The foundation, planning and building of new towns in the 13th and 14th centuries in Europe : an architectural-historical research into urban form and its creation

Boerefijn, W.N.A.

Citation for published version (APA):
Boerefijn, W. N. A. (2010). The foundation, planning and building of new towns in the 13th and 14th centuries in Europe : an architectural-historical research into urban form and its creation
6 THE USE OF GEOMETRY IN THE DESIGN OF NEW TOWN PLANS IN THE HIGH-PERIOD OF TOWN FOUNDATION

As became clear in the previous chapters, the layout of newly founded towns of the 12th to 14th centuries was largely planned. A crucial question is, of course, how these towns were planned, or more particularly, how they were designed. Unfortunately, there are barely sources from the time of the creation of the towns that inform us on the process of their planning. From the few written sources we know a little about the organisation and execution, but almost nothing about the design stage: how and why specific forms and dimensions were chosen.

Since the middle of the mid-20th century, however, various hypothetical theories concerning the design of the town plans have been forwarded in the scholarly literature. Some of these theories clearly contradict one another, so the subject remains very much open to debate. Therefore, a number of the proposed theories will be discussed in this chapter in order to present an overview of the proposed methods of design and to analyse whether or not they are likely to really have been used in the planning of towns. In particular, it regards theories of geometric and arithmetic methods of design for regular town plans.

Relatively close attention will be given to theories on the design of the plans of the bastide of Grenade-sur-Garonne and the terre nuove fiorentine, since these towns form part of the groups of towns that were studied in the first three chapters, and because they are interesting examples of different hypothesised methods of plan design that have been given relatively much attention in the scholarly literature of the past decades. These cases are also interesting because they serve very well to contrast the two different basic methods of design: complex geometric design and arithmetic (or ‘simple geometric’) design. In particular the terre nuove have come to stand in the spotlights as subject of debate over these methods in urban planning, as many different theories have been proposed for their design. These towns will therefore be treated extensively, in order to create some clarity in the confusion of the various theories on this particular subject.1 (par.6.4)

This chapter is particularly concerned with regular town plans. As already pointed out in chapters 1 and 2, there were also newly founded towns with plans that can hardly or not be characterised as ‘regular’. The irregularities in these plans were generally caused by convenient adaptation to the specific circumstances of the landscape. These towns will barely be considered here, since this chapter is specifically concerned with the more abstract sort of design that took place in the mind of the planner and the subsequent translation of this design into reality. Such abstract design may also have been part of the planning of irregular layouts, but unfortunately that is very hard or even impossible to reconstruct from those layouts with the current methods.2

The chapter will open with short introductions of the two basic possible methods of design, the one using arithmetical proportions and the other one using complex geometric constructions. (pars.6.1 and 6.2) After that a number of different hypotheses concerning specific town plans will be critically discussed (par.6.3). Then, a relatively large part of the chapter will treat different hypotheses that have been proposed by different authors for the design of the terre nuove plans (par.6.4.1) and the analysis on which of these is most likely (6.4.2).3 Subsequently it will be discussed how this design method possibly worked, what it was inspired by, and why it was used. (pars.6.4.3-6.4.4) The chapter ends with an analysis of how many misunderstandings on the subject could have come about and a short general conclusion. (pars.6.5, 6.6)

---

1 According to Fernie, such a comparative study of different proposed design methods for the same structure is what is presently needed in order to bring the discussion on the application of geometry in ‘medieval’ architectural design to a next level. (Fernie 2002, p.9) Fernie explicitly refers to buildings; the discussion is, however, equally relevant to urban design.

2 Only in par.6.3.4 irregular plans will be considered. Critics might oppose that irregularity was a crucial element in ‘medieval’ aesthetics, and that therefore these plans particularly ought to be examined with respect to design principles. In pars.8.6 and 11.1 however, it will be demonstrated that the theory of irregularity as an aesthetic principle in ‘the middle ages’ is a romantic misconception dating from the 19th century.

The creation of irregular elements in urban plans will be regarded in par.9.6.2.

3 In appendix B the analysis is described in more detail, with all the relevant dimensions given in tables, as well as the calculated percentile differences between measurements and theoretical dimensions.
6.1 Arithmetic design of urban plans by use of rational dimensions, and reconstruction of original town plans by metrological analysis

In most of the plans of towns that were newly founded in the 12th to 14th centuries, one can recognise an inclination towards regularity and orthogonality. It is obvious that there must have been a very basic idea of geometric order behind this, involving straight lines, right angles, equality of distances and, quite often, even symmetry. In many cases, it is clear that these straight lines, right angles and equal distances were not very precisely set out as such, but that in the basic idea of the plan structure in the minds of the planners these elements must have been present.

Ever since the late 19th century, scholars have tried to reconstruct the original designs of new town plans from the high-period of town foundation. Various scholars claim that plans which are more or less regular have been laid out by use of very simple geometry, by setting out straight boundaries at right angles with regular dimensions that were determined as rational, mostly rounded, numbers of the then current units of measurement. Below, this will be designated as ‘arithmetic design’ or ‘simple geometric design’.

In order to reconstruct the original plans that were set out in this way, scholars have used the method of ‘metrological analysis’. This method involves taking measurements of the relevant dimensions in the plans, either from maps or from the actual built form, in order to identify the dimensions as they were originally set out. If it appears that the dimensions can be consistently identified as rational numbers of the unit of measurement that was used in the region of the town and its time of origin, they are taken for the dimensions that were actually set out as such originally. By use of this method, reconstructions have been drawn up of supposed original plans of various new towns from the 12th to 15th centuries.4 (fig.6.1) The method of

---

metrical analysis is, however, not infallible. The reliability of the results varies with the accuracy of the analysis, depending among others on the accuracy of measuring of the relevant dimensions\(^5\) and on whether or not there are clear indications about the dimensions of the then current units of measurement.\(^6\) Below, the method of ‘metrical analysis’ is used for the reconstruction of the method of design for the original plans of Grenade-sur-Garonne and the terre nuove.\(^7\)

With the reconstruction of the supposed original dimensions from measured dimensions, there mostly is a difference between the measured dimensions and the ‘theoretical dimensions’ that is tolerated as deviation. Different scholars who made such reconstructions, have tolerated different amounts of deviation. It is obvious that the larger the relative deviation (the percentile difference between measured and reconstructed values) that is tolerated for a reconstruction, the less likely it generally is to be correct. With this, one should take in account, however, that there always was a difference between the dimensions as they were meant (the design) and as they were set out (the layout). And this difference would probably only increase in the course of time, as streets, buildings, fences or ditches were changed or renewed. Consequently, there necessarily is a difference between what was originally intended, what was originally realised and what is measured at present. Hence, there always is a grey area of deviation that has to be accepted: the tolerance. The point is, however, that it is impossible to draw a solid fact-based limit of what is acceptable in this respect, wherefore it remains debatable whether reconstructed dimensions are plausible or not. This is even more so when it is not factually known what the original unit of measurement exactly was. This is often the case, as even standard units of measurement, such as the foot, varied with place and time, and sometimes even with the use it was employed for.\(^8\)

6.2 Design of urban plans by complex geometry

There is also another general theory, which is largely conflicting with the theory of arithmetic design. This alternative theory is based on the idea that urban plans were designed by use of more or less complex geometric methods. The use of this complex geometry for the design is not directly visible in the form of the plan structures, but mostly it is supposed to have been the basis of their dimensions.\(^9\)

It is almost generally accepted that in ‘the middle ages’, geometry was one of the basic instruments in architectural design in general. At least since the 19\(^{th}\) century, scholars have suggested that regular geometric figures or geometric adaptations were used to create specific forms and to determine proportions in the design of ‘medieval’ architecture.

There are contemporary documents that support this idea.\(^10\) (fig.6.2) But there is still no consensus about how, to what extent and, more importantly for the general history of culture, for what reason geometry would have ruled architectural design.\(^11\) Since the 19\(^{th}\) century, scholars have made reconstructions of hypothetical geometric design systems of nearly all the major architectural monuments of the past, overlaying drawings of plans and elevations with complex geometric figures and systems which are believed to have determined the form, place and dimensions of the main elements of the designs.\(^12\)

---

\(^5\) It can make a big difference, for instance, whether the measurements are taken in reality or from a plan drawing. In the latter case the accuracy of the plan is, of course, crucial. The reliability of the method may also vary with the amount of measurements taken: the width of a street, for instance, often varies along its course, for which reason it is preferable to measure it at a number of different places and to take the average as presumable original dimension.

\(^6\) Cf. Schich 1993, p.108.

\(^7\) See pars.6.3.3, 6.4.2 and app.B.

\(^8\) The length of the foot could vary from c. 27.5 to c.34 cm. (Hecht 1971/2, pp.89-91)

\(^9\) It can make a big difference, for instance, whether the measurements are taken in reality or from a plan drawing. In the latter case the accuracy of the plan is, of course, crucial. The reliability of the method may also vary with the amount of measurements taken: the width of a street, for instance, often varies along its course, for which reason it is preferable to measure it at a number of different places and to take the average as presumable original dimension.

\(^10\) One of these sources, for instance, is the 15\(^{th}\) century tractate Geometria Deutsch by Mathias Roriczer, master mason of Regensburg. (Hecht 1983) (see fig.6.2)

\(^11\) In general: Binding 1985; Binding 1993; Bucher 1979; Nareidi-Rainer 1982; Shelby 1983; pp.209-212. The interpretation of many of these sources is still a matter of debate. (Hecht 1959-1971; Surdell 1993) The only sources that are very clear about the use of geometry, for proportioning elements of gothic churches and designing decorations, are from the 15\(^{th}\) and 16\(^{th}\) centuries, written in Germany. (Hecht 1983, pp.214-215) One of these sources, for instance, is the 15\(^{th}\) century tractate Geometria Deutsch by Mathias Roriczer, master mason of Regensburg. (Shelby 1983) (see fig.6.2)

\(^12\) There are also some earlier examples of reconstructed geometric design methods. Most famous among these, and possibly also the very first one, is Cesariano’s reconstruction of the design of Milan cathedral from 1521. (Hecht 1983, pp.244-245; 1970, pp.121-122, 165)
6.3 Various theories of complex design geometry in the plans of newly created towns

In the following paragraphs the more interesting theories of design by way of complex geometric methods, and especially those that regard towns discussed in the first three chapters of this study, will be examined. The first examples of such theories will be treated only briefly. Since I could not get hold of accurate plans of the towns in question, I have not been able to thoroughly check the hypothesised design systems. However, despite this lack of exhaustive examination, more superficial analysis has led to a number of relevant comments.

13 Hecht 1969, pp.272-309; see also Naredi-Rainer 1982, p.216, n.239.
14 Nonsense theories concerning the meaning of architectural design by way of geometry can be found, for instance, in Boer 1948; Burgers 1996; Charpentier 1966; Van der Eerden 1997; Freckmann 1985; Kottmann 1971; Lesser 1957; Mössel 1926; Schneider Berenberger 1988. See also Hecht 1969; Kruft 1985, p.40; Naredi-Rainer 1982, p.216, n.239.
15 See par.11.1.
16 For instance, Zagrodzki (s.d.), discussing Polish towns, but claiming general relevance; Buselli 1970, on Pietrasanta in Tuscany; Spagnesi & Properzi 1972 and Fiore 1975, on Cittaducale in Abruzzo; Bucher 1972, on the bastide of Grenade-sur-Garonne; Higounet 1984, on the bastide of Vianne; Guidoni 1992 (II), extending Zagrodzki’s theories to several bastides; Morelli 1994, on Pontedera in Tuscany; Schütte (see Nitz 1996, pp. 65, 88) on Göttingen in Germany; Fernie and Gauthiez (both in Gransden 1998) on Bury St. Edmunds and a number of towns in Normandy. An especially interesting case regards the Florentine terre nuove of Grenade-sur-Garonne and Vianne, because on these towns no less than nine different authors have launched different theories on the ground plan design (Baldari 1980; Bartoli 2003; Bertocci 2003; Buselli 1970; Carli 1981; Friedman 1988; Guidoni 1970; Van den Heuvel 1983; Higounet 1982). Humpert and Schenk (2001) published a book on design geometry in ‘mittelalterliche Städtbau’ which regards towns, mainly newly founded ones in different parts of Europe, foremost southern Germany.
17 Meckseper 1991, p.68. See for instance the theory of Higounet in par.5.4.1.4.
18 I will not consider more complex, and in my opinion more fantastical and unlikely, theories of new town planning on the basis of geometric design systems, which are in their place derived by complex geometric manipulation of figural representations (as for instance, of an eagle; see Boerefijn 1999, pp.75-78). Nor will I go into Guidoni’s theory according to which important buildings such as gates, towers, churches, monasteries and town halls were arranged according to geometric schemes. (see Guidoni 1970, pp.175-177; 1972, pp.135-144; 1976 (II), pp.79, 85, 120.)
6.3.1 Pythagorean triangles in the design of bastide plans

Charles Higounet proposed a theory on the way certain elements of the town plan of the bastide of Vianne (1284) were designed. According to this theory the right angles in the plan were created by use of the geometric ‘tool’ of the so-called ‘pythagorean triangle’. This triangle has sides of which the proportions are $3 : 4 : 5$, which implies that the angle between the two shorter sides is $90^\circ$. This construction is very easy to create with a rope that is divided into 12 equal parts by markings. It is known that surveyors and architectural planners used such ropes, divided by knots, from antiquity until the 16th century at least. According to Higounet the use of such a rope for setting out the plan of Vianne is proven by the fact that one can still find this triangle in the dimensions of the market place and of the wall circuit. 

