begin{aligned} & U(v|r, b(\cdot, X)) \\ = & \frac{1}{1 - F(r)} \int_r^v u(v - y + \Phi(y|v, X)) dF(y) \quad (4.13) \\ & + \frac{1 - F(v)}{1 - F(r)} u(\varphi(b(v, X) - X)) \end{aligned}$$

$$\begin{aligned} = & \frac{1}{1 - F(r)} \int_r^H u(\max(v - y, 0) + \Phi(y|v, X)) dF(y) \\ = & E[u(\max(v - y, 0) + \Phi(y|v, X))|r]. \quad (4.14) \end{aligned}$$

Comparing the expected utility U in (4.14) to the expected utility W in (4.11) for $r = r(X)$, we find that the “gamble” Φ is entirely due to the premium offered, with the special case $\Phi \equiv 0$ corresponding to the EA.

Our next theorem establishes a *net-premium* effect in the EPA, which shows that as long as the second-stage EPA continues, the current screening level r reveals each finalist’s conditional equilibrium expected utility for Φ —that is, when Φ is evaluated in *isolation* of other random payoffs. This result holds for arbitrary utility function u and premium function φ , and is independent of the bidder’s private values. As can be seen from the proof of this theorem, the net-premium effect is essentially an “envelope theorem effect” as a consequence of incentive compatibility, which reduces to a revenue equivalence result in the present context when the bidders are risk neutral.

Theorem 6 (Net-premium effect) *For any utility function u and premium function φ , the second-stage EPA equilibrium implies that for all $X \in [L, H)$, $r \in [r(X), H]$, and $v \in [r, H]$,*

$$E[u(\Phi(y|v, X))|r] = u(\varphi(b(r, X) - X)). \quad (4.15)$$

Proof. Let b be an EPA-SE. In the second stage of the EPA, for any bottom price X and screening level $r \geq r(X)$, the conditional expected

utility of a finalist who has value v and who bids as though his value is z equals $\bar{U}(v, z|r, b(\cdot, X))$ (see (4.2)). Differentiating \bar{U} with respect to v gives

$$\begin{aligned} & \bar{U}_1(v, z|b(\cdot, X)) \\ = & \frac{1}{1 - F(r)} \int_r^z u'(v - b(y, X) + \varphi(b(y, X) - X)) dF(y). \end{aligned}$$

Because \bar{U} is maximized at $z = v$ and $U(r|r, b(\cdot, X)) = u(\varphi(b(r, X) - X))$, incentive compatibility and the envelope theorem imply

$$\begin{aligned} & U(v|r, b(\cdot, X)) \\ = & U(r|r, b(\cdot, X)) + \int_r^v \bar{U}_1(z, z|b(\cdot, X)) dz \\ = & U(r|r, b(\cdot, X)) \\ & + \frac{1}{1 - F(r)} \int_r^v \int_r^z u'(z - b(y, X) + \varphi(b(y, X) - X)) dF(y) dz. \end{aligned}$$

Interchanging the order of integration we obtain

$$\begin{aligned} & U(v|r, b(\cdot, X)) \\ = & U(r|r, b(\cdot, X)) \\ & + \frac{1}{1 - F(r)} \int_r^v \int_y^v u'(z - b(y, X) + \varphi(b(y, X) - X)) dz dF(y) \\ = & u(\varphi(b(r, X) - X)) \\ & + \frac{1}{1 - F(r)} \int_r^v u(v - b(y, X) + \varphi(b(y, X) - X)) dF(y) \\ & - \frac{1}{1 - F(r)} \int_r^v u(y - b(y, X) + \varphi(b(y, X) - X)) dF(y). \quad (4.16) \end{aligned}$$

On the other hand, $U(v|r, b(\cdot, X))$ has a direct expression given in (4.1). Thus, subtracting (4.1) from (4.16) yields

$$\begin{aligned} & E[u(\Phi(y|v, X))|r] \\ = & \frac{1 - F(v)}{1 - F(r)} u(\varphi(b(v, X) - X)) \\ & + \frac{1}{1 - F(r)} \int_r^v u(y - b(y, X) + \varphi(b(y, X) - X)) dF(y) \\ = & u(\varphi(b(r, X) - X)). \end{aligned}$$

■

The net-premium effect is useful for gaining insight into the competitive bidding behavior in the English premium auctions. Notice that the right-hand side of (4.15) equals the bidder's utility for the premium if he quits at the current screening level r . As long as the bidder has value $v > r$, however, he will have no incentive to quit because staying in the auction gives him the same level of expected utility for the premium. In addition, from (4.14) we can see that there is an additional $\max(v - y, 0)$ to be possibly gained in case the opponent has a value $y \in [r, v)$. In equilibrium, this reasoning is common knowledge, and it therefore offers both finalists the comfort to sit back and relax, watching their premium grow up with the price. It is also common knowledge that this bidding process will continue until one of the bidders no longer expects to win.

Another useful implication of the net-premium effect is that at the time when the first stage has just ended with a bottom price X , both finalists derive a conditional expected utility for the premium that must be equal to zero. This is because $b(r(X), X) = X$ and thus at the screening level $r(X)$, $E[u(\Phi(y|v, X))|r(X)] = 0$. A special case is where the bidders are risk neutral; then, the net-premium effect reduces to an equivalent statement of the revenue equivalence principle (e.g., Myerson (1981)) that $E[\Phi(y|v, X)|r(X)] = 0$. It should be stressed, however, that the net-premium effect is an *isolated* premium effect. Only in the special case of risk neutrality does the effect imply that the bidders will be indifferent

about the premiums. In general, the premium will affect the expected utilities of both risk averse and risk loving bidders in an EPA.⁹

Just as the revenue equivalence principle offers a useful tool for comparing welfare implications of various auction policies, the net-premium effect offers a handy tool for the subsequent analysis of the premium effects on the expected revenue and expected utilities of the seller and bidders in an EPA.

In what follows, unless specified otherwise we let $E(\cdot)$ denote the expectation under distribution $[1 - F(v)] / [1 - F(r)]$, where r is the screening level derived from the bottom price at the start of the second stage of the EPA.

Theorem 7 *For arbitrary number $n (> 2)$ of the bidders, and for arbitrary premium function φ , the expected revenue in any EPA-SE is lower (higher) when the bidders are more risk averse (loving).*

Proof. Let \hat{u} be another utility function satisfying the same assumptions as u , with an absolute risk aversion measure satisfying $-\hat{u}''/\hat{u}' > -u''/u'$ at all relevant income levels. Let \hat{b} and \hat{X} denote the bid function and the bottom price when the bidders' preferences are represented by \hat{u} , and define $\hat{\Phi}$ similar to Φ as in (4.12).

Let r denote the third highest value from among the n bidders' values. It is clear that in an EPA equilibrium, r will be the screening level at the start of the second stage that is independent of the utility functional forms. (The bottom price at which the first stage ends can be different as the utility function changes.)

Denote by R and \hat{R} the conditional expected payment of a finalist entering the second stage who has utility function u and \hat{u} , respectively. We

⁹This follows from the simple fact that for non-quasi-linear utility functions, in general

$$E[u(\max(v - y, 0) + \Phi(y|v, X)) | r] \neq E[u(\max(v - y, 0)) | r] + E[u(\Phi(y|v, X)) | r]$$

have

$$\begin{aligned}
& R(v|X) \\
&= \frac{1}{1-F(r)} \int_r^v [b(y, X) - \varphi(b(y, X) - X)] dF - \frac{1-F(v)}{1-F(r)} \varphi(b - X) \\
&= \frac{1}{1-F(r)} \left(\int_r^v [y - \Phi(y|v, X)] dF - (1-F(v)) \varphi(b(v, X) - X) \right) \\
&\hat{R}(v|\hat{X}) \\
&= \frac{1}{1-F(r)} \left(\int_r^v [y - \hat{\Phi}(y|v, \hat{X})] dF - (1-F(v)) \varphi(\hat{b}(v, \hat{X}) - \hat{X}) \right)
\end{aligned}$$

Subtracting gives

$$\hat{R}(v|\hat{X}) - R(v|X) = E(\Phi(y|v, X)) - E(\hat{\Phi}(y|v, \hat{X})).$$

We know from Theorem 6 that

$$E\hat{u}(\hat{\Phi}(y|v, \hat{X})) = Eu(\Phi(y|v, X)) = 0.$$

Since \hat{u} is more risk averse than u , the above equations imply

$$E(\Phi(y|v, X)) - E(\hat{\Phi}(y|v, \hat{X})) < 0$$

and hence $\hat{R}(v|\hat{X}) < R(v|X)$. Because the bidders are symmetric ex ante, this implies the conclusion of the theorem straightforwardly. ■

The reason why revenue decreases in the bidders' risk aversion can be seen from the expressions of R and \hat{R} : each finalist's conditional expected payment is the difference of the expected value of his opponent (in the event of winning) and the expected premium. Since the former has nothing to do with the utility functional forms, this difference is solely explained by the difference in the expected premiums. It follows then from the net-premium effect that the more risk averse bidders will command more expected premium in equilibrium, resulting in a lower expected payment.

The next corollary is an immediate consequence of Theorem 7.

Corollary 2 *Given any number $n (> 2)$ of the bidders, adding a premium φ to an EA increases the expected revenue when bidders are risk loving, and decreases the expected revenue when bidders are risk averse.*

Proof. In an EA, for arbitrary utility function u , each bidder bids up to his true value v . This strategy leads to the same expected revenue in an EA when bidders are risk neutral. By the revenue equivalence theorem, the expected revenue is also the same in an EPA where the bidders are risk neutral. Therefore, by Theorem 7, the expected revenue under any premium φ is higher (lower) than that without a premium when bidders are risk loving (averse). ■

A straightforward implication of this corollary is that the problem of designing the “optimal premium function” that maximizes expected revenue, with the number of participants given, involves only a corner solution when the bidders are risk averse. In this case, the optimal premium function should be a constant zero.¹⁰

The results obtained so far are quite general, as they hold without much restriction regarding the shape of the distribution, premium, and utility functions (except for the uniqueness of the EPA-SE). The next two theorems require the assumptions (A1)-(A3). As shown in Theorem 5, these assumptions imply that the EPA-SE is unique. We first present a lemma that will be useful for Theorems 8-9.

Lemma 5 *Suppose that the assumptions (A1)-(A3) hold. Then, in the (unique) EPA-SE, $[1 - \varphi'(b(v, X) - X)]b_1(v, X) < 1$ for all $X \in [L, H)$ and $v \in (r(X), H)$.*

Proof. Fix $X \in [L, H)$. We apply Lemma 3(i) to

$$h(v) = 1 - [1 - \varphi'(b(v, X) - X)]b_1(v, X).$$

¹⁰If the buyers are risk preferring, the revenue maximizing premium function may not exist unless we impose some functional structure on φ . For instance, if we restrict φ to be linear such that $\varphi(b - X) = \alpha(b - X)$, where the constant α is the seller's choice variable that is restricted to be no greater than $1/2$, then it can be shown that $\alpha = 1/2$ maximizes expected revenue.

Differentiating gives

$$h'(v) = -(1 - \varphi') b_{11} + \varphi'' b_1^2.$$

Since $h(H) = 1 - \frac{1 - \varphi'(H-X)}{1 + \varphi'(H-X)} > 0$ and $\varphi'' \leq 0$ (by (A1)), it suffices to show that at any $v \in (r(X), H)$, $h(v) = 0$ implies $b_{11} > 0$ and hence $h'(v) < 0$. Differentiating b_1 gives

$$\begin{aligned} b_{11}(v, X) &= \frac{u(\varphi) - u(v - b + \varphi)}{u'(\varphi)\varphi'(b - X)} \left(\frac{f(v)}{1 - F(v)} \right)' \\ &+ \frac{u'(\varphi)\varphi' b_1 - u'(v - b + \varphi)(1 - (1 - \varphi') b_1)}{u'(\varphi)\varphi'} \frac{f(v)}{1 - F(v)} \\ &- \frac{u(\varphi) - u(v - b + \varphi)}{u'(\varphi)\varphi'} \left(\frac{u''(\varphi)}{u'(\varphi)} \varphi' + \frac{\varphi''}{\varphi'} \right) b_1 \frac{f(v)}{1 - F(v)}. \end{aligned}$$