In my opinion, it is not unlikely that the 12-parted rope was indeed used in setting out the right angles in the plan. Higounet’s argumentation, however, does not prove this, since the use of the 12-parted rope

---

19 Higounet 1984, pp.11, 14-15. This is part of a booklet on the creation of Vianne.
20 Binding 1993, p.341; 2002, pp.139-142. The pythagorean triangle was called norma. It could also be constructed with different ropes or sticks (two at least), as long as the relation $3 : 4 : 5$ of the sides would be secured.
in creating right angles does not logically lead to large scale pythagorean triangles being part of the resulting design. It is likely that many architectural creations were set out by use of such a rope, but these creations rarely, and only by coincidence, had the dimensional relation of the rectangle sides of 3 : 4. In the case of Vianne, it is unlikely that the rope actually determined the absolute dimensions of the market place. In fact, the market place is square, and it is likely that the sides of the rectangle, made up out of the inner square together with the enclosing roads on two of its four sides, approximate the dimensional relation of 3 : 4 only by coincidence. If this relation was actually created with the 12-parted rope, it must have been circa 105 m. long, which is most unlikely since this is much longer than are the lengths of the ropes that are known to have been used for measuring and setting out specific distances.\textsuperscript{21} For the larger triangles which, according to Higounet, would determine the dimensions of the wall circuit in relation to the market square, it is even more unlikely, since it would imply the use of a rope of c. 510 m., or two ropes, divided into five parts, of c. 212 m. each.

Higounet’s theory becomes all the more unlikely when his plans are analysed more closely. It appears that the pythagorean triangle fits the market place of the cadastral plan of 1837 quite well indeed. But from Higounet’s diagrammatic reconstructed plans, it appears much more likely that the 3 : 4 relation has another cause than the use of the pythagorean triangle. The plans suggest that there originally were six house lots facing the market place on every side. Just like the streets bordering the place, they would have been 16 rases (c. 7.8 m.) wide. This would make 96 rases for the width of the six house lots and 128 rases for the six house lots and the two street; this relates as 3 : 4. Hence, it seems more likely that the choice for six standard house lots along the sides of the square and the choice for streets of the same width as the house lots were the reason for the 3 : 4 proportion, rather than the use of the 3 : 4 : 5 triangle.\textsuperscript{22} Apart from that, the corners of the wall circuit are not accurately determined by the angles of the supposed pythagorean triangles: only the southwestern corner fits well within the hypothetical scheme, while the two on the long northern side of the town are both situated circa 10 m. away from the angles of the triangles.

Des pite these flaws, Higounet’s theory has had its believers. A very similar scheme can be found in the book on bastides by Laurent, Malebranche and Séraphin, but now relating to the north-bank town of Ville-neuve-sur-Lot.\textsuperscript{23} (fig. 6.4) Comparing the diagrammatic figure with a 20th-century plan\textsuperscript{24}, it appears that the

\textsuperscript{21} For a discussion on the possible length of the surveying ropes of the period, see par. 6.4.3.2.

\textsuperscript{22} According to Higounet the lots facing the market place measured 16 x 48 rases (c. 7.8 x 23.4 m.), while other lots could also be 24 rases wide; main streets were 16 rases wide and back streets 8 rases. (Higounet 1984, p. 15) Analysing the plan depicted by Higounet (see fig. 6.3), which is based on the 1837 cadastral plan, it seems well possible that there originally were six house lots on every side of the square. It should be considered, however, that this does not correspond to the supposed original width of the house lots of 24 rases given in the coutumes of 1287. (See Higounet 1984, p. 11)

\textsuperscript{23} Laurent, Malebranche & Séraphin 1988, p. 31.

\textsuperscript{24} The best plan I could find is depicted in Calmettes 1986, p. 53.
pythagorean triangles can be fitted in the plan quite accurately indeed. But it seems unlikely that they were actually used for the design or the laying out of the plan, largely for the same reasons as with regard to Vianne.

Hence, it may be concluded that the theory of the dimensioning of urban plans by way of the pythagorean triangle is highly unlikely to be right, even though this triangle may actually have been used for determining right angles.25

6.3.2 Rotated squares in the design of bastide plans

According to Enrico Guidoni, the planning of the bastides is directly related to the creation of the gothic cathedrals: the methods of ‘the great tradition of gothic design’ would also have been used in the design of many bastides.26 In Guidoni’s opinion, the orthogonal structures of bastides would have been dimensioned in the proportion 1:√2 and structured by a grid of squares that lies in diagonal direction with respect to the urban grid. According to Guidoni this would have been a common method in the gothic design tradition.27 It is not explained how this exactly worked or why this rather complex design method would have been used, but there are accompanying figures that illustrate the idea. (figs.6.5, 6.6, see also figs.2.40, 2.39)

From these, it appears that Guidoni thinks that the proportioning method of the so-called ‘rotating square schema’ was used. This geometric method works by constructing a square and taking its diagonal, which relates to the sides of the square as √2:1, as the side of a larger square of which the diagonal may be taken as the side of a larger square, and so on. This method has been taken for a commonplace principle of design in ‘the middle ages’ by many scholars, designated as ‘rotating square schema’, ‘Vierung über Ort’ or with the quasi medieval term ‘ad quadratum’.28 The idea is largely based on the evidence of techniques used in three architectural treatises, so-called Musterbücher, by three authors from southern Germany of the late 15th and early 16th century.29 These treatises, however, discuss the design and execution of buildings, and more particularly ecclesiastic architecture and specific elements such as pinnacles. (see fig.6.2) They do not regard the planning of urban structures.

In my opinion, however, the use of this ‘rotated square method’ has been accepted far too easily as a more or less generally adopted technique in architectural design ‘in the middle ages’. Quite a number of scholars in the past century have drawn up hypotheses claiming the use of this method, but too often they could only demonstrate it by distorting the actual dimensions or by postulating more complex rather than simpler methods of design.30 This is also true for hypotheses on the design of town plans.31 Since there is no historical evidence of the use of this method in town plan design at all, it ought to be demonstrated to be logical and to correspond well to actual dimensions in town plans, in order to allow the theory to be acceptable. Guidoni fails to do this.

The diagonal squares of the theoretical design method that are drawn in Guidoni’s illustrations, only have a limited correspondence to the plans. In the figure of Daxman (fig.6.5), the inner diagonal square is drawn so that it exactly contains the square market hall, which stands in the middle of the market place. The side of this diagonal square relates to the side of the market hall as √2:1. The meaning of this square is, however, unclear since it does not relate logically to any other part of the plan. Presumably, the four diagonal squares are intended to form a geometrical series with every larger square having sides twice as long as those of the smaller one.32 Close inspection of the figure, however, shows that this is only approximately the case. The second smallest diagonal square has its corners near the points where the two central axes of the town cross the back boundaries of the street blocks that front on the market place. The intention probably is to show that the place of these boundaries was marked by the diagonal square. This is not truly the case, how-

25 See also par.6.3.4.
29 It regards the Buchlein der Fialen Gerechtigkeit (1480) and the Geometria Deutsch (1487/88) by Mathias Roriczer, a short treatise on the design of pinnacles by Hans Schmuttermayer (shortly after 1488), and a treatise by Lorenz Lechler (or Lacher, 1516). (see Hecht 1970, vol.22, pp.163-188)
31 Zagrodzki proposed that this method was used for the design of various plans of new towns from the 13th to 15th centuries in Poland and Czechia. (Zagrodzki S.D.; Zagrodzki 1966) This clearly inspired Guidoni in the ideas and figures described here. When Zagrodzki’s illustrations are studied closely, it appears that the dimensions of the various squares of the theoretical design method often do not relate geometrically exact to one another, and that some squares are not even truly square. In my opinion, many lines of the theoretical grid are too far from the actual building lines in the plans to be rightly identified with them. Moreover, some of the various squares have no actual relevance in the town plans at all.
32 This was at least the idea of Zagrodzki’s theoretic design method (Zagrodzki S.D.; Zagrodzki 1966), which most probably formed Guidoni’s main inspiration, and various other theories on gothic architectural design that were inspired for a great deal by the sources mentioned in n.29 (see also fig.6.2).
ever, as the distance between the two opposite boundaries is circa 5% greater in the one direction than in the other, which is also visible at the upper corner of the diagonal square. The next larger diagonal square does not appear to mark any relevant place or proportion in the plan, and neither does the largest one. The same holds true for the largest non-diagonal square – which actually is a deformed square. It is probably intended to define the limits of the town plan, but it does not mark any actual boundaries in the plan with precision; on the northern end a substantial part of the town is even 'cut off '. Guidoni’s figure suggests that the plan formed a somewhat deformed square, while actually the outline was elongated, with angles that were only approximately right. All in all, the whole figure appears to be quite senseless.

This is even more so with the figure concerning Tournay. (fig. 6.6, cf. fig. 2.39) It is probably intended

---

33 I have tried to check this with the redrafted cadastral plan that is depicted on a small scale in Lauret, Malebranche & Séraphin 1988, p.67. It appeared that the street blocks are not exactly arranged in the form of a square since there is circa 5% difference between the two directions of the rectangle.

34 In the same book, Guidoni also depicts schematised plans of the bastides of Marciac and Cologne, which suggest geometric schemes underlying their design. (Guidoni 1992 (II), pp.126-128) Both these figures suffer from the same flaws as the ones regarding Damazan and Tournay, which is that the schematic (quasi-) geometric figures do not fit well onto the plans, and that they follow no clear logic, as one would expect from a geometric method of design. (cf. plans in Lauret, Malebranche and Séraphin 1988, p.73 and Lavedan & Hugueney 1974, p.264)
that the smallest non-diagonal square has a diagonal which is just as long as the sides of the market square (hence, the sides relate as 1: \(\sqrt{2}\)). That may be, but it is irrelevant, since this non-diagonal square appears to have no logical relation to any other part in the plan. The largest diagonal square is probably intended to have sides as long as the streets that are laid out in a square along the backsides of the street blocks surrounding the market square. Again, it appears that this diagonal square has no logical relation to any other part in the plan, and therefore it makes no sense at all.

### 6.3.3 Complex geometry in the design of the bastide of Grenade-sur-Garonne

In an article about ‘medieval architectural design methods’ Francois Bucher proposed a hypothetical geometric method of design for the plan of Grenade-sur-Garonne.\(^{36}\) (fig. 6.7, cf. fig. 2.22) The regular grid plan of the town’s historical core contains street blocks of three different sizes. In the one direction the streets are laid out at equal distances, but in the other direction they are not equally spaced. According to Bucher, the spacing of these latter streets was fixed by geometrically determined distances. In the centre of the town there is a row of square blocks, which is interrupted by the market square. The length of the ‘larger blocks’, lying in the rows to the northwest and southeast of the squares, would have been given by the length of the diagonal of the square, so that the length relates to the width as \(\sqrt{2}:1\).\(^{37}\) The length of the ‘largest blocks’ (lying in rows northwest and southeast of the ‘larger blocks’) would have been generated by the auron. The auron, or golden-section rectangle, is dimensioned by rotating the diagonal of the half-square.\(^{38}\) (see fig. 6.7) According to Bucher, this would be analogous to the geometric techniques that were often used in ‘medieval architectural planning’ (especially for churches). Bucher refers to the Grenade plan as a clear example of this design method, but he does not explain it exactly, nor does he try to prove his theory by giving actual measurements. He does mention, however, that the same principle can be found in the plans of Sainte-Foy-la-Grande (fig. 2.21) and the new towns founded by the Zähringer dynasty in northwest Switzerland and southwest Germany.\(^{39}\)

David Friedman has tried to check whether Bucher’s theory actually fits Grenade’s town plan. After taking measurements, Friedman found that the first part of the theory seems to be correct, as the length of the ‘larger blocks’ is almost equal to the diagonal of the square blocks.\(^{40}\) Although Friedman seems to have misunderstood the second part of Bucher’s theory, he is right in finding that the length of the ‘largest blocks’ does not correspond with Bucher’s theory.\(^{41}\) Hence, Friedman concluded that Bucher’s theory is substantiated only as far as the relation of the square blocks to the ‘larger blocks’ is concerned.

It seems that Kostof has, in his turn, misunderstood Friedman. He depicts the geometric scheme as if five different sizes of blocks in Grenade were all related as follows: the diagonal of the square block is equal to the long side of the larger block, whose diagonal, in its turn, is equal to the long side of the next larger block, and so on.\(^{42}\) Kostof’s illustration of this system is very schematic (fig. 6.8) and has not much to do with the actual plan of Grenade, since there are not five regular rows of blocks of progressively greater length, but only three.

Randolph understood Bucher in still another way, when he wrote that the street blocks east and west of the square have long sides that equal the diagonal of the square. Randolph probably means the blocks to the southeast and northwest of the square. This is in agreement with Friedman’s findings and probably is what

---

\(^{35}\) The non-diagonal square actually has a logical geometric relation to the smallest diagonal square, but this square has no logical function in (or relation to) the plan, and therefore it is senseless.

\(^{36}\) Bucher 1972, p.43.

\(^{37}\) Cf. par. 6.3.2.

\(^{38}\) Bucher does not explain the dimensions in detail, but he is probably hinting at the rectangle of which the proportions are 1: 0.618 (which is the ‘golden section’). This proportion has the special quality that the small part (the minor) relates to the greater part (the major) as the greater part to the whole of the two. Many scholars regard this ‘golden section’ as a mathematical tool that has been a general principle in (architectural) design ever since antiquity. As far as I know, however, knowledge of the ‘golden section’, its geometric construction and arithmetical relation, can only first be unambiguously detected in Luca Pacioli’s De divina proportione of 1479. For the period between this work and antiquity, knowledge of the ‘golden section’, let alone its use in architectural design, is still conjectural.

\(^{39}\) Since Bucher is unclear about what he exactly intends, there is no point in trying to verify his ideas concerning Sainte-Foy-la-Grande and the Zähringer new towns. These Zähringer new towns were founded between the early 12th and 13th centuries. Most famous among them are Bern and Freiburg-im-Breisgau (Divonne, 1993; Schwinkepöer, 1980).