By Lemma 4 and (A3), the first term on the right-hand side of this equation is strictly positive. If $(1 - \varphi') b_1 = 1$ at some $v \in (L, H)$, then substituting into the above equation we have

$$\begin{aligned} &b_{11}(v, X) \\ &> \left[1 - \frac{u(\varphi) - u(v - b + \varphi)}{u'(\varphi)\varphi'} \left(\frac{u''(\varphi)}{u'(\varphi)} \varphi' + \frac{\varphi''}{\varphi'} \right) \right] b_1 \frac{f(v)}{1 - F(v)} \\ &\geq \left[1 - \frac{u(\varphi) - u(v - b + \varphi)}{u'(\varphi)} \frac{u''(\varphi)}{u'(\varphi)} \right] b_1 \frac{f(v)}{1 - F(v)} \quad (\text{by (A1)}) \\ &\geq 0 \quad (\text{by (A2)}) \end{aligned}$$

Lemma 3(i) then implies that $h > 0$, and hence for all $v \in (r(X), H]$, we have $(1 - \varphi'(b(v, X) - X)) b_1(v, X) < 1$. ■

The role of this lemma is to establish that the “gamble” $\Phi(y|v, X)$ is an increasing function of y . An important implication of this property is that the premium reduces the riskiness of payment and therefore the riskiness of revenue.

Theorem 8 *Suppose that the assumptions (A1)-(A3) hold. Then, adding a premium φ to an EA increases the expected utility of the risk averse bidders, and decreases the expected utility of the risk loving bidders.*

Proof. Since the bidders who drop out in the first stage of an EPA have a zero expected utility regardless of any premium, we focus on the two finalists' conditional expected utilities at the start of the second stage. Fix v and X , and define $A(\theta|v, X) = Eu(\max(v - y, 0) + \theta\Phi(y|v, X))$ for $\theta \in [0, 1]$. Differentiating A with respect to θ gives

$$A'(\theta|v, X) = E[u'(\max(v - y, 0) + \theta\Phi(y|v, X))\Phi(y|v, X)].$$

Differentiate Φ with respect to $y \in (r(X), v)$ and $y \in (v, H]$, respectively, gives

$$\begin{aligned} & \Phi'(y|v, X) \\ = & \begin{cases} 1 - [1 - \varphi'(b(y, X) - X)]b_1(y, X) > 0 & \text{if } y \in (r(X), v) \\ 0 & \text{if } y \in (v, H] \end{cases} . \end{aligned}$$

where the inequality follows from Lemma 5. It is also easy to see that $\max(v - y, 0) + \theta\Phi(y|v, X)$ is a decreasing function of y for $\theta \in [0, 1]$. Now by Theorem 6, $Eu(\Phi) = 0$ implies that $E(\Phi) > 0$ if $u'' < 0$ and $E(\Phi) < 0$ if $u'' > 0$. Consequently,

$$\begin{aligned} & A'(\theta|v, X) \\ & \begin{cases} > E[u'(\max(v - y, 0) + \theta\Phi(y|v, X))] E(\Phi) > 0 & \text{if } u'' < 0 \\ < E[u'(\max(v - y, 0) + \theta\Phi(y|v, X))] E(\Phi) < 0 & \text{if } u'' > 0 \end{cases} . \end{aligned}$$

where the first two inequalities follow from the established fact that u' and Φ are positively (negatively) correlated when $u'' < 0$ ($u'' > 0$). Because $A(0|v, X) = W(v|r(X))$ and $A(1|v, X) = U(v|b(\cdot, X))$, we obtain

$$U(v|b(\cdot, X)) \begin{cases} > W(v|r(X)) & \text{if } u'' < 0 \\ < W(v|r(X)) & \text{if } u'' > 0 \end{cases} .$$

■

Intuitively, because $\Phi(y|v, X)$ and $-y$ are negatively correlated, adding a premium to an English auction reduces the risk of payment. In addition,

for risk averse bidders the expected value of Φ is positive. These two effects are both favorable and hence the EPA is more attractive to risk averse bidders than the EA. The opposite holds for risk loving bidders.

Back to the Example considered in Section 4.3, we see that the bid functions depicted in Figure 4.1 confirm the results of Theorem 8. Using the risk neutral bidders' bid function as the reference, we see that the premium induces the risk loving bidders to bid "too high." Thus, the risk lovers will pay the seller a higher expected "net price" for the premium, resulting in lower expected utilities in comparison with the no-premium case (where they bid the true values). Likewise, the risk averse bidders bid "too low," and hence the seller's expected revenue is lower, and the bidders are uniformly better off with, rather than without, a premium.

Our last theorem extends the result of Theorem 8 to the case where the seller may be risk averse or risk loving.

Theorem 9 *Suppose that the assumptions (A1)-(A3) hold. Then, adding a premium φ to an English auction decreases a risk loving seller's expected utility if the bidders are (weakly) risk averse, and increases a risk averse seller's expected utility if the bidders are (weakly) risk loving.*

Proof. Let u_S denote the seller's utility function for income, and let $v^{(2)}$ and r denote the second and third highest values from among the n bidders, respectively. W.l.o.g. we normalize $u_S(0) = 0$. In either an EA or an EPA, r is revealed as soon as there are two bidders remain. Hence, we focus on the beginning of the second stage expected utilities conditional on r . The seller's utility is then uniquely determined by the realized value of $v^{(2)}$. Conditional on knowing $v^{(2)} \geq r$ in the second stage, the density function of $v^{(2)}$ is $2 [1 - F(v^{(2)})] f(v^{(2)}) / [1 - F(r)]^2$. It follows, $E(\cdot)$ denotes the expectation taken with respect to this density function.

In an EA when only two bidders remain, the seller's conditional expected utility is thus

$$E(u_S|\text{EA}) = \frac{2}{[1 - F(r)]^2} \int_r^H u_S(v^{(2)}) [1 - F(v^{(2)})] dF(v^{(2)}).$$

Likewise, in the second stage of an EPA the seller's conditional expected utility for the total net payment equals

$$\begin{aligned} & E(u_S | \text{EPA}) \\ = & \frac{2}{[1 - F(r)]^2} \times \\ & \int_r^H u_S(b(v^{(2)}, X) - 2\varphi(b(v^{(2)}, X) - X)) [1 - F(v^{(2)})] dF(v^{(2)}) \\ = & \frac{2}{[1 - F(r)]^2} \int_r^H u_S(v^{(2)} + \Psi(v^{(2)})) [1 - F(v^{(2)})] dF(v^{(2)}), \end{aligned}$$

where $\Psi(v^{(2)}) = b(v^{(2)}, X) - 2\varphi(b(v^{(2)}, X) - X) - v^{(2)}$. Corollary 2 implies that the expected value of Ψ satisfies

$$E[\Psi(v^{(2)})] \begin{cases} \geq 0 & \text{if } u'' \geq 0 \\ \leq 0 & \text{if } u'' \leq 0 \end{cases}.$$

By Lemma 5, $(1 - \varphi')b_1 < 1$. Thus, for $\theta \in [0, 1]$,

$$\begin{aligned} \Psi'(v^{(2)}) &= b_1(v^{(2)}, X)(1 - 2\varphi') - 1 < 0, \\ [v^{(2)} + \theta\Psi(v^{(2)})]' &= 1 + \theta [b_1(v^{(2)}, X)(1 - 2\varphi') - 1] > 0. \end{aligned}$$

Now define $B(\theta|v^{(2)}) = E[u_S(v^{(2)} + \theta\Psi(v^{(2)}))]$. Similar to the proof of Theorem 8, we have

$$\begin{aligned} B'(\theta|v^{(2)}) &= E[u'_S(v^{(2)} + \theta\Psi(v^{(2)})) \Psi(v^{(2)})] \\ & \begin{cases} < E[u'_S(v^{(2)} + \theta\Psi(v^{(2)}))] E(\Psi) \leq 0 & \text{if } u'' \leq 0 \text{ and } u''_S > 0 \\ > E[u'_S(v^{(2)} + \theta\Psi(v^{(2)}))] E(\Psi) \geq 0 & \text{if } u'' \geq 0 \text{ and } u''_S < 0 \end{cases} \end{aligned}$$

Consequently, if $u'' \leq 0$ and $u''_S > 0$, the seller prefers to choose the EA (i.e., $\theta = 0$), and if $u'' \geq 0$ and $u''_S < 0$, the seller prefers to choose the EPA (i.e., $\theta = 1$). ■

The premium effect on the seller's expected utility is ambiguous if both the seller and bidders are either simultaneously risk averse or simultaneously risk loving. By a continuity argument, however, it can happen that

when all players are risk averse, the seller also prefers the EPA to the EA as long as he is sufficiently more risk averse than the bidders. This follows from the strict preference of the seller for the EPA when the bidders are risk neutral, in which case the premium effect in reducing the revenue risk is predominant for the seller. Conversely, when all players are risk loving, it can happen that neither the seller nor the bidders would like to have the premium practice provided that the bidders are close to risk neutral.

4.5 Concluding Discussion

This chapter has studied a general English premium auction (EPA) model in a symmetric private values setting. The existence and uniqueness of the symmetric equilibrium for the class of EPA is established, along with some in-depth analyses of the effects of premium in relation to the bidders' risk preferences. When the premium is viewed as an additional "gamble" to an otherwise standard English auction, a remarkable "net-premium" effect emerges from our study. This effect implies that whatever the premium function is specified prior to the auction, and whatever is the bidders' risk preferences, the equilibrium expected utility for the premium, if calculated at the start of the second stage and in isolation of other random payoffs, must be equal to zero. This result considerably simplifies our comparative statics analysis, highlighting the reason why the premium enhances revenue if the bidders are risk loving, and the bidders prefer to have a premium if they are risk averse.

Under plausible conditions (the assumptions (A1)-(A3)), we find in Lemma 5 that the premium, in general, reduces the riskiness of the payment in the English auction. This result generalizes a similar finding in Goeree and Offerman (2004) that the premium reduces the variance of payment, calculated under a uniform distribution and linear premium rule. When the bidders exhibit constant absolute risk aversion (not necessarily risk averse), we also derive a closed-form solution for the equilibrium bid function for arbitrary distribution functions.

We conclude from this study that a seller facing ex ante symmetric bidders may consider a premium auction in two general situations. The first situation is where the seller is risk averse and where he has some good reason to believe that the bidders are approximately risk neutral or, at least, not “too” risk averse. In this situation the premium will play a (marginally) positive role in attracting entry, while at the same time reducing revenue risk. As long as the benefit of risk reduction outweighs the potential cost of a lower expected revenue, the premium auction will be preferred by both the seller and the buyers. The second situation arises where the seller is approximately risk neutral, where he is not concerned about entry and where he believes that the bidders will behave like risk seekers. Then the premium induces the bidders to overbid, resulting in a higher expected revenue. As long as the seller is not “too” risk loving himself, he will derive a higher expected utility in a premium auction.

The model presented in this chapter has assumed that the buyers have independent private values. This may be a reasonable assumption when the auctioned good is for private consumption. In other situations, allowing the bidders to have affiliated information and interdependent values will be more adequate (e.g., Milgrom and Weber, 1982). A natural extension of the present study is then to examine the potential effects of the premium tactics in the Milgrom-Weber general symmetric model with risk averse or risk loving players. Another line of extension is to endogenize the entry decision of the potential bidders when they face certain costs to participate in the auction.

5

Conclusion

The various results obtained in this thesis suggest that auction design is not a matter of “one size fits all” (Klemperer, 2002). The relative performance of different auction policies will be context-dependent, and any type of auction can perform better or worse, from either the seller’s or the buyers’ viewpoint, than another in a different context. Thus, “art is nearly always auctioned off according to the English rules, whereas job contracts are normally awarded through sealed bids.” (Maskin and Riley, 2000). This “context dependency” also reminds us to pay more attention to the context details while modelling auctions. For example, the previous studies commonly compare different auctions under the same reserve price. This assumption may be valid, in general, only if the seller has no ability to commit to a reserve price higher than his own value prior to an auction.

In other situations, as shown in Chapter 2, endogenizing the seller’s choice of reserve prices could lead to drastically different conclusions. Chapter 2 has analyzed the effects of buyer and seller risk aversion in first and second-price auctions in the classic setting of symmetric and

independent private values. The seller's optimal reserve price has been shown to decrease in his own risk aversion, and more so in the first-price auction. The reserve price also decreases in the buyers' risk aversion in the first-price auction. Thus, greater risk aversion increases ex post efficiency in both auctions—especially that of the first-price auction. At the interim stage, the first-price auction is preferred by all buyer types in a lower interval, as well as by the seller.