\(^{40}\) When measurements are taken from the centre line of the streets, the square blocks have sides of 64 m. and diagonals of 90.5 m., which comes very close to the long side of the ‘larger blocks’, being 90.4 m. according to Friedman. (Friedman 1988, pp.132, 135, n.39)

\(^{41}\) Friedman 1988, pp.132, 135, n.39. It does not make much difference that, in my opinion, Friedman did not understand correctly what Bucher exactly meant (instead of the auron he takes the diagonal of the ‘larger block’ as the side of the ‘largest block’), because either way the theories do not correspond to the real measurements (116.6 - 119.4 m. measured by Friedman; 110.8 m. calculated by Friedman; 103.55 m. calculated for the auron based on the square with 64 m. sides). It appears that Friedman’s measurements do correspond fairly well to the ones I measured in the cadastral plan of Grenade (see below), which are in general circa 0.5% larger, however.

\(^{42}\) Kostof 1991, p.128.
Bucher intended. However, Randolph considered the blocks north and south of the square to be aurons. Here
he seems to have been referring to the square blocks in the central NE-SW row, not the ‘largest’ blocks to
which Bucher was referring.43 It is obvious, however, that these blocks are not
aurons but squares.

All in all, this is a curious case of uncritical acceptance (Friedman’s contribution excepted) and misun-
derstandings piled one on top of the other, based on an unsubstantiated, almost casual, remark by Bucher.
Many other scholars also seem to have accepted Bucher’s theory without reservation.44 Therefore, it is appro-
priate to examine the plan of Grenade thoroughly in order to check whether, or to what extent, Bucher’s
theory really fits with the actual dimensions in the plan.

6.3.3.1 Metrological analysis of the plan of Grenade-sur-Garonne

When the plan of Grenade is closely studied, it appears that it is not as regular as might seem at first sight.45
The streets from northwest to southeast, for instance, are slightly curved, and the streets in the other direc-
tion (SW-NE) converge slightly towards the southwest.46 Consequently, the dimensions of the street blocks
show some variety; therefore the averages have been calculated. Thus, the average lengths of the three types
of blocks appeared to be 55.05 m., 82.44 m., and 110.11 m.47 From these dimensions it appears the ‘larger
blocks’ were planned to be one and a half times as long as the square blocks, and the ‘largest blocks’ twice as
long.48 Compared to Bucher’s theory this seems almost disappointingly simple.

43  Randolph 1994, p.300. In referring to the blocks ‘to the east and west’ of the square Randolph probably means the ‘larger blocks’ (actually NWW and SSE, to be exact),
which is apparent from the non-oriented illustration he took from Friedman.
44  Recently, Lilley, Slater and Scrase contributed to a discussion in the journal Urban Morphology (1998, nr.2, pp.82-93; 1999, nr.2, pp.107-111; 2000, nr.2, pp.104-106) on
the use and function of geometry in town plan design. Although they had conflicting standpoints on several aspects, they all agreed that Grenade was designed by use of
complex geometry in the way that Bucher suggested. See also Lilley 2002, pp.161-162.
45  I have used the cadastral plan on the scale 1:1250, which is kept in the office of the regional cadastre in Toulouse Colomiers, entitled
Grenade-sur-Garonne, section C, feuille 3.
46  The extremes of the NW-SE-streets lie circa 1-1.7 m. farther southwest from a straight tangent line along their centre. This can also be easily observed when looking along
the streets. On the southwest side of town the SW-NE-streets that separate ‘larger’ from ‘largest blocks’ are circa 5.2 m. closer to each other than on the northeast side.
Hence, the blocks on the northeastern side of town are slightly longer than the ones on the southwestern side.
47  The first dimension is calculated from 12 measurements for the length of the square blocks from northwest to southeast, averaging 55.08 m., and 37 measurements of the
width of all the blocks from northeast to southwest, averaging 55.02 m., ranging from 53.87 m. to 56.37 m.; the second dimension is calculated from 22 measurements,
ranging from 80.12 m. to 84.37 m.; and the third from 21 measurements, ranging from 108.25 m. to 112.25 m. By comparison Friedman found dimensions of 55.4 m., with
variations of only 20 cm., 82.4 m. and 109.2 m. (probably on the southeast side) / 112 m. (probably on the northwest side). (Friedman 1998, p.239, n.39) Lavigne mentions
measurements of 56 m., 85 m. and 110 m. (Lavigne 1996, p.192) In both cases it is not explained what exactly they have measured, so it is unclear how the differences arise.
48  Taking 55.05 m. as the basis, the ‘larger block’ would theoretically measure 1.5 x 55.05 m. = 82.57 m., and the ‘largest block’ 2 x 55.05 m. = 110.10 m. Thus the differences

fig. 6.7: Figurative depiction of the method of determining the dimensions of the blocks in the design of the plan of
Grenade-sur-Garonne, according to Bucher. (After: Bucher
1972) The central block is square, the larger ones next to it
are proportioned by the diagonal (1 : $\sqrt{2}$), and the larg-
est blocks by the auron (2 : 1+$\sqrt{5}$). Verification with the
actual dimensions in the modern town plan shows that it
is highly unlikely that this hypothetical design method
was actually used.

fig. 6.8: Figurative depiction of the method of determin-
ing the dimensions of the blocks in the design of the plan
of Grenade-sur-Garonne according to Kostof. (After: Kostof
1991) Starting from the square block, the long side of the
larger block is given by the diagonal of the square; the long
side of the next larger block is given by the diagonal of the
preceding block, and so on. Verification with the actual
dimensions in the modern town plan shows that it
is highly unlikely that this hypothetical design method was
actually used.
In the charte du parage it is stated that the bastide’s foundation provided for 3000 households, which required 3000 house lots, 3000 garden plots and 2000 fields of arable land.49 The house lots were to measure 5 x 15 brasses, costing 5 denier rent a year. According to Cédric Lavigne these dimensions would be circa 8 x 24 m.50 However, given the calculated sizes of the street blocks in the cadastral plan, it seems more likely that they measured circa 9.175 x 27.525 m., so that the different types of blocks would have contained 12, 18 and 24 lots, lying back to back. The brasse would measure 1.835 m. in that case. Trying to verify this from the actual lots in the plan, one is confronted with the fact that the plan does not immediately suggest an initial division into house lots of equal size. One should consider, however, that 700 years have passed, and while the boundaries between public and private space seem to have remained much the same, the structure of ownership of private land and private buildings has changed considerably over this period. However, there still are many lots that approximately have the dimensions of 9.175 x 27.525 m. In length they extend to half the width of the blocks, and in width they are one-sixth of the side of the square blocks, one-ninth of the length of the ‘larger blocks’ and one-twelveth of the ‘largest’ ones. Most of the present-day lots, however, are smaller, probably owing to subdivision of the original standard lots. There are also some larger ones, which probably were amalgamated.51 The church with its yard occupies the space of twelve standard lots of one of the ‘largest blocks’, taking up a square plot that covers half the block.

The streets of Grenade’s town centre, measured from the cadastral plan, are for the most part 8.55 m. wide on average. The only regular exceptions in this respect are the two streets that separate the ‘larger blocks’ from the ‘largest’ ones. These streets are only c. 7.02 m. wide on average. This is probably because they were not intended as residential streets, since the house frontages were oriented to the streets transverse to these two.52

In conclusion, the evidence seems to contradict Bucher’s theory of complicated geometric design. Instead, the dimensions in the present-day plan show simple arithmetic proportions, which were most probably generated by the choice for particular numbers of theoretically identical lots within the different street blocks. Alternatively, it may not have been the choice for particular numbers of lots, but rather the choice for particular proportions that determined the dimensions of the blocks. It seems likely that there was some significance in the use of the simple arithmetic proportions of 1:3 (the house lots) and 1:1, 2:3 and 1:2 (the street blocks). Apparently, these simple proportions were for some reason preferred above random proportions or less bold proportions such as 4:5 or 8:13.

So, the layout of the town of Grenade-sur-Garonne was probably not designed by way of complex geometric constructions. Instead, the number of desired households for the new town was determined, and a standard lot size was chosen with dimensions in rational numbers of the traditional local unit of measurement, in such a way that the dimensions of length and width would have a clear and simple relationship and the lot would have a useful size for a normal urban household. Subsequently, these lots were arranged in three different sizes of street blocks, so that the smallest ones would be square, the larger ones would have the dimensional relation 2:3 and the largest ones 1:2. One of the square blocks in the centre of the layout was left open to create space for the market place.53 The streets were given widths as required by their intended functions, presumably four and five brasses wide, and a square piece of land with the size of twelve house lots was reserved for the town church.

49 This document is in the archives of the Haute-Garonne district, no. 108 H 15 (expédition), and is published in French in Rivals 1986, pp.78-88.
50 Lavigne 1996, p.192. It is not mentioned on what source this is based.
51 One must always be careful not automatically to suppose that the intended lots, as described in the foundation document, were indeed laid out and distributed correspondingly. Research elsewhere in Europe has shown the probability of those standard lots being also issued in halves, one and a halves, or multiples. (see par.9.11) Hence, it is possible that the many lots in Grenade that cover fractions or multiples of the original standard lot size were originally created in these dimensions. (see also Slater 2001, pp.50-51)
52 Slater 2001, p.50. The width of the widest streets is the average of 45 measurements, ranging from 7.5 m. to 9.25 m.; the width of the two narrower streets is calculated from 13 measurements ranging from 6.60 m. to 7.75 m. The widths may originally have been intended to be five and four brasses (9.175 m. and 7.34 m.). The differences may reflect inaccuracies in the plan or in my measurements; for an accurate analysis it would be better to measure the width of the streets in reality. It is also possible that the differences are partly caused by the rebuilding of the houses, over and over again during seven centuries, by which there would be a tendency for public space to be encroached upon.
53 This was quite common in the bastides of southwest France, where the market place preferably seems to have been made square or nearly square.
The basic problem with which this paragraph began, was the question as to whether it is true that the town plan of Grenade was designed by use of complex geometry, as has been claimed by several scholars. After comparing this geometric hypothesis with measurements in the modern town plan, it appears that this is unlikely. Instead, there is a much simpler interpretation that explains the measured dimensions much better and which fits with the standard lot size mentioned in the paréage document. Apparently, though, this simple and obvious ‘arithmetic’ explanation has been less attractive to students of the design of Grenade’s plan up to now.

Apparently the idea of a complex geometry lying at the basis of ‘medieval architectural design’ is so dear to many people, or is found so natural, that Bucher’s hypothesis, with its defects and unsubstantiated statements, is taken for the obvious truth. As discussed above, this case is far from unique in that respect: many theories have been put forward over the last 150 years or so that suggest complex geometric figures underlying ‘medieval architectural design’, which are based on very poor and often far-fetched sources, and which mostly are attested inadequately.

6.3.4 Complex geometry in the design of various town plans according to Humpert and Schenk

It is necessary nor useful to treat all the theories of complex geometric town plan design in the period under consideration that are proposed in the literature of the past hundred years or so. One of the most recent, however, may not be omitted here. It regards the book with the pretentious title *Entdeckung der mittelalterlichen Stadtplanung. Das Ende vom Mythos der ’gewachsenen Stadt* (‘Discovery of medieval town planning. The end of the myth of the ‘grown town’) by Klaus Humpert and Martin Schenk, which treats the plan design of many different towns from between the 11th and the 15th century, mainly newly founded ones from southern Germany.55

In this book, recent plans of the various towns are dissected in order to analyse the way their components can be made to fit in theoretical geometric schemes by which the plans would have been designed and set out according to Humpert and Schenk. The schemes particularly concentrate on the irregularities in the plans, such as winding streets and curved sections of wall circuits, oblique angles and asymmetries, in order to ‘rationalise’ these elements. The various geometric constructions add up to highly complex networks of points, lines and arcs that would have been the underlying basis of the plan design, more or less like a sowing pattern of a garment.56 (figs.6.9, 6.10) The difference is, however, that the complexity of a sowing pattern usually has a clear goal, which is to create a garment that fits well, whereas the goal of the theoretical schemes of Humpert and Schenk, and the ‘resulting’ irregular forms, is not explained by them.

In fact, it is difficult to conceive of an explanation for such methods of town plan design, as there is no systematic logic in the schemes: in one and the same plan dimensions for construction lines appear to have been chosen without logical reason - mostly in numbers of feet rounded off at decimals -, for example 1300 here, 180 there and 40 at another place. Hardly any of the separate stages within the ‘reconstructed design processes’ follow logically from earlier ones. In this way Humpert and Schenk succeed in describing the form of the plans more or less accurately by use of dimensions and geometric manipulations, but due to a lack of logical coherence between the various dimensions and manipulations this is most unlikely to have been the actual method of design.

Another problem is that the authors claim that the dimensions and arcs were set out on the land by ropes with lengths up to no less than 2200 metres. This is, however, highly unlikely. Even with the strong, thin and light ropes that exist at present, it would be almost impossible to manage this, but in the early 15th century a rope of more than circa 45 m. long was already thought of as difficult to handle because of its elasticity and the variability of the length, which depended on the temperature and the moisture of the air.57 It is clear that Humpert and Schenk have hardly studied the techniques, methods, knowledge or ideology of the concerned period. They argue that this would be the function of historians, which they themselves are not; their ambi-

---

55 Humpert & Schenk 2001. Also, a number of towns from antiquity and some highlights from the history of architecture, such as temples and cathedrals, are superficially discussed. (Humpert & Schenk 2001, pp.258-343)

56 Eight towns are treated laboriously by Humpert and Schenk, dealing with various details of their plans. They use many figures to illustrate the component geometric schemes, but they never depict a figure that contains all the points, lines and arcs which would have been the components of the theoretical design scheme of the whole plan. Such an illustration would make a very complicated figure with an intricate tracery of straight lines and circle segments, of which hardly any lines or points clearly agree with the actual town plan.