Another example of “context dependency” concerns the use of premiums in auctions. In a premium auction, the seller pays a cash reward to a number of highest bidders according to some pre-specified rule. The premium is believed to encourage participation in the auction and to enhance competition among the bidders. However, as shown in Chapter 4, whether or not the premium will increase the seller's (or the buyers') expected utility critically depends on the risk preferences of the participants. Moreover, the possibility of bidder collusion, as studied in Chapter 3, can also make the premium auction more attractive to the seller than other standard auctions.

Collusion is often a primary concern for the seller in practice. If the bidders can manage to form a cartel and act like a single bidder, they can seriously harm the seller's revenue. Some auction experts (e.g., Klemperer, 2002) even believe that collusion and other competition policy related issues, like predation and entry deterrence, are more relevant for practical auction design than risk-aversion, affiliation of signals, or budget-constraints. To the least, as case law shows that collusion in auctions is not just a theoretical possibility (e.g., Krishna, 2002), and as competition law enforcement does not seem to sufficiently deter bidders from collusion in some circumstances, it makes more sense “to create an environment that discourages collusion in the first place than trying to prove unlawful behavior afterwards.” Motta (2004). Chapter 3 has demonstrated the desirability of premium auctions when it comes to fighting collusion.

This thesis has been focusing on the independent private values auctions only. It can be expected that some results obtained so far are extendable to

the informationally more general settings, whereas some are not. Indeed, for the cases where bidders have interdependent values and affiliated signals (Milgrom and Weber, 1982), how the bidders' risk preferences would affect the seller's choice of reserve prices remains an open issue that deserves serious studies in the future (e.g., Levin and Smith, 1996). In some other cases, the object for sale may carry substantial *ex post* risk (e.g., a business license, a mineral right, or a troubled bank). More research is needed as to understand how risk and risk attitudes of the bidders interact (e.g., Eso and White, 2004). Incorporating heterogeneity in bidders' risk preferences is another important area for future research.

6

Summary

This thesis studies the roles of reserve price and premium tactics in a number of standard and non-standard auctions, extending the existing literature to the cases where the participants may be risk averse or risk preferring, and where the bidders may have the possibility to collude. The main contributions are Chapters 2 , 3 and 4, summarized as follows.

Chapter 2 focuses on the effects of buyer and seller risk aversion on the seller's optimal reserve price in standard Dutch or first-price auctions (FPA) and English or second-price auctions (SPA). Sharp results are obtained by restricting attention to the otherwise simplest setting, that of symmetric and independent private values. It is shown that when the seller and/or the buyers are risk averse, the seller's optimal reserve price will be lower in the FPA than in the SPA. Risk aversion thus makes the FPA, in general, more ex post efficient than the SPA. In either auction, a more risk averse seller will set a lower reserve price. Thus, the more risk averse the seller, the more ex post efficient are both auctions. In addition, the seller sets a lower reserve price in the FPA if the bidders are more risk averse. The general conclusion of this chapter is that risk aversion can be

a disguised blessing in terms of ex post efficiency, because it induces the seller to lower the reserve price and leads to a higher probability that the object is allocated to the one who values it most.

Chapter 3 examines how premium auctions may deter bidder collusion. The main idea is that a premium auction may discourage “strong” bidders (e.g., those who have a serious interest in acquiring the object) to form a cartel, because “weak” bidders (e.g., “fortune hunters”) can be attracted to the auction in quest of the premium and bid aggressively to spoil the potential profits of the cartel. The collusive properties of the first-price, English, and English premium auctions (EPA) are derived and then investigated using a laboratory experiment. The experiment confirms the theoretical prediction that the EPA is less conducive to collusion than the other auction formats. The EPA is therefore likely to outperform the English and first-price auctions in generating a higher expected revenue.

Chapter 4 develops a theory of the EPA for the canonical case in which risk averse or risk loving bidders with symmetric private values compete. The aim of this chapter is to sharpen and enrich the current understanding about the premium auctions, as these have been studied by far only under the assumption of risk neutrality. The chapter establishes the existence and uniqueness of the EPA symmetric equilibrium, and shows that, in general, the premium reduces the riskiness of revenue and induces all bidders to bid higher than their values. However, the *net* expected revenue in the premium auction strictly decreases in the bidders’ risk aversion. These results suggest that in the symmetric private values settings, revenue maximization is not likely to be the seller’s motive for the use of premium auctions when the bidders are risk averse. Instead, reducing revenue risk and encouraging more entry are plausible reasons for the use of premium tactics in practice, as the EPA is always more attractive to the risk averse buyers than the English auction.

7

Samenvatting (Summary in Dutch)

Dit proefschrift bestudeert de tactische inzet van bodemprijzen (reserve prices) en premies in enkele standaard en niet-standaard veilingtypen en breidt de bestaande literatuur op dit gebied uit tot die gevallen waarbij deelnemers (verkopers en bidders) risicomijdend of risicozoekend kunnen zijn en bidders de mogelijkheid hebben samen te spannen. De belangrijkste bijdragen van dit proefschrift zijn te vinden in de hoofdstukken 2, 3, en 4, hieronder samengevat.

Hoofdstuk 2 richt zich op de gevolgen van risicomijding bij bidders en verkopers op de optimale bodemprijs in de standaard Hollandse of eerste-prijs veiling en de Engelse of tweede-prijs veiling. De auteur heeft duidelijke resultaten kunnen afleiden door haar aandacht te beperken tot een eenvoudig model met symmetrische en onafhankelijke private waarderingen. Zo wordt aangetoond dat indien de verkoper en/of bidders risicomijdend zijn de voor de verkoper optimale bodemprijs lager ligt voor een eerste-prijs veiling dan voor een tweede-prijs veiling. Risicomijding maakt daarmee in het algemeen de eerste-prijs veiling ex-post efficiënter dan de tweede-prijs veiling. Bij beide veilingen zal een risicomijdende verkoper

een lagere bodemprijs vaststellen. Daaruit volgt dat hoe risicomijdender de verkoper is, des te efficiënter ex post beide veilingen zijn. Daarnaast bepaalt de verkoper een lagere bodemprijs in de eerste-prijs veiling als de bieders meer risicomijdend zijn. De algemene conclusie van dit hoofdstuk is dat risicomijding een verkapte zegen kan zijn met betrekking tot ex post efficiëntie, omdat het verkopers aanzet om de bodemprijs te verlagen en leidt tot een hogere kans dat het te veilen object terecht komt bij degene die er de hoogste waardering voor heeft.