57 Guerreau 1995, p.90; see also below par.6.4.1.2.
Humpert and Schenk try to demonstrate that the concerned towns and cities are purposefully planned by use of complicated geometric schemes, by describing the irregularities in the plan forms in terms of geometry. This ‘demonstration’, however, does not give insight into the way new towns were actually shaped. In fact, regularity, and not irregularity, is the clearest characteristic of spatial planning. Of course it is true, as shown in chapters 1 to 3, that many newly planned towns contain forms that are not regular: streets are not entirely straight, angles are not wholly right, distances often are not rationally related, lots are not equal, and the outline form has no geometrically regular shape. One can be quite certain, however, that these irregularities normally have other causes than complicated methods of geometric design. They may have been caused by adaption to specific circumstances, such as the shape of the natural and cultural landscape. It is also well possible that they were not planned at once, being created in various phases in the course of time.

It is illustrating how little attention Humpert and Schenk give to the form of the pre-existing landscape as a contributor to the creation of specific plan forms. In a number of cases they even reverse the influence that the landscape logically exerts on urban form. In the town of Weiden, for instance, the curved line of part of the town wall parallel to the shore of a river, would not have been determined by the course of the river, but according to Humpert and Schenk the river and the town wall would have been laid out according to a geometric design. Likewise, the Canal Grande in Venice would have been laid out in large geometric arcs.

---

58 Humpert & Schenk 2001, p. 379. The professional background of the authors is in architectural design and town planning.
59 For reactions on the bold ideas of Humpert and Schenk regarding this aspect with respect to the towns of Speyer and Lübeck (Humpert & Schenk 2001, pp.150-171, 20, 46), see Untermann 2004, pp.12-13 and Gläser 2004. See also par.9.6.2.
because that was how the designer wanted it to be. I believe that this case does not need to be commented on in more detail in order to disqualify it.

Humpert and Schenk also believe that they can prove the use of the Pythagorean triangle in the setting out of town plans. This triangle would have formed half of a rectangle with sides of which the proportions are 3 : 4. According to the authors, this rectangle underlies most historical architectural designs. This does not necessarily mean that the rectangle can be seen in the final design: it would only have been the basis of different sorts of geometric manipulations that result in the final design. The authors even believe that with this rectangle they have found a universal principle of design, which was followed all through the ages. However, the theory does not hold under critical examination. In the main part of the presented examples, regarding buildings rather than urban plans, the rectangle is only part of a larger modular grid of squares from which it is highlighted more or less arbitrarily. In the cases where Humpert and Schenk claim the use of the Pythagorean rectangle in urban design, it appears to have hardly any coincidence with the actual town plan at all.

As with the other proposed theories of complex geometric town plan design treated in the paragraphs above, the authors silently pass over the question why these complicated geometric methods would have been used. Apparently, they do not find this a very important matter. In my opinion, however, this question is of prime importance. In paragraph 6.4.4, this matter will be attended to.

6.4 Complex geometry in the town plan design of the terre nuove fiorentine

In the previous paragraph a number of theories regarding town plan design by use of complex geometry have been disproved. This does not mean, however, that complicated methods of design were not used at all in
the planning of urban ground plans in the high-period of town foundation. A rather crucial case is the one of the terre nuove fiorentine. Although the plans of these towns are as boldly orthogonal as that of Grenade-sur-Garonne, the proportioning of some of the dimensions seems to have been designed by use of a geometric method that is even more complicated than are some of the theories discussed above.

In chapter 3 it is already considered that the terre nuove fiorentine have highly regular orthogonal plans and that those of Castelfranco di Sopra, San Giovanni Valdarno, Scarperia, Terranuova Bracciolini and Giglio Fiorentino have in common the otherwise very rare aspect of diminution of lot length: the length of the house lots (or the width of the rows) decreases in stages from the central main street outward. The number of different stages and number of parallel house rows within one stage varies between the towns, but the basic principle is present in all five of them. This diminution of the lot length, together with the remarkable regularity and the strong correspondences in the plans of the five new towns, have led several scholars to suppose that - despite the various differences between the plans - there is a common geometric principle in the method of design used for the distinct plans. Nine different authors have proposed nine methods of determining relevant dimensions for one or more of the concerned town plans. One of these proposals is that the plans were not designed by complex geometric methods, but arithmetically, by simply taking rational numbers of the local unit of measurement for the crucial dimensions, much like my proposal for Grenade-sur-Garonne discussed above.

With this, the terre nuove are a very important case in the discussion on the methods of town plan design in the concerned period, and therefore they deserve full attention in this context. Hence, their plans and the various theories proposed for their design will be elaborately discussed in the following pages. In addition, a theory of design will be proposed which is adjusted from some of the existing theories and which fits best with the evidence.

### 6.4.1 Theories regarding the method of design of the terre nuove-plans

On the following pages, the various hypotheses of design methods will be discussed. For the sake of brevity, only the more plausible theories will be treated in detail: the theories that are unlikely to be correct are only briefly outlined in paragraph 6.4.1.4 and are more accurately analysed in appendix B, paragraph 2. Further below, in paragraph 6.4.2, the three more plausible theories will be compared with the modern town plans and will be analysed in detail, in order to check their probability. For the sake of brevity and clarity direct reference will be made to the illustrations, since they play an important role in the analysis of the various theories.

#### 6.4.1.1 Guidoni

Enrico Guidoni proposed a theory on the design method of the terre nuove-plans in his book *Arte e urbanistica in Toscana 1000-1315* of 1970. In this book many hypotheses are propounded concerning architectural design geometry in this period, many of which do not seem very convincing at first sight. With regard to the Florentine new towns, however, he propounded a theory that would have great influence on the ideas of other scholars.

Guidoni proposes a compound design method that partly works on the principle of regular polygons that govern the outline proportions and inner structure of the town plans, and partly with squares that determine the proportions of the piazzas, as well as the inner street blocks at Castelfranco. This system is applied, in different forms and different degree of complexity, to the plans of Castelfranco, San Giovanni and Terranuova. (figs. 6.11–6.13)

The more important part of Guidoni’s hypothesis is formed by the proportioning of the perimeters and

64 See pars. 3.9.2.1, 3.9.2.6.
66 As will become clear in the following paragraphs, Guidoni’s theories concerning the design method of the terre nuove-plans have had a considerable influence on the ideas of Carli, Baldari, Friedman and Van den Heuvel.
67 The two other terre nuove, according to Guidoni, are not proportioned by this geometry, since they were not designed by the same designer, the architect Arnolfo di Cambio. Guidoni’s attribution to Arnolfo is based on the 18th-century artists biographer Giorgio Vasari, who wrote that he made the designs for Castelfranco and San Giovanni. It may, however, be doubted if this attribution is right. (see par. 7.8) Guidoni’s argument is strongly aimed at convincing the readers that Arnolfo brought important innovations in architectural design methods. (Guidoni 1970, p. 219)
the inner structures by means of regular polygons. In Guidoni’s reconstruction of the plan of Castelfranco a hexagon describes the relation between width and length of the perimeter of the town. (fig. 6.11) In the plan of San Giovanni the proportions of the perimeter would have been determined by two hexagons and the spacing of streets and lots in lateral direction would have been determined by the regular geometric figure of a dodecagon, which is formed by two hexagons that are rotated by 30 degrees in regard to one another. (fig. 6.12) The outer corners of these hexagons determine the place of the axis of the main street and six building lines in the longitudinal direction of the town.68 But there are more striking agreements between this geometric figure and the plan to be remarked. When it is studied closely, eight of the twelve points that fix the intersections of the radiating lines and the sides of the two rotated hexagons, appear to mark four more relevant lines in the town structure: these are the inner alignments of the primary longitudinal parallel streets (to both sides of the main street), and the outer parallel streets (the original longitudinal wall streets).69 It is unclear if Guidoni meant these points to be relevant in his theory, since he does not spend many words to explain his rather schematic figures. This also counts for the four points at the intersections of the sides of the two hexagons, that mark the backside of the third row of houses (counted from the main street). All in all this rotated double hexagon seems to determine the positions of eleven lines that play a relevant role in San Giovanni’s plan structure.

Guidoni’s theory regarding Terranuova contains a similar geometric system, but instead of the central rotated double hexagon of San Giovanni, there is a rotated double dodecagon, which gives 24 points regularly spaced along the circumference of a circle. (fig. 6.13) As with the hypothetical system of San Giovanni, two connected hexagons determine the perimeter dimensions. The eleven parallel lines that go through the angle points of the polygon, mark the middle axis of the town, the inner alignments of the six parallel streets, the inner alignment of the original wall streets and the position of the longitudinal stretches of the town wall. It remains unclear, however, why Guidoni has illustrated the central polygon in the form of two rotated dodecagons, instead of a polygon with 24 sides or four rotated hexagons (which would seem more coherent with the rest of his theories).

These different hypothetical design systems proposed by Guidoni, fit very well onto the town plans in his illustrations, and therefore seem quite convincing. (figs. 6.11–6.13) But these town plans are no more than schematic reconstructions of the supposed original design, made by Guidoni himself.70 So, in order to check if the theoretical geometric figures really correspond to the actual plans of the towns, they need to be compared. This will be described below in paragraphs 6.4.2 and more detailed in appendix B, but here it can already be unveiled that the comparison will show that the idea of the use of hexagons and dodecagons will

---

68 The only angle points of the dodecagon that do not seem to mark relevant points in the urban structure are the outer two, but from Guidoni’s figure it might be understood that these points would have determined the outside of the ditch that surrounded the town. It is impossible to tell if this is right, since no traces are left of this ditch.

69 The longitudinal wall streets originally were very wide, 18 braccia (10.51 m.) according to the 16th-century plan of San Giovanni by Piero della Zucca. (fig. 3.13) Later these streets were largely occupied, until the 19th century mainly by gardens and later on by rows of houses, sheds and barns, which used the town wall as rear wall.

70 Guidoni does not mention where these schematic reconstructions come from or what they are based on.
appear to be quite likely. Firstly, however, a number of other theories by other authors will be introduced, since they are related to Guidoni’s theories.71

6.4.1.2 Friedman

In an article from 1974 David Friedman first unfolded his theory on the design method of the town plans, which was largely based on Guidoni’s. In his book on the terre nuove from 1988 he made some adjustments to his theory and described further specifications. With respect to Terranuova Friedman proposes almost the same geometric system as Guidoni does, only Friedman describes it as a circle of which the circumference is divided into 24 equal parts with arcs of 15°, instead of a double rotated dodecagon.72 (fig. 6.14) The 15° division points, however, mark the same relevant lines in the plan structure as the angular points of Guidoni’s dodecagons. According to Friedman this circle has a radius of 83.4 m., with which it would also determine the inner alignments of the two secondary cross streets. By doubling this radius, the position of the short sides of the town wall would be determined. Basically, this comes to the same relevant value as Guidoni’s theory of the two hexagons. (fig. 6.13) But while Guidoni thought that it would determine the distance between the inner alignments of the perpendicular wall streets, Friedman would have it that it marks the outer alignments.

Regarding San Giovanni, Friedman’s theory is a strongly reduced version of Guidoni’s. (compare figs. 6.12 and 6.15). Friedman does not explain why he reduced it, but it is likely that he assumed that the rest of Guidoni’s theory did not correspond with the plan very well. Friedman has calculated a relationship between the distances from the central axis of the town to the inner alignments of the parallel streets and the longitudinal wall streets.73 He describes the mutual distances between these relevant lines as the sine-values of angles of

71 The very sketchy theory regarding Firenzuola, published by Carli in 1981 (Carli 1981), is also partly based on Guidoni’s ideas. This hypothesis will not be considered here, since it is obvious that the plan of Firenzuola, as we know it, is not the originally intended plan. (see par. 3.8.4)


73 The original longitudinal wall street has the same inner alignment as the present second parallel street, because it is essentially the same street, but partly built on at the side of the town wall in the past centuries.
30 and 60 degrees, given that the radius of the imaginary circle is 96.68 m. and that the zero-degree-line is the central axis of the town. This may seem rather complicated, but in fact this comes to the angular points of a dodecagon of which the radius is 96.68 m. and the centre is in the is the very midpoint of the town plan, much like Guidoni’s theory. The difference, however, is that in this way there are only five relevant lines in the urban structure marked by the geometric system, whereas in Guidoni’s system there seem to be eleven points that would mark relevant lines in the plan. In order to decide which of these theories is most likely, they will both be compared with the dimensions of the present-day town plan in paragraph 6.4.2 and, more elaborately, in appendix B.

6.4.1.3 Pirillo

In 1989 Paolo Pirillo proposed a new hypothesis, with which he reacted to the earlier theories. Instead of a geometric method of dimensioning, Pirillo wrote that the plans of Terranuova and San Giovanni were designed on the basis of simple numerical dimensions made up of units of the then prevailing unit of length measurement, the Florentine braccio da panna (0.5835 m.).74 The situation bears likeness to the case described above in paragraph 6.3.3, where an arithmetic method of design was proposed for Grenade-sur-Garonne instead of Bucher’s theory of complicated geometric design.75

According to Pirillo, the house lots originally were 10 braccia wide, and all other relevant dimensions were, similarly, determined by simply taking round numbers of braccia.76 Thus, at Terranuova the piazza would measure 70 x 90 b., the main street would be 15 b. wide and the lots would be 30, 25 and 20 b. long. Pirillo does not mention a fourth row of house lots. The back streets would be 7.5 b. wide, and the cross streets 10.77 He also writes that the total circumference of the town was not directly dimensioned in this way, but was only a result of the internal dimensioning, since it measures the non-rounded numbers and the non-rational relation of 274 x 565 b.78 In paragraph 6.4.2 it will be checked to what degree these dimensions

---

74 According to Pirillo, the Florentine braccio da panna was 0.5835 m. (Pirillo 1989, p.13) According to Finoello Zervas it was on average 0.587397 m. (Finoello Zervas 1979, pp.6–10) I take the ancient braccio for 0.5836 m., as has commonly been done since the 19th century, and Friedman takes it as 0.584 m. (Friedman 1988, p.236, n.2)

75 Pirillo 1989.

76 The width of 10 b. for the original lots is generally accepted (see Friedman 1988, pp.74-75), and is also mentioned in Piero della Zucca’s plan of San Giovanni from 1553. (see fig.3.13 and Friedman 1988, p.349, doc.23)


78 Pirillo 1989, p.20.
fig. 6.14: Figurative depiction of the geometric proportioning method of the plan of Terranuova according to Friedman (1974), on the basis of a schematic modern plan. The internal structure of the plan is dimensioned by way of a circle, the circumference of which is divided into 24 equal parts. The relevant points on the circumference determine the positions of parallel lines that mark the centre of the main street and the central cross street, as well as the inner alignments of the streets parallel to the main street (the outer ones along the town walls have since been built over) and the longitudinal town walls. The circle also gives the inner alignments of the lateral cross streets. This geometric system is largely taken over from Guidoni (Guidoni 1970, see fig. 6.13).

fig. 6.15: Figurative depiction of the geometric proportioning method of the plan of San Giovanni according to Friedman (1988). This figure shows the sine-triangles with angles of 30 and 60 degrees and how they fit the plan. These triangles implicitly give the same proportions as can be found in the two rotated hexagons of Guidoni’s scheme (fig. 6.12), but Guidoni’s scheme also comprises other relevant proportions.
correspond to the actual dimensions measured in the plan of Terranuova. Regarding San Giovanni, Pirillo claims that the lots are 40, 30 (including the width of the back alley), 25, en 20 braccia (including back alley) long, while the main street and the cross street are 20 b. wide and the piazza is 160 b. long, just like the rows of house lots. This will also be checked with the present-day plan in paragraph 6.4.2.

6.4.1.4 Other theories on the design of the terre nuove plans

Six more authors have proposed theories of geometric design of more or less complex nature for the terre nuove. These theories largely appear to be less plausible than the ones presented above. Therefore, they will only be discussed briefly here. In appendix B, however, most of them are analysed more thoroughly. The reader who is particularly interested in this subject should read that appendix for additional information.

Charles Higounet proposed that the dimensions of the outline forms of the terre nuove plans are simply made up of large squares with sides of 100 or 150 brachiata (162.5 or 243.75 m.) length. San Giovanni originally would have measured 100 x 300 brachiata, for Scarperia and Terranuova it would have been 100 x 200, and Castelfranco would have measured 150 x 150 brachiata. Analysis of the present-day plans, however, shows that these dimensions are largely incorrect, with deviations from the actual dimensions of up to 20%. Besides this considerable flaw, Higounet’s theory is also rather unlikely because of the fact that contemporary Florentine documents show that, instead of the brachiata, the braccia (c. 0.5836 m.) was generally used as the standard unit of measurement in Florentine building operations in the concerned period.

Franco Buselli proposed a theory for a complex geometric method of design for the town of Pietrasanta, which was founded in 1255 by the city-state of Lucca in northwest Tuscany. Pietrasanta’s plan is likely to have been a model for the terre nuove fiorentine, being basically rectangular and having relatively long parallel rows of rather narrow house lots. In fact, Buselli claims that the Florentine new towns were also designed by use of certain elements of the theoretical Pietrasanta design system. The proportions of the perimeters of San Giovanni and Terranuova would have been determined by the geometric relation of the so-called ‘golden section’. Buselli also claims that the width of the central market places of both towns would have been determined by manipulating the same triangle that was used to arrive at the golden section proportions for the perimeter, and the exact place of the parallel front streets in the Terranuova plan would have been determined by golden section proportions. Comparison of these claims with the present-day town plans leads to the conclusion that they must be largely incorrect. Furthermore, it is in fact unlikely that the golden section was actually used as a proportioning method in the 13th century, since the technique was probably not known at the time.

Eugenio Baldari proposed a hypothetical geometric design method for the plan of San Giovanni. It was largely based on Guidoni’s proposal, which Baldari called ‘theoretical model’ (similar to fig.6.12), but was supplied with a second ‘executive model’. (fig.6.16) It seems that Baldari intended that Guidoni’s ‘theoretical model’ was only used in the design stage, on the drawing board, while the ‘execution model’ was used as the method for the actual laying out of the town plan by the surveyor. Baldari believes that the perimeter proportions of the town were determined by a large octagon, which has twice the size of an

80 Higounet 1962.
81 For instance: San Giovanni measured 462.50 x 190.44 m. (see app.B, n.14, while according to Higounet it would be 487.5 x 162.5 m., which gives deviations of 5.4% and 17.2% (of the smallest values) respectively. Higounet does not explain from where he got these dimensions, but they clearly are wrong. For the dimensions of the other towns, see par.3.9.2.2.
82 See above, n.74.
84 See fig.3.4.
85 Buselli 1970, p.34.
86 See app.B, par.2.2. For the plans that were used, see app.B, n.3.
87 See above, n.39.
88 Baldari 1980.
89 This idea of a twofold design method also seems to stem from Guidoni, who writes that the design geometry often forms a starting point in the design, but that the actual dimensions may be much more pragmatically chosen. (Guidoni 1970, pp.215, 219, 223) In connection with the terre nuove (and the designs of Arnolfo di Cambio) he does, however, not suppose such a procedure. (Guidoni 1970, p.233)
inner octagon that fixed the outer alignments of the inner street blocks.\textsuperscript{90} These inner street blocks fitted in squares of which the diagonals were directly connected to the two small outer squares that regulated the proportions of the market place according to Guidoni.

Comparison with the present-day plan shows that these ideas fit fairly well onto the plan.\textsuperscript{91} It is most likely, however, that this is largely coincidental. The three diagonal squares that would have given the proportions of the piazza are most unlikely, as they do not have dimensions that are logically related. The two larger squares that comprise the inner street blocks seem coincidental, while the rest of the proportions of the plan do not seem to be determined by squares in any way. The same holds true for the octagons.\textsuperscript{92} This makes that Baldari’s theory does not appear very likely. Only the geometric explanation of the perimeter proportions is in itself plausible. But since the geometric construction apparently does not determine other relevant proportions in the plan in a logically coherent way, the agreement between the perimeter proportions and this geometric construction is probably coincidental.

\footnotesize
\textsuperscript{90} In the illustration this does not show very clearly, since the outer octagon is not wholly depicted. \\
\textsuperscript{91} For the plan that was used, see app. B, n.3. \\
\textsuperscript{92} I have tried to fit other squares and other octagons on the plan, but I have not found relevant coincidences.
Charles van den Heuvel suggested some further applications of Friedman’s basic theory. He claimed that a circle with a 30-degree-division (just like a dodecagon) would show significant correspondence with a number of relevant lines in the design of the Scarperia plan. Van den Heuvel gave no comparison of dimensions or illustrations of his suggestion; he only gave short indications in which way Friedman’s ideas could be applied differently in order to get more relevant results. Below, in paragraph 6.4.2 these indications will be checked in comparison with an accurate plan of the town.

Giglio Fiorentino had so far escaped becoming subject of theories of geometric design, since the dimensions mentioned in the document of 1350 appear so straightforwardly arithmetically determined as round numbers of braccia. In 2003, however, Maria Cecilia Bartoli proposed that Giglio’s plan was partly proportioned by a complex geometric method. She proposed that the basis of the design was the piazza, which probably measured 70 x 70 braccia. From this square the length of the town would have been determined by taking the length of the piazza plus four times its diagonal. In this hypothesis the $\sqrt{2}$ relation of the ‘rotated squares’ is once again proposed as a method of determining proportions. According to Bartoli the diagonal of the 70 x 70 b. square was approximated as 100 b., so that a length of 470 b. was the result, which is exactly the length mentioned in the document. The width of the town would subsequently have been determined by taking twice the diagonal (200 b.) as the height of an equilateral triangle, which would give sides of 230.94 b. According to Bartoli this was approximated as 232 b., to which 14 b. were added for the width of the main street. This would give the 246 b.-width of the town which is mentioned in the document.

In my opinion, this way of determining the outline proportions is very unlikely. The method is inconsistent and illogical. The subsequent steps in the process are not logically related. Also, the idea that the square piazza was the basis of the whole plan is difficult to reconcile to the description of 1350, in which the piazza is described as measuring 90 x 70 b. It is far more likely that the outline dimensions given in the description of the Giglio project of 1350, simply resulted from adding the individual dimensions of the internal plan elements, which were rationally determined and preferably rounded off at tens.

Stefano Bertocci suggested a proportioning method for some aspects of San Giovanni’s plan. As many others did before him, he postulates that the proportional relation of the side and the diagonal of the square ($\sqrt{2}$) are crucial in ‘gothic’ architecture. In his opinion, the outer perimeter of the town must have been proportioned by taking the short side as the side of a square to which the diagonal of the square was added to arrive at the long side of the perimeter. This fits very well to my measurements in the present-day plan. Bertocci believes that the inner perimeter of the town was dimensioned in another way. Apparently, he thinks that the inner perimeter was not simply the outer perimeter minus the width of the town wall, which would seem logical.
Instead, the internal perimeter would have been proportioned by taking the short side as the side of a square, the diagonal of which is rotated in two opposite directions, in this way creating overlapping rectangles (with proportions a : $\sqrt{2}a$) of which the diagonals were again rotated in order to arrive at the length of the inner perimeter. This also fits the dimensions that I measured. However, this method of design seems most illogical. If the town walls had the same dimensions on all four sides, which is most likely, it is very unlikely that the inner dimensions had been determined in a very complex way apart from the outer dimensions, which were determined in another complex way. Hence, it must be coincidental that the proposed methods for determining the outer and inner perimeter correspond to the (partly reconstructed) actual dimensions. Since their geometric constructions do not determine other relevant proportions in the plan in a logically coherent way they both seem unlikely to actually have been used for the proportioning of the town.

Bertocci’s third figure is meant to illustrate the way eight out of the 24 wall towers were spaced along the perimeter. (fig. 6.18C) This rather complex method seems very unlikely to actually have been used, since it only regards the positions of a limited part of the wall towers, and since it is much more likely that they were simply spaced evenly along the perimeter sides.

---

104 The difference would be only 0.52%. (see appendix B, par. B.2.6)
6.4.2 Analysis of the theories of Guidoni, Friedman and Pirillo in comparison to the terre nuove plans

Thus, the theories of Guidoni, Friedman and Pirillo are left as possible methods of design for the terre nuove plans. In order to check to what degree these theories actually correspond to the plans of the towns, they must be compared in detail. The design methods are compared to the modern plans and to one another in figures as well as in the measurements of the basic dimensions. This is done in appendix B, pars. B.3.1 to B.3.4. This paragraph is a summary of that detailed analysis.

Regarding the case of Terranuova, there is a strong correspondence between the modern plan and the design system proposed by Friedman. (fig. 6.14) This is demonstrated by figure 6.19, in which the theoretical figure is laid over the digitised modern plan. Friedman’s theory, which is an adaptation of Guidoni’s roughly worked out proposal (fig. 6.13), fits the plan very well, unlike the other relevant theories of design methods by Higounet and Buselli. It appears, however, that the polygon (or radially divided circle) fits the plan better when the radius is 143.75 braccia. (83.89 m.) instead of Friedman’s theoretical radius of 142.91 b. (83.4 m.). The angular points of the polygon pinpoint the axis of the main street, the inner alignments of the streets parallel to the main street, as well as the position of the longitudinal parts of the town wall.

---

105 Additionally, a suggestion by Van den Heuvel as to the application of Friedman’s theory on Scarperia (see par.6.4.1.4) is also still to be considered.
106 These figures are created digitally, by digitising the most accurate paper plans that I could get hold of (see app. B, n.3), and overlaying these with the geometric figures of the theoretical design methods by use of Computer Aided Drafting (Autocad). For a more detailed description of the advantages and possibilities of computer aided drafting in reconstructing design methods, see Stenvert 1991.
107 I have come to this unusual layout in order not to make the text of this chapter too heavily burdened with numbers and details that are not important in the context of the chapter as a whole. But for the study of the design of the terre nuove specifically the numbers and details may be important, and therefore the complete analysis is added in the appendix. Those readers who are specifically interested in the design of the terre nuove plans should read appendix B instead of paragraph 6.4.2.
108 See par.6.4.1.4 and app. B, pars. 2.1, 2.2. See also Boereijn 1994, pp.141-173.
109 This would reduce the percentile difference between theoretical dimensions and my measurements to 0.43%. (see app. B, par.3.1)
It seems illogical, however, that the radius, as theoretically generative value of the whole design, would be 143.75 braccia. It would be more logical that the length of the radius was determined as a rounded number in braccia. Therefore, it seems likely that a radius of 144 braccia (which is only 0.17% more) was chosen. The number 144 was a very self-evident number back in those days, as it is 12 times 12, and the duodecimal system was as common in quantification and calculation as the decimal system is now. Apart from that, the number 144 also bore important symbolic implications, and the dimension of 144 braccia may even have referred to the biblical description of the Heavenly Jerusalem.

With this, it would still be possible that Pirillo’s theory of arithmetic design fits the town plan even better, which would make it appear even more likely to be right. It seems, however, that my measurements correspond better to Friedman’s theory (with the adjusted radius of 144.4 braccia) than to Pirillo’s. From this it may be concluded that the hypothesis of complex geometric design of Friedman (and Guidoni) fits better with the actual dimensions measured in Terranuova and therefore seems more likely to be right. That is, if one accepts that the ‘designer’ did not simply choose for the easiest and most obvious method to lay out a town plan.