Hoofdstuk 3 onderzoekt hoe premieveilingen bieders kunnen ontmoedigen samen te spanning. Het centrale idee is dat een premieveiling 'sterke' bieders (zij die serieus in het te veilen object zijn geïnteresseerd) kan ontmoedigen om een kartel te vormen omdat 'zwakke' bieders (zoals gelukzoekers), aangetrokken door de premie, agressief bieden en zo de winsten van het kartel drukken. Eerst is theoretisch geanalyseerd welke van drie veilingtypen (eerste-prijs veiling, de Engelse veiling en de Engelse premie veiling) het meest kwetsbaar is voor samenspanning. Vervolgens werd dit onderzocht in een laboratoriumexperiment. Het experiment bevestigt de theoretische voorspelling dat de Engelse premieveiling minder gevoelig is voor samenspanning dan de andere veilingtypen. Het is daarom waarschijnlijk dat een Engelse premieveiling tot hogere verwachte opbrengsten zal leiden dan een standaard Engelse veiling en een eerste-prijs veiling.

Hoofdstuk 4 ontwikkelt een theorie voor een Engelse premieveiling met risicomijdende dan wel risicozoekende bieders met symmetrische private waarderingen. Het doel van dit hoofdstuk is om het begrip van premieveilingen aan te scherpen en te verrijken, omdat deze tot nu toe voornamelijk zijn bestudeerd onder de vooronderstelling van risiconeutraliteit. Het hoofdstuk bewijst het bestaan en de uniciteit van een symmetrisch evenwicht voor de Engelse premieveiling en laat zien dat, in het algemeen, een premie de risicograad van de opbrengsten vermindert en alle bieders aanzet meer te bieden dan hun waardering. Desondanks zijn de netto verwachte opbrengsten van een premieveiling strikt dalend in de risicomijding van de bieders. Deze resultaten suggereren dat in een opzet met symmetrische

private waarderingen wanneer bieders risicomijdend zijn, het maximaliseren van de opbrengsten waarschijnlijk niet de reden is voor het gebruik van premies. In plaats hiervan zijn het verminderen van de risicograad van de opbrengsten en het stimuleren van toetreding wel plausibele redenen om premies in de praktijk toe te passen, omdat de Engelse premieveiling altijd aantrekkelijker is voor een risicomijdende koper dan de standaard Engelse veiling.

8

Bibliography

1. Abdulkadiroğlu, A. and Chung, K.S. (2003). “Auction design with tacit collusion.” Working paper, Columbia University.
2. Armstrong, M. (1996). “Multiproduct nonlinear pricing.” *Econometrica* 64, 51-75.
3. Aoyagi, M. (2003). “Bid rotation and collusion in repeated auctions.” *Journal of Economic Theory* 112, 79-105.
4. Aoyagi, M. (2007). “Efficient collusion in repeated auctions with communication.” *Journal of Economic Theory* 134, 61-92.
5. Athey, S. (2001). “Single crossing properties and the existence of pure strategy equilibria in games of incomplete information.” *Econometrica* 69, 861-889.
6. Ayres, I. and Cramton, P. (1996). “Pursuing deficit reduction through diversity: How affirmative action at the FCC increased auction competition.” *Stanford Law Review* 48, 761-815.

7. Blume, A. and Heidhues, P. (2008). "Modeling tacit collusion in auctions." *Journal of Institutional and Theoretical Economics* 164, 163-184.
8. Boone, J., Chen, R., Goeree, J.K., and Polydoro, A. (2009). "Risky procurement with an insider bidder." *Experimental Economics* 12, 417-436.
9. Che, Y.-K. and Kim, J. (2006). "Robustly collusion-proof implementation." *Econometrica* 74, 1063-1107.
10. Che, Y.-K. and Kim, J. (2009). "Optimal collusion-proof auctions." *Journal of Economic Theory* 144, 565-603.
11. Cox, J.C., Smith, V.L., and Walker, J. M. (1982). "Auction market theory of heterogeneous bidders." *Economics Letters* 9, 319-325.
12. Cox, J.C., Smith, V.L., and Walker, J. M. (1988). "Theory and individual behavior of first-price auctions." *Journal of Risk and Uncertainty* 1, 61-99.
13. Engelbrecht-Wiggans, R. (1993). "Optimal auctions revisited." *Games and Economic Behavior* 5, 227-239.
14. Eso, P. and White, L. (2004). "Precautionary bidding in auctions." *Econometrica* 72, 77-92.
15. Eso, P. and Futo, G. (1999). "Auction design with a risk averse seller." *Economics Letters* 65, 71-74.
16. Goeree, J.K. and Offerman, T. (2004). "The Amsterdam auction." *Econometrica* 72, 281-294.
17. Goswami, G., Noe, T., and Rebello, M. (1996). "Collusion in uniform-price auctions: Experimental evidence and implications for treasury auctions." *Review of Financial Studies* 9, 757-785.

18. Graham, D.M., and Marshall, R.C. (1987). "Collusive bidder behavior at single-object second-price and English auctions." *Journal of Political Economy* 95, 1217-1239.
19. Hale, J. K. (2009). *Ordinary Differential Equations*. Mineola, N.Y. Dover Publications. (Originally published: New York: Wiley, 1969)
20. Hamaguchi, Y., Ishikawa, T., Ishimoto, M., Kimura, Y., and Tanno, T. (2007). "An experimental study of procurement auctions with leniency programs." Competition Policy Research Center Discussion Paper, Atomi University.
21. Harris, M. and Raviv, A. (1981). "Allocation mechanisms and the design of auctions." *Econometrica* 49, 1477-1499.
22. Holt, C.A. (1980). "Competitive bidding for contracts under alternative auction procedures." *The Journal of Political Economy* 88, 433-445.
23. Hu, A., Matthews, S.A., and Zou, L. (2010). "Risk aversion and optimal reserve prices in first and second-price auctions." *Journal of Economic Theory* 145, 1188-1202.
24. Hu, A., Offerman, T., and Onderstal, S. (2010). "Fighting collusion in auctions: An experimental investigation." *Journal of Industrial Organization*, forthcoming.
25. Isaac, M.R. and Walker, J. (1985). "Information and conspiracy in sealed bid auctions." *Journal of Economic Behavior and Organization* 6, 139-159.
26. Jeon, D.S. and Menicucci, D. (2005). "Optimal second-degree price discrimination and arbitrage: On the role of asymmetric information among buyers." *RAND Journal of Economics* 36, 337-360.