Friedman’s theory for San Giovanni is a strongly reduced version of Guidoni’s. (compare figs. 6.12 and 6.15). His theory corresponds quite well to my measurements. With a slightly larger theoretical radius of 97.65 m. instead of 96.68 m., it fits my measurements even better.\footnote{The number twelve had a high symbolic value in contemporary numerology (see par. 8.1.3), and this value was probably even higher for 12 x 12. The dimension of 144 braccia may have referred to the Heavenly Jerusalem as described in the bible in the Revelations of St. John (21:17), where an angel measures the walls, which are 144 ells (whether this is the width or the height is not mentioned). This dimension must already have been a symbolical dimension in the bible, since it is unlikely that the height or the width of a real city wall would have been 144 ells.}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig_620.png}
\caption{Graphic verification of the geometric proportioning method of the plan of San Giovanni according to Guidoni, superimposed on the digitised modern plan. It appears that there is quite a strong correspondence between the modern plan and the design system proposed by Guidoni. The geometric figure seems to mark the centre of the main street, the inner and outer alignments of the parallel streets, the backside of the third and fourth rows of lots from the main street and the outside of the longitudinal sides of the town wall. The figure also appears to mark the proportions of the width and length of the town by the two outer hexagons. But it is not very logical that the width is taken from town wall to town wall, whereas the length is taken from the inner alignments of the transverse wall streets, making this part of the theory questionable. (cf. figs. 6.12, 6.15 and par. B. 3.2)}
\end{figure}

\footnote{The difference between my measurements and the dimensions in Pirillo’s theory is 2.47% on average, and between my measurements and Friedman’s theory 0.48%. (see app.B, par.3.1)}

\footnote{The average difference would be reduced to 0.23%. (see app.B, par.3.2)
However, Guidoni’s theory seems to determine more relevant boundary lines in the plan. From figure 6.20, in which Guidoni’s theoretical figure is laid over the digitised modern plan, it appears that there is quite a strong correspondence between the plan and the hypothetical design method if the radius is chosen as 189 b. (110.30 m.). The geometric figure seems to pinpoint the centre of the main street, the inner and outer alignments of the parallel streets, the backside of the third and fourth rows of lots (counted from the main street) and the outside of the longitudinal sides of the town wall. It appears that an addition can be made to the geometric figure by which it also comes to mark the backside of the house rows right next to the main street. (see fig. 6.21)

Pirillo’s theory concerning San Giovanni is that the house lots are 40, 30 (including the width of the back alley), 25, en 20 braccia (including the back alley) deep, and that the main street would be 20 b. wide. Two 16th-century documents give measured values of relevant dimensions in the town plan. Comparison of Pirillo’s theoretical dimensions with these two plans and my measurements shows that the dimensions proposed by Pirillo are not far from the dimensions measured in the 16th century, but only the width of the main street from Gentile and Batista, and the length of the fourth house lot from Della Zucca correspond precisely to Pirillo’s dimensions. Therefore, it does not seem very likely that Pirillo is right. The lot sizes according to Guidoni’s geometric theory are closer to my measurements than Pirillo’s. These lot sizes are on average also closer to the dimensions given by the 16th-century surveyors than to Pirillo’s.  

113 This radius is obviously not determined by a symbolical number, as was probably the case at Terranuova. The only special dimension of this number, as far as I can see, is that it is three to the fifth power. 

114 The three diagonal squares, which would have determined the proportions of the piazza according to Guidoni fit the plan quite well, in a way that they correspond to what presumably was the original form of the piazza. (fig. b9) The piazza is, however, not very symmetrical or regular in detail, so the agreement is not very clear. More problematical, however, is the fact that this construction with the different sizes of squares is not logical, or geometrically defined, since the size of the middle square is not related to the size of the other two squares in any logical way. Therefore, it seems unlikely that this was in fact the method of design or of laying out the dimensions of the piazza. 

115 This addition is made up by the lines that cross through the centre point and through the four intersection points of the sides of the two rotated hexagons closest to the main street. The four points where these lines touch the outer circle mark the backsides of the first rows of lots. These points can be regarded as the corners of a 24-sided polygon, as at Terranuova. This polygon is not depicted as such in this figure, but it would have the same radius as the hexagons, in this case 189 b. 

116 The two plans with inscribed measurements in braccia are made by maestri Gentile and Batista and by the surveyor Piero della Zucca. (fig.3.13) (Friedman 1988, pp.10, 11, 347-351. Archivio di Stato di Firenze, Piante dei Capitani di parte, cartone XVIII, no.28 ; Archivio di Stato di Firenze, Cinque conservatori del Contado, 358, fol. 602 bto)

117 See appendix B, table VII.
It seems most likely therefore, that the design geometry proposed by Guidoni, possibly with the small addition described above, was indeed used to find the relevant dimensions.

For the plan of Castelfranco di Sopra, Guidoni proposed a much simpler geometric figure underlying its design. (fig. 6.11) It does not appear very likely, however, that this was actually used. The central diagonal square makes no sense and the larger square does not contain the central street blocks with much accuracy – all the more so because the angles are not truly right.118 (fig. 6.22) The outer hexagon, which theoretically describes the relation between the width and the length of the original perimeter, seems to fit quite well on the digitised modern plan. It can be objected, however, that it is not very logical, as the length of the town is given between the outer alignments of the wall streets on the northeast and southwest sides, whereas the width is given between the inner alignments of the wall streets on the other two sides (theoretically; only on the southeast side still verifiable).119

However, close study of the dimensions in the plan, shows that a reduced version of Guidoni’s proposals for Terranuova and San Giovanni accurately pinpoints some important boundary lines in the plan. (fig. 6.23) The figure is a dodecagon with a radius of 234.07 b., and its angular points mark the axis of the main street, the outer building lines of the (remaining parts of the) second parallel streets to both sides of the main street, and the outer building line of the wall street near to the southeast gate.

Thus, it seems quite well possible that this geometry was used in the design of the plan for Castelfranco. Not many dimensions are given by the dodecagon (only two actually) but the similarity of this geometry to that found for San Giovanni, Scarperia (see below) and Terranuova seems to confirm that this geometry really played a role in the design of the dimensions.120

For the terra nuova of Scarperia I have also checked if there are relevant correspondences between the theoretical geometric design methods that were encountered above and the digitised modern plan. Guidoni, Friedman and Pirillo did not extend their theories to this town.121 Nonetheless, it seems possible that a similar method of design was employed here, as is suggested by Van den Heuvel.

---

118 The difference varies between 2 and 5%.
119 When the outside of the lateral wall street would be taken as relevant line for the side of the hexagon to mark, the length of the hexagon along the axis of the main street would be 3.38% too much. See appendix B, par. 3.3.
120 If a special significance is to be found in the number 234 (rounded from the dimension of 234.07 b.) for the length of the radius, the only option I can think of, is that the numbers 2, 3 and 4 are arranged as in the elemental numerical series.
121 According to Guidoni this town is much less interesting because of an unspecified ‘loss of tension’ in comparison to Castelfranco, San Giovanni and Terranuova (Guidoni 1970, p. 229); and Friedman assumes that similar methods of design as he proposed for San Giovanni and Terranuova were not used here because only the house lots on the main street are larger than the three rows of lots behind them. (Friedman 1988, p. 120) See app. B, par. 3.4.
The plan of Scarperia is less regular, particularly in its outline, than Terranuova and San Giovanni, and Castelfranco as it was until around the 17th century. It appears, however, that, just like with Castelfranco, a dodecagon fits on the modern digitised plan in such a way that its angular points mark the axis of the main street, the outer alignments of the parallel streets and the outer alignments of (the remnants of) the secondary parallel streets. As described in chapter 3, the secondary parallel streets were originally probably intended to be the longitudinal wall streets. Just as with regard to Castelfranco, this geometry is relatively limited in its relevance for the plan as a whole, but the method corresponds closely to it, and thereby also to the design systems of San Giovanni and, less directly, of Terranuova. (cf. figs.6.22, 6.21, 6.19)

So, it seems very likely that the town plans of San Giovanni, Castelfranco, Scarperia and Terranuova were partly dimensioned by use of geometric figures. Guidoni’s theory of regular polygons that determined the distance between alignments of rows of lots and streets that lie parallel in the longitudinal direction of the plans, seems very likely. The other part of Guidoni’s theory, concerning geometric squares that give the proportions of the piazzas, appears rather unlikely, however.

Although the basic principle is the same in all four cases (Friedman calls it ‘sine geometry’) there are considerable differences in complexity and refinement. The simplest form is the dodecagon found at Castelfranco and Scarperia, which determines the relative distances between the axis of the main street, the outer alignment of the parallel streets, and the outer alignment of the wall streets (or the streets that were probably planned as such at Scarperia). (figs.6.23-6.24) The same principle is employed in San Giovanni, but there it is extended into a more complicated figure that determines more relevant dimensions. (fig.6.21) There it is not just the angular points of the polygon that determine the place of relevant lines in the plan, but also the crossing-points of the construction lines of the two hexagons. The relevant points lie on the circumferences of three imaginary co-centrical circles, determining the relative positions of eleven, or possibly thirteen.

122 See par.3.8.3.
123 See app.6, par.3.4. The radius of the dodecagon is calculated as best fitting to the measurements when it is 125.68 b. But 125 b. would be the logical dimension to suppose for this radius. Although the number is not known for any specific symbolic meaning, it may have been relevant for the choice of this radius that 125 is 5 x 5 x 5.
124 See par.3.8.3.
125 It should be considered, however, that the more complicated system similar to the one proposed for San Giovanni (fig.6.21), based on the same outer radius of 125.68 b., also seems to pinpoint more relevant lines in the Scarperia plan. (see graphic verification in Boereijn 1994, fig.10.31). It regards the inner building lines of the parallel streets, the backsides of the lots facing the main street (impossible to verify, however, as this feature is rather irregular in Scarperia) and the backsides of the third rows of lots from the main street (also difficult to verify because of irregularity). Unlike San Giovanni, however, the inner building line of the original lateral wall streets is certainly not determined by this system, for which reason it appears less likely.
126 The same geometric figure of the two rotated hexagons may also have been drawn or set out instead of dodecagons for the cases of Castelfranco and Scarperia, and four rotated hexagons may have been made instead of a 24-sided polygon for Terranuova. The way the polygons were probably constructed will be discussed in par.6.4.3.2.
The design system of Terranuova also determines eleven alignments in the plan, between which, just like in San Giovanni, the length of a house lot and the width of a street or alley had to be encompassed, whereas the ultimate dimension determined the width of the wall street and the wall. In its geometric construction, however, the figure of Terranuova is rather different from that of San Giovanni. Being a regular polygon with 24 angular points that determine the relevant lines, this system is simpler and looks more ‘logical’ than the San Giovanni system does. In Terranuova, the length of the radius of the polygon also seems to have determined the length of the whole town as well as the inner alignments of the transverse streets.

The way in which this design geometry works, may seem unnecessarily complicated. This may be an obstacle for accepting it as the actual method of design. It should be considered, however, that the geometric and arithmetic design systems that are used by present-day architects, are often also unnecessarily complicated, and would often also be very difficult to reconstruct if one would not have any information about their form or principle. Concerning the reconstructed design systems of the terre nuove, it certainly is a strong argument in favour of the hypothesis that the same principle can be found in more or less the same way in all the four town plans. Would one have found such geometry in just one of the towns, the chance would be greater that it depended on coincidence, but now this chance is very small.

The town plan of Giglio Fiorentino, the last of the Florentine new towns, was designed in a different way. The document describing its layout mentions only absolute dimensions, which are not reconcilable with similar geometric design by use of regular polygons. Because of its simplicity, this seems to be a more obvious method of design, and therefore Pirillo proposed that a similar simple method of arithmetic design was also used for Terranuova and San Giovanni. It appeared however, that the values of this simpler method

---

127 These are: the axis of the main street, the two backsides of the first rows of lots, the two inner building lines of the parallel streets, the two backsides of the third rows of lots, the two inner building lines of the longitudinal wall streets, the outsides of the longitudinal town walls, and possibly also, but with a greater difference between theoretical and measured position, the two outer building lines of the parallel streets.

128 It is possible, however, that the radiuses of the polygons also played a role in determining the length of the plan of San Giovanni, Castelfranco and Scarperia, but more indirectly. See app.B, n.94.

129 For instance, the famous 20th-century architect Le Corbusier used proportions from the “golden section” and anthropomorphic forms in his dimensioning system called modulor. This design method may serve as an example of a method that would probably never have been correctly reconstructed without information from sources outside the buildings themselves, in this case the writings and drawings of the architect. (Le Corbusier 2000)
correspond less well to the dimensions actually measured in the plans than the values from the proposed geometric design methods.

There remains one aspect related to the question of geometry versus ‘numerical dimensioning’, about which something more needs to be said. The geometric figures that were found to be liable to have been used in the design of the four Florentine new towns did not provide all dimensions. They determined specific relevant lines in the plans, the space between which still had to be divided, in order to determine the width of streets and the length of lots. It is only a matter of course that this was done in such a way that the width of the street as well as the length of the lot were as convenient as possible for their purpose. With this, it is most likely that one of the dimensions was taken in whole braccia (or possibly halves, for small dimensions such as the width of the alleys). In this case, it seems most obvious that the width of the streets was given this rounded value, so that this fixed dimension could be ‘guarded’ easily. It was in the public interest to protect this space from being encroached on by private structures, for which reason it would be convenient when the width of the street or alley had a specific value that was easy to measure.139

From my measurements in the 1:500 plans it is, however, not determinable whether the dimensions of the streets were more likely to have been set out in whole braccia or those of the lots, as both generally diverge considerably from the round number. This could be due to inaccuracy in my measurements or in the plans that I consulted, or to small deviations in the building lines caused by changes over the past seven centuries, or even to inaccuracy in the original laying out of the town plan. From Friedman’s measurements on the spot in Terranuova one can conclude, however, that the widths of the streets are much closer to whole numbers of braccia than the lengths of the lots.131 It is tempting to conclude from this, that this would be the case in the other towns as well, but the dimensions measured by the 16th-century surveyors in San Giovanni do not confirm this.132 So, to be able to say more about this problem, one would first need more data from measurements on the spot. Finally, one must also consider the possibility that although the geometric method of design led to dimensions in irrational numbers, these may all have been rounded to whole numbers (or halves or smaller rational parts) of braccia.

The hypothesis of design by use of regular polygons would be supported if it would be possible to recognise some sort of logical development in this method through time. This is, however, not the case.133 Castelfranco and San Giovanni were founded at the same time, but their geometry is rather different in complexity and relevance for the plans, while Scarperia, which was founded more than six years later, repeats the simpler model of Castelfranco. Terranuova, more than thirty years later, has still another figure underlying its layout, less complex than San Giovanni’s, but with just about as much relevance for the plan. Subsequently, Giglio was designed without using such a geometric method for dimensioning its layout. Looking at the layout of the elements in the plan instead of at the dimensions, it is clearly visible that Giglio has all the typical aspects of the terre nuove plans. The plan design of Giglio also appears to be especially related to the Terranuova design. The back streets, which in Terranuova replaced the alleys of the earlier plans, were used again in Giglio, and the form and dimensions of the piazzas of the two were identical.134 But the method of dimensioning is very different. The highly complex geometry was traded for a very simple way of establishing dimensions in numerical values of the local standard value for longitudinal measurement, the Florentine braccia. Especially rounded numbers dividable by ten were favoured. The preferred relation between dimensions seems to have been the simplest one conceivable: 1 : 2. The main street (14 b.) was related thus to the adjoining lots (28 b.), while the same relation also bound the parallel streets with the adjoining lots (10 : 20 b.) and the middle class lots with the lower class lots (20 : 10 b.). Thus, the layout of Giglio followed the model of the earlier terre nuove in its basic layout, but its dimensions were determined in a much simpler way.

6.4.3 The inspiration for the geometric method used in the terre nuove plan designs and the way it was handled

Now that it is found likely that the terre nuove plans were dimensioned by a complex geometric method, it must be asked where this method could have come from and how it was exactly handled in the process.

---

130 In contemporary sources concerning the maintenance of spatial order in towns, one can often find specific minimal widths of streets mentioned. In Florence, for instance, the minimal widths in the 14th century, would be 12 b. for main streets and 8 b. for other streets (cf. table VII), while in Siena the minimal width for communal streets was 6 b. (Braunfels 1968, pp.102-103)

131 See appendix B, table II.

132 See appendix B, table VII.

133 This also holds true for the development of the elementary structure of the town plans, not considering specific dimensions.

134 Or at least almost identical. See discussion app. B, par.4.
of design. In order not to get lost in a-historical speculation the hypothetical design geometry must be compared with the mathematical knowledge in the period and the technical possibilities. The proposed design method will appear more probable if it can be related to knowledge of which it is known that it existed at the time. Subsequently, in paragraph 6.4.6, the question why the complex method of design was used will be attended to.

In most cases of proposed complex geometric design methods of urban plans, as treated in paragraphs 6.3 and 6.4, authors seem only to have been interested in the reconstruction of a geometric system. The questions of what inspired the use of the complex methods or how the geometry was exactly handled are rarely posed, let alone answered. Friedman is the only author who really goes into the questions of how the geometry was handled in practice and where it may have come from, and therefore his book will be much referred to in the following paragraph. But, just like the other authors, he leaves unasked the question why this complex geometry was used.

6.4.3.1 Possible sources of inspiration for the proposed geometric design method of the terre nuove plans

If it is true that the rather complicated proportioning method by use of polygonal geometry (or ‘sine geometry’) was used for the terre nuove plans, it would seem likely that it was not completely newly invented. The designer must probably have been inspired by a similar use of geometry in other designs or practical applications.

The most obvious place to search for a similar design method as a source of inspiration would be in earlier town plan designs. The town of Pietrasanta, founded by the city-state of Lucca in 1255, is the most likely source of inspiration for the urban form of San Giovanni and Scarperia. (compare figs. 3.4, 3.12, 3.20) But in Pietrasanta the lots of the original layout all seem to have had a more or less equal length (c. 18.5 m.138), for which reason it seems unlikely that a similar design method would have been used, since the essence of this method is that it generates a diminution of the distance between the parallel lines in the design system, which in the terre nuove is reflected in the length of the lots. According to Spagnesi and Properzi the plan of Cittaducale, which was founded in 1308 in central Italy, would have been proportioned in a method very similar to Guidoni’s theory for San Giovanni. In its basic layout of streets, rows of house lots and central piazza, the town plan bears resemblance to Terranuova in particular, but unlike the terre nuove the original lots all seem to have had the same length. Hence, it appears unlikely that a similar design method would have been used there.139 In fact, I know no other towns of the period that seem to be designed by use of a similar proportioning method.

The next best place to search for a source of inspiration would seem the wider field of contemporary architectural design in general. There is no doubt that more or less complicated geometry was used in the design of architectural elements such as window tracery, rib vaulting, polygonal apses and towers, circular stairs, etcetera. For these elements, however, no similar geometric design methods seem to have been used as those that have appeared likely for the terre nuove plans.137 According to Guidoni, however, a more or less similar method was used in roughly the same period for the proportioning of palazzo facades in Tuscany. Among others, the facades of the Palazzo dei Priori in Volterra, the Castello of Poppi and the Palazzo Vecchio in Florence would have been proportioned by use of a geometric method that he calls the ‘quarter circle’.138 (fig. 6.25) This method is based on a quarter of a circle that is divided into equal parts along the circumference. When parallel horizontal lines are drawn through the division points along the circumference, their mutual distance decreases progressively. According to Guidoni the architect Arnolfo di Cambio was the first to have employed this method systematically.139 The ‘quarter circle’ and the proposed design method of the

135 This appears from my measurements in the plan in the scale 1 :1000, Provincia di Lucca, Comune di Pietrasanta, foglio n. 17, levato anno 1951, riprod. anno 1956, that I obtained from the Uffizio tecnico of the commune of Pietrasanta.

136 Spagnesi & Properzi 1972, p.56. In the proposed theory it also seems very illogical that it has no relevance to the outer street blocks. Guidoni also opposes to the suggestion that a similar method would have been used for Cittaducale. (Guidoni 1992, p.95). Jäger (2004) proposed a method by which a number of dimensions in the plan design of the Maltese city of La Valletta would have been determined. The method is similar to Friedman’s proposals for Terranuova and San Giovanni. As far as I can see, the hypothesis does not seem unlikely, but it has little or no relevance for the terre nuove, as La Valletta is more than two centuries younger.

137 Mössel (1900) proposed a design method for the facade of the Münster church in Freiburg (Germany, 13th–14th centuries), which is more or less similar to the method for the terre nuove plans. According to the verification of this method by Hecht it appears, however, that the theoretical dimensions generated by the proposed geometry do not correspond well to the actual dimensions of the facade. (Hecht 1969, pp.288–291, 303) Apart from that, another important objection is that the proposed method lacks coherence and logic.


139 Guidoni 1970, pp.217–224. Guidoni introduced this method in a chapter on the development of Tuscan architecture around the year 1300, in which Arnolfo di Cambio would have played a very important role. It is in the same context that he introduced his theory of the complex geometric design method of the terre nuove, the first of which would have been designed by Arnolfo.
Looking outside the discipline of architecture, one may find similar geometric figures as those that seem to have been used for the design of the *terre nuove* in the contemporary fields of geography and astronomy. The motif of the circle divided into twelve equal parts along its circumference is probably almost as old as the discipline of astronomy. The figure represented the division of the heavens, of time, and implicitly of the cosmos, into equal parts. An implicit geometrical element of this figure was used for the construction of among others sundials. It corresponded to the geometry in the proposed design method of the *terre nuove* plans since parallel lines through the regularly spaced points along the circumference generated progressively receding mutual distances between the lines. The astronomical instrument of the astrolabe, which could be used for observation, measuring and calculation, made use of the same sort of geometry. Some astrolabes had a so-called sine scale, which was used for measuring and calculation. It consisted of a circle or quarter circle in which parallel lines were drawn through equally spaced points along the circumference. Essential element once more, is the progressively receding distance between the lines.

In contemporary charts for sea navigation, so-called portolan charts, one may find compass roses depicted that are strongly reminiscent to the polygonic figures that probably were used for the design of the *terre nuove*. (fig. 6.26) The basic principle is, once more, that the spatial world is divided in equal radial parts

---

140 In the book *Architecture Toscana* by Grandjean de Montigny and A. Famin (Paris 1874) the facades of a number of palaces are drawn, seemingly rather accurate. I have measured these designs and found that in a number of cases it is possible that the ‘quarter circle’ was used for proportioning. In the facade of the Palazzo Strozzi in Florence, the basis is taken on the plinth and the radius reaches to the basis of the arches that carry the crenelation. For the Castello in Poppi I obtained accurate drawings in the scale 1:50 from prof. Domenico Taddei of the architectural department of the University of Florence. Again, the ‘quarter circle’ (divided in five equal parts instead of four) corresponds quite well to the dimensions that I measured in the plan. Unfortunately I have not been able to get hold of a good drawing of the Palazzo dei Priori in Volterra.

141 Following Friedman, one might call this ‘sine geometry’.

142 For the Palazzo Vecchio I obtained copies of drawings in the scale 1:100 kept in the Florentine Archivio Storico Comunale (inv.nrs. 26/74 and 4494 made by Luigi Zumbella). From these, it appears that Guidoni’s theory fits well, when the basis is taken on the plinth and the radius reaches to the basis of the arches that carry the crenelation. For the Castello in Poppi I obtained accurate drawings in the scale 1:50 from prof. Domenico Taddei of the architectural department of the University of Florence. Again, the ‘quarter circle’ (divided in five equal parts instead of four) corresponds quite well to the dimensions that I measured in the plan. Unfortunately I have not been able to get hold of a good drawing of the Palazzo dei Priori in Volterra.

143 It is improbable that the ‘quarter circle’ would have been set out in scale 1:1 on the vertical plane of the facades, since this would be very difficult. It seems more likely that the design would have been drawn in a horizontal plane, whether or not on reduced scale, by use of geometric figures, from which the dimensions would have been measured. This would be similar to the method of design and execution of the facade of the Palazzo Sensodini in Siena (in 1340), as described by Toker. (1985, pp.82-85)

144 The basic principle is, once more, that the spatial world is divided in equal radial parts.
from a centre with a regularly divided circumference. The purpose of such compass roses is to make it easy to navigate on the basic heavenly directions. Portolan charts were no obscure phenomena solely known by seafarers, as it appears that the Florentine writers Dante, Petrarca and Giovanni Villani all knew such charts. The cleric Opicinus de Canistris (1296 - before 1352), who left a number of wonderful scriptures and drawings in which he records his view of the cosmos, even seems to have ascribed a special, more or less magical, significance to the compass rose. It seems that he interpreted it not so much as a figure that helps to order the spatial world, but rather as some hidden reality that lies behind the visible world.

In this context, it may not be omitted to discuss the fact that in the architectural treatise De Architectura libri X by Vitruvius, from the first century A.D., the compass rose is described as instrument for correctly setting out urban plans in the field. Vitruvius extensively describes the way the streets ought to be oriented with regard to the direction of the winds in order to prevent unhealthy and unpleasant winds to take a hold of the people in the streets. He also explains the way a compass rose must be constructed in the very centre of the future urban layout in order to come to the right orientation of the streets. The idea of constructing a compass rose in the centre of the urban layout is reminiscent to the polygonal figures that probably were used for the design of the terre nuove, which were also centred on the very heart of the urban layout and which look like a sort of overdimensioned compass roses. A significant difference, however, is that the polygonal figures of the proposed design method do not mark eight directions from the centre, as Vitruvius’ compass rose does, but 12 or 24. A more important difference is that the design method marked the distance between the streets and alleys rather than their direction. In the plans of the terre nuove no regularity can be recognised with regard to their orientation that might be based on the Vitruvian rules: only the topography of the landscape and the roads in it seem to have determined the orientation of the plans. Hence, the correspondence between the compass rose and the figures of the design method is only superficial. It is well possible that the planners of the terre knew Vitruvius’ treatise, but it is impossible to tell whether the design geometry is somehow inspired

147 Friedman 1988, pp.133-138. The number of the parts is not necessarily twelve, but it always is a multiple of four, because the four elementary directions of the compass are the essential basis of the division.
150 Vitruvius 1962, book 1, ch.6.
151 This passage probably led to the misinterpretation of the Vitruvian street plan as having a radial structure in the 15th century. (see par.10.3, n.82)
152 Kramsky 1967, pp.36-41. See par.10.3.
for instance, painted in the rose windows of cathedrals or in the vaults of churches and spaces with a ceremonial function, as for instance the great cupola of San Marco in Venice. Such diagrams of cosmic schemes were also drawn in manuscripts, such as those made by Opicinus de Canistris, which were already referred to above. In the one depicted in figure 6.27, for instance, among others the signs of the zodiac, the major prophets, the tribes of Israel, the disciples, the winds and the four evangelists take their place in the scheme. These elements represent among others time (the months of the year), the celestial regions, the old and the new testament, and thereby the elementary Christian history of the world including the future apocalypse, the regions of the world, and the regions of Christendom (by the disciples). The dodecagon or wheel with 12 spokes can also be seen as reference to the Heavenly Jerusalem, which was clearly described as square (actually even cubic) in the bible (see par.8.1.1), it is likely that the image of heaven as being circular played a role in its centre. (Edson 1997, p.45)

It is more likely, in my opinion, that the polygonal figures used in the plan design of the terre nue were inspired by symbolic figures of cosmic significance. These figures were related to the compass roses, as the cosmic symbols implicitly also referred to the spatial world with its directions. The figure of the wheel with 12 spokes, whether in the form of a circle or a polygon, is known to have been a symbol for the cosmos since ancient times. The radial partition into 12 or, as a derivative, 24 parts, could refer to the stages of cyclical time, to the different parts of the world, the signs of the zodiac, or to images from the Judeo-Christian tradition, such as the 12 tribes of Israel, the 12 disciples or the 24 elders. In all cases, the basic idea is that different parts make up a whole that is essentially complete and perfect. Such schemes can be found, for instance, in the rose windows of cathedrals or in the vaults of churches and spaces with a ceremonial function, as for instance the great cupola of San Marco in Venice. Such diagrams of cosmic schemes were also drawn in manuscripts, such as those made by Opicinus de Canistris, which were already referred to above. In the one depicted in figure 6.27, for instance, among others the signs of the zodiac, the major prophets, the tribes of Israel, the disciples, the winds and the four evangelists take their place in the scheme. These elements represent among others time (the months of the year), the celestial regions, the old and the new testament, and thereby the elementary Christian history of the world including the future apocalypse, the regions of the world, and the regions of Christendom (by the disciples). The dodecagon or wheel with 12 spokes can also be seen as reference to the Heavenly Jerusalem, which was a sort of Christian utopia. The Heavenly Jerusalem that was described in the bible has 12 gates, one for every tribe of Israel, and had a square plan. But in many images of the 6th to 16th centuries it is depicted in a circular or polygonal form, with the twelve gates equally spaced along the circumference. It is not impossible that the polygonal figures of the proportioning method of the terre referred to this image of the Heavenly Jerusalem.