27. Kagel, J.H. (1995). "Auctions: A survey of experimental research." In: J.H. Kagel and A.E. Roth (eds.), *The Handbook of Experimental Economics*, Princeton, NJ, Princeton University Press, 501-585.
28. Kagel, J.H. and Levin, D. (2001). "Behavior in multi-unit demand auctions: experiments with uniform price and dynamic Vickrey auctions." *Econometrica*, 69, 413-54.
29. Kagel, J.H. and Levin, D. (2005). "Multi-unit demand auctions with synergies: Behavior in sealed-bid vs. ascending-bid uniform price auctions." *Games and Economic Behavior* 53, 170-207.
30. Klemperer, P. (2002). "What really matters in auction design." *Journal of Economic Perspectives* 16, 169-190.
31. Klemperer, P. (2004). *Auctions: Theory and Practice*. Princeton, NJ, Princeton University Press.
32. Krishna, V. (2002). *Auction Theory*. San Diego, Academic Press.
33. Krishna, V. (2009). *Auction Theory*. 2nd Ed. San Diego, Academic Press.
34. Krishna, V. and Maenner, E. (2001). "Convex potentials with an application to mechanism design." *Econometrica* 69, 1113-1119.
35. Kwasnica, A. (2000). "The choice of cooperative strategies in sealed bid auctions." *Journal of Economic Behavior and Organization* 42, 323-346.
36. Kwasnica, A. and Sherstyuk, K. (2007). "Collusion and equilibrium selection in auctions." *Economic Journal* 117, 120-145.
37. Laffont, J. and Martimort, D. (1997). "Collusion under asymmetric information." *Econometrica* 65, 875-911.

38. Laffont, J. and Martimort, D. (2000). "Mechanism design with collusion and correlation." *Econometrica* 68, 309-342.
39. Levin, D. and Smith, J. L. (1994). "Equilibrium in auctions with entry." *The American Economic Review* 84, 585-599.
40. Lind, B. and Plott, C.R. (1991). "The winner's curse: Experiments with buyers and with sellers." *The American Economic Review* 81, 335-46.
41. Marshall, R.C. and Marx, L.M. (2007). "Bidder collusion." *Journal of Economic Theory* 133, 374-402.
42. Maskin, E. and Riley, J. (1983). "The gains to making losers pay in high bid auctions." In R. Engelbrecht-Wiggans, M. Shubik, and R. Stark (eds.), *Auctions, Bidding, and Contracting*, New York University Press, 205-230.
43. Maskin, E. and Riley, J. (1984). "Optimal auctions with risk averse buyers." *Econometrica* 52, 1473-1518.
44. Maskin, E. and Riley, J. (2000). "Asymmetric auctions." *Review of Economic Studies* 67, 413-438.
45. Maskin, E. and Riley, J. (2003). "Uniqueness of equilibrium in sealed high-bid auctions." *Games and Economic Behavior* 45, 395-409.
46. Matthews, S.A. (1980). "Risk aversion and the efficiency of first and second price auctions." Mimeo. Northwestern University.
47. Matthews, S.A. (1983). "Selling to risk averse buyers with unobservable tastes." *Journal of Economic Theory* 30, 370-400.
48. Matthews, S.A. (1987). "Comparing auctions for risk averse buyers: A buyer's point of view." *Econometrica* 55, 633-646.

49. McAfee, R.P. and McMillan, J. (1987). "Auctions with entry." *Economic Letters* 23, 343-347.
50. McAfee, R.P. and McMillan, J. (1992). "Bidding rings." *American Economic Review* 82, 579-599.
51. Milgrom, P. (2004). *Putting Auction Theory to Work*. Cambridge, UK, Cambridge University Press.
52. Milgrom, P. and Segal, I. (2002). "Envelope theorems for arbitrary choice sets." *Econometrica* 70, 583-601.
53. Milgrom, P. and Shannon, C. (1994). "Monotone comparative statics." *Econometrica* 62, 157-180.
54. Milgrom, P. and Weber, R. J. (1982). "A Theory of auctions and competitive bidding." *Econometrica* 50, 1089-1122.
55. Motta, M. (2004). *Competition Policy*. Cambridge, UK, Cambridge University Press.
56. Myerson, R.B. (1981). "Optimal auction design." *Mathematics of Operations Research* 6, 58-73.
57. Pesendorfer, M. (2000). "A study of collusion in first-price auctions." *Review of Economic Studies* 67, 381-411.
58. Phillips, O.R., Menkhaus, D.J., and Coatney, K.T. (2003). "Collusive practices in repeated English auctions: Experimental evidence on bidding rings." *American Economic Review* 93, 965-979.
59. Porter, R. and Zona, D. (1993). "Detection of bid rigging in procurement auctions." *Journal of Political Economy* 101, 518-538.
60. Porter, R. and Zona, D. (1999). "Ohio school milk markets: An analysis of bidding." *RAND Journal of Economics* 30, 263-288.

61. Pratt, J.W. (1964). "Risk aversion in the small and in the large." *Econometrics* 32, 122-136.
62. Riley, J.G. and Samuelson, W.F. (1981). "Optimal auctions." *The American Economic Review* 71, 381-392.
63. Robinson, M.S. (1985). "Collusion and the choice of auction." *RAND Journal of Economics* 16, 141-145.
64. Sade, O., Schnitzlein, C., and Zender, J.F. (2006). "Competition and cooperation in divisible good auctions: An experimental examination." *Review of Financial Studies* 19, 195-235.
65. Scherer, F.M. (1980). *Industrial Market Structure and Economic Performance*. 2nd Ed. Chicago, Rand-McNally.
66. Sherstyuk, K. (1999). "Collusion without conspiracy: An experimental study of one-sided auctions." *Experimental Economics* 2, 59-75.
67. Sherstyuk, K. (2002). "Collusion in private value ascending price auctions." *Journal of Economic Behavior and Organization* 48, 177-195.
68. Skrzypacz, A. and Hopenhayn, H. (2004). "Tacit collusion in repeated auctions." *Journal of Economic Theory* 114, 153-169.
69. Smith, J.L. and Levin, D. (1996). "Ranking auctions with risk averse bidders." *Journal of Economic Theory* 68, 549-561.
70. Stiglitz, J.E. and Weiss, A. (1981). "Credit rationing in markets with imperfect information." *The American Economic Review* 71, 393-410.
71. Topkis, D. (1978). "Minimizing a submodular function on a lattice." *Operations Research* 26, 305-321.

72. Vickrey, W. (1961). "Counterspeculation, auctions and competitive sealed tenders." *Journal of Finance* 16, 8-37.
73. Waehrer, K., Harstad, R.M., and Rothkopf, M.H. (1998). "Auction form preferences of risk-averse bid takers." *The RAND Journal of Economics* 29, 179-192.
74. Wilson, R. (1987). "Bidding." In: Eatwell, J., Milgate, M., and Newman, P. (eds.), *The New Palgrave: A Dictionary of Economics*. London: MacMillan Press.

The Tinbergen Institute is the Institute for Economic Research, which was founded in 1987 by the Faculties of Economics and Econometrics of the Erasmus University Rotterdam, University of Amsterdam and VU University Amsterdam. The Institute is named after the late Professor Jan Tinbergen, Dutch Nobel Prize laureate in economics in 1969. The Tinbergen Institute is located in Amsterdam and Rotterdam. The following books recently appeared in the Tinbergen Institute Research Series:

436. S.T. TRAUTMANN, *Uncertainty in Individual and Social Decisions: Theory and Experiments.*
437. R. LORD, *Efficient pricing algorithms for exotic derivatives.*
438. R.P. WOLTHOFF, *Essays on Simultaneous Search Equilibrium.*
439. Y.-Y. TSENG, *Valuation of travel time reliability in passenger transport.*
440. M.C. NON, *Essays on Consumer Search and Interlocking Directorates.*
441. M. DE HAAN, *Family Background and Children's Schooling Outcomes.*
442. T. ZAVADIL, *Dynamic Econometric Analysis of Insurance Markets with Imperfect Information.*
443. I.A. MAZZA, *Essays on endogenous economic policy.*
444. R. HAIJEMA, *Solving large structured Markov Decision Problems for perishable-inventory management and traffic control.*
445. A.S.K. WONG, *Derivatives in Dynamic Markets.*
446. R. SEGERS, *Advances in Monitoring the Economy.*
447. F.M. VIEIDER, *Social Influences on Individual Decision Making Processes.*
448. L. PAN, *Poverty, Risk and Insurance: Evidence from Ethiopia and Yemen.*
449. B. TIEBEN, *The concept of equilibrium in different economic traditions: A Historical Investigation.*
450. P. HEEMEIJER, *Expectation Formation in Dynamic Market Experiments.*
451. A.S. BOOIJ, *Essays on the Measurement Sensitivity of Risk Aversion and Causal Effects in Education.*
452. M.I. LÓPEZ YURDA, *Four Essays on Applied Micro econometrics.*
453. S. MEENTS, *The Influence of Sellers and the Intermediary on Buyers' Trust in C2C Electronic Marketplaces.*
454. S. VUJIĆ, *Econometric Studies to the Economic and Social Factors of Crime.*
455. F. HEUKELOM, *Kahneman and Tversky and the Making of Behavioral Economics.*
456. G. BUDAI-BALKE, *Operations Research Models for Scheduling Railway Infrastructure Maintenance.*
457. T.R. DANIËLS, *Rationalised Panics: The Consequences of Strategic Uncertainty during Financial Crises.*
458. A. VAN DIJK, *Essays on Finite Mixture Models.*
459. C.P.B.J. VAN KLAVEREN, *The Intra-household Allocation of Time.*
460. O.E. JONKEREN, *Adaptation to Climate Change in Inland Waterway Transport.*

461. S.C. GO, *Marine Insurance in the Netherlands 1600-1870, A Comparative Institutional Approach*.
462. J. NIEMCZYK, *Consequences and Detection of Invalid Exogeneity Conditions*.
463. I. BOS, *Incomplete Cartels and Antitrust Policy: Incidence and Detection*
464. M. KRAWCZYK, *Affect and risk in social interactions and individual decision-making*.
465. T.C. LIN, *Three Essays on Empirical Asset Pricing*.
466. J.A. BOLHAAR, *Health Insurance: Selection, Incentives and Search*.
467. T. FARENHORST-YUAN, *Efficient Simulation Algorithms for Optimization of Discrete Event Based on Measure Valued Differentiation*.
468. M.I. OCHEA, *Essays on Nonlinear Evolutionary Game Dynamics*.
469. J.L.W. KIPPERSLUIS, *Understanding Socioeconomic Differences in Health An Economic Approach*.
470. A. AL-IBRAHIM, *Dynamic Delay Management at Railways: A Semi-Markovian Decision Approach*.
471. R.P. FABER, *Prices and Price Setting*.
472. J. HUANG, *Education and Social Capital: Empirical Evidences from Microeconomic Analyses*.
473. J.W. VAN DER STRAATEN, *Essays on Urban Amenities and Location Choice*.
474. K.M. LEE, *Filtering Non Linear State Space Models: Methods and Economic Applications*.
475. M.J. REINDERS, *Managing Consumer Resistance to Innovations*.
476. A. PARAKHONYAK, *Essays on Consumer Search, Dynamic Competition and Regulation*.
477. S. GUPTA, *The Study of Impact of Early Life Conditions on Later Life Events: A Look Across the Individual's Life Course*.
478. J. LIU, *Breaking the Ice between Government and Business: From IT Enabled Control Procedure Redesign to Trusted Relationship Building*.
479. D. RUSINOVA, *Economic Development and Growth in Transition Countries*.
480. H. WU, *Essays on Top Management and corporate behavior*.
481. X. LUI, *Three Essays on Real Estate Finance*
482. E.L.W. JONGEN, *Modelling the Impact of Labour Market Policies in the Netherlands*
483. M.J. SMIT, *Agglomeration and Innovations: Evidence from Dutch Microdata*
484. S.VAN BEKKUM, *What is Wrong With Pricing Errors? Essays on Value Price Divergence*