---

153 The polygons of the proposed design method of the terre also bears resemblance to figures that according to Vitruvius were used for the design of theatre stages. (book V, ch. 6 and 7) It regards circles of which the circumference was divided in 12 equal parts by three squares (Greek theatre) or four triangles (Latin theatre). (Isler S.D.) It does not seem likely, however, that these figures have inspired the planners of the terre, since the architectural products and the purpose of the figures were very different.

154 Serra 1954, pp.118-126.

155 Ever since antiquity, circular diagrams (whether or not radially subdivided) were used as models to visualise cosmic relations. One of the most famous and influential sources where this can be found is the encyclopedia of Isidore of Seville from the 6th century. (Suckale 1981, pp.264-265, 270, 284) The circle in itself served as a symbol for divine order in the universe, for virtue, for the heavens, etc. (see par.8.1.1, n.11) When this circle was radially subdivided into 12 parts (whether or not making it polygonal) it could also be seen as the cosmos (Marconi 1973, p.48) or be interpreted as the majesty of the Holy Trinity in all four directions of the world (Krautheimer 1969, p.123). The number 12 was a number of the greatest significance in cosmic thought and, derived from it, in practical arithmetic, since the duodecimal system was crucial in systems of quantification, as well as in subdivisions of all sorts of bodies, as for instance in the military and political organisation of towns and their administration. (see par.8.1.2)

156 Suckale 1981.

157 Soloman 1936, vol.1, pp.166, 246, cf. vol.11; Soloman 1953; Marconi 1973, pp.42, 48. Other elements that Opicinus adopted in his 12-part schemes were, among others, the ‘small prophets’, body parts, the months, the patriarchs and the parts of the credo. (Soloman 1956, vol.5, p.164) Another example is a 12-parted circular scheme in a manuscript of Isidore’s De natura rerum, which only contains the 12 main winds, but which is explicitly cosmological in its meaning, since the words mundus and kosmos mark its centre. (Edson 1997, p.45)

158 Revelations 21:9-27; Ezekiel 48:30-35.

159 The tradition of the depiction of the Heavenly Jerusalem in circular form probably stems from Rome, whereas the tradition of the square depiction (see fig.8.1) probably comes from the Beatus manuscripts from the Iberian peninsula. (Schiller 1991, vol.5, p.172, plates 748-757; see also Braunfels 1953, pp.49-50, Frugoni 1991, pp.20-21) Since the Heavenly Jerusalem is clearly described as square (actually even cubic) in the bible (see par.8.1.1), it is likely that the image of heaven as being circular played a role in the circular depiction. (Finotto 1972, p.95)
So, the figures of the proposed geometric design methods for the terrenuove plans, containing polygons or circles that are regularly divided along the circumference, may have carried symbolic meaning as reference to, or connection with, the higher order of the divinely created universe (the cosmos) or the redemption (the apocalypse) and the Christian image of heaven (the Heavenly Jerusalem). In this way, the design method, of which the essential geometric figures are invisible in the built form but which generated a number of essential proportions in the plans, may have connected the towns to the whole cosmos and the essential Christian utopia. In my opinion, this is the most likely motivation for the use of the specific geometric figures in dimensioning the terrenuove plans. If this hypothesis is right, it is still debatable to what extent this meaning was explicitly known to patrons, planners, inhabitants and public.

With this, however, it is not impossible that the proposed design geometry was also inspired by similar methods used in architectural design. No such methods are directly known from sources, but the ‘quarter circle’ method proposed by Guidoni for the proportioning of palazzo facades in 13th-14th-century Tuscany may have been important in this respect.

### 6.4.3.2 How was the design geometry handled?

There are several possible ways in which the polygonal figures may have been handled in order to arrive at the dimensions they generate in the plans of the terrenuove. These different possibilities will be described in this paragraph and it will be argued which is the most likely way. This is, however, far from easy, since it is a rather complicated technical matter.

The most obvious method seems that the polygonal figures were set out on the land by use of two ropes of equal length and pegs or stones for marking relevant points.\(^{160}\) This may have worked as follows. First the centre was determined, which was to become the centre of town, and a straight line was drawn right through it, having a rope’s length in both directions. This line would become the central axis of the main street and the whole urban structure. A hexagon was created by stretching the ropes from the centre point and the end points of the line in both directions to common end points, giving four angular points of the hexagon while the two end points of the central line make up the other two angular points.\(^{161}\) The hexagon could be ‘extended’ into a dodecagon by stretching the ropes from the centre point through the middle of the sides of the hexagon, so that the other six angular points were found.\(^{162}\) The dodecagon could be extended into a polygon with 24 sides in a similar way.\(^{163}\)

It is not known whether planners of architectural works or surveyors actually did set out such large-scale geometric figures on the ground. Information on the contemporary knowledge of geometry of such people can be retrieved from treatises that were written at the time, so-called practica geometriae and abacusscripts. In the 13th and 14th centuries the number of these treatises increased rapidly, as knowledge of geometry and arithmetic was increasingly spread. Some of these treatises were written for particular professional

---

\(^{160}\) It is known from various sources that surveyors used ropes and pegs in the period under consideration. (Binding 1985; Binding 1993, pp. 344-348)

\(^{161}\) Finding the centre points of the sides is easy, as they implicitly have the length of the rope, and the middle point of a rope is easy to find by folding it double. It is also possible to find the angular points of a dodecagon by making a right angled cross with arms of a rope’s length, which gives the first four angular points, and determining the intersection points of the five (imaginary) circles around the centre point and the four angular points. This method could not be used, however, for the design method that probably was used at San Giovanni, as this essentially involved the intersections of the construction lines of the two hexagons that implicitly make up the dodecagon.

\(^{162}\) For the proposed design method of San Giovanni (fig.6.21), the intersections of the sides of the two hexagons are found by stretching the ropes along the sides. And the points that mark the back boundary of the rows of lots facing the main street are found by stretching the rope from the centre point through the relevant intersections of the sides of the hexagons.

---

1. fig. 6.28: Figurative depiction of the method of construction of a regular hexagon by the use of two ropes or chains of equal length. From points A, B and C on the red baseline (which is twice as long as the rope), the ropes are stretched to four common end points (D, E, F, G), which form the four other corner points of the hexagon besides A and C.
groups, as for instance bankers or surveyors. The treatises for this latter professional group were generally called practica geometria, which means ‘practical geometry’. Apart from more general mathematics, they particularly treated geometry. A part of these treatises regards the measuring of the area of pieces of land by dividing complex shapes into various geometric shapes that could be measured and of which the areas could be calculated. It is a known fact that around 1300 both the practica geometriae were relatively widely known, and that many copies circulated in Florence. In the late 13th and 14th centuries many inhabitants of Florence also followed courses in practical mathematics.

So, it is well possible that the planners of the terre nuove were familiar with the contents of the practica geometriae. However, the geometric theorems and methods with regard to surveying that are treated in these books solely regard measuring and calculating distances and areas; they do not concern the setting out or designing of (town) plans or, for that case, complex geometric figures.

In his article of 1970, Friedman deemed it most probable that the polygons were set out by use of ropes or chains. In his book of 1988, however, he describes a very different way in which the design geometry would have been handled in his opinion. In this version the polygons were not actually set out on the ground: the relevant distances would have been calculated by use of a trigonometric table of chords and subsequently they would have been measured out on the ground. Tables of chords originally were used for astronomic calculations, but their use was extended to other applications related to sine geometry after they were described in specific scholarly works and some of the mathematic abacus manuscripts in the 13th century.

In my opinion, however, the use of the table of chords seems rather unlikely, as this application of the table is unknown for the period and since it is a very complicated process to calculate the dimensions by the use of the table. It seems more likely that the figures of the compass roses or cosmic symbols, or possibly the figures on astrolabes or the proportioning method of palazzo facades, directly inspired the setting out of more or less similar figures on the ground for the proportioning of the town plans.

Friedman’s main argument in favour of the use of the table of chords, is that there is no evidence that circles or other geometric figures were set out on such a large scale as in the terre nuove plans in the period under consideration. Although it is true that there is no evidence, it is not impossible that such large-scale figures were actually set out. An objection is that the large scale, with basic distances of more than 80 m., would lead to inaccuracies, as the ropes that were made at the time were rather elastic and varied in length with the humidity and temperature, and chains of such length would be very heavy. But still, it would not be impossible. When a surveyor measured pieces of land, he also used ropes of considerable length. So probably, an experienced surveyor may have been able to handle a rope in such a way that its elasticity would not be a great

---

163 Van Egmond 1976, pp. 13, 31. Many of the practica geometriae were not particularly written for surveyors or other kinds of professionals, however. In fact, the contents often barely appear to be aimed at practice at all. (see par. 7.1)
164 Van Egmond 1976, pp. 6, 100, 115, 146, 148, 152, 156; Friedman 1988, pp. 135, 130.
165 Van Egmond 1976, p. 6. In 1343, Giovanni Villani wrote that six abacus schools existed in Florence, having about 1000 to 1200 students, which would be circa one percent of the population of the city.
166 Surveying techniques were required for setting out the geometric figures on the land and for setting out the town plans that partly were determined by the figures. For designing the plans such may not have been necessary. It is well possible, however, that both were done, or directed by, the same person(s). (see par. 7.5). It should be considered here, that building masters in the period sometimes also appear to have been experienced in surveying. (Binding 1993, pp. 240-243).
167 Friedman 1970, n. 24. Friedman also mentioned the possibility that the polygons were set out by an instrument for measuring angles. This is less likely in his opinion, since this method is more liable to inaccuracy and since it is not known whether such an instrument for measuring angles in the horizontal plane was ever used before 1432, when it was first described by Alberti. (Friedman 1988, p. 256, n. 33)
169 The table of chords was based on antique knowledge described in Ptolemy’s Almagest. It was extensively described in Latin around 1220 by the highly influential mathematician Leonardo Fibonacci, after which it was copied many times. (Friedman 1988, pp. 123-126)
170 In fact, Friedman uses the same argument the other way around: no similar use of polygons is recorded to have been used by ‘medieval masons’, and therefore he prefers the option of the tables of chords. (Friedman 1988, pp. 142, 265, n. 56). The fact that the diameters of the various polygons for the different town plans do not structurally represent whole numbers in pertiche (one pertica measured 3 braccia = 2.918 m.) or multiples of the 42 pertiche = (one braccio = 2.918 m.) or multiples of the 42 pertiche from Fibonacci’s table of chords, makes Friedman’s theory less probable, since it would require many rather difficult calculations. The radii of the proposed figures would be: Castelfranco: 234 b. (136.56 m.); San Giovanni: 189 b. (110.30 m.); Scarperia: 125 b. (72.95 m.); Terranuova: 144 b. (84.04 m.). So, only the 125 b. radius for Scarperia measures a whole number in pertiche. The use of the table appears even less plausible when the relevant points within one design method are not all on the same circumference, but on three concentric circumferences, as in the proposed geometric system for the design of San Giovanni’s plan. (see fig. 6.21) If the table of chords was used here, the relevant points must have been calculated for three different radii (theoretically of 189 b., 156.88 b. and 109.45 b.), which must have been very hard or even impossible.
171 It must be considered that even when the tables would have been used, the regular polygons must still have been the essential basis of the design method, which must have been envisaged in some way, probably in a drawing. It seems rather complex and therefore unlikely that an initial design was made with an actual polygon, after which the table of chords had to be adapted to the radius of the imaginary circle (or polygon) on the real scale of the town (i.e., a different length for every town) and that subsequently the calculated relevant distances were set out, whereas the whole polygonal figure could also have been set out relatively easily.
172 Friedman 1988, pp. 135, 256-258, n. 33.
Friedman, would have been calculated with the help of a ‘table of chords’, and were not actually set out as seen in this figure. 

1974 (fig.6.14), but here the principle is depicted with the chords of the angles of 30, 60, 90, 120, and 150 degrees which, according to Friedman, would have been calculated with the help of a ‘table of chords’, and were not actually set out as seen in this figure.

It is not known how its form was laid out, but it is most likely that long ropes were circled around a centre point.

As mentioned above, the great length of the ropes may have made it quite difficult to set out the figures with a satisfactory amount of accuracy. Therefore, it does not seem unlikely that the figures were set out at a reduced scale from the same centre point. It is possible that the relevant distances of the reduced scale were sized to the full scale by multiplying the radius or the distance from the central axis (perpendicular to it) with a specific factor. This would be quite simple, by ‘overturning’ the rope of the specific dimension given by the reduced-scale figure a specific number of times along straight (sight)lines. In order not to get too much inaccuracy, the figure must have had a minimal diameter of circa 10 m.177

The most obvious reduced scale for setting out the figures seems to be the half scale. In order to multiply the relevant distances from the half scale to the full scale, they only have to be doubled by ‘turning over’ the rope once. Construction on this scale would even make some steps in the construction of the figure redundant. In the half-scale figure the outermost points already mark the outer building lines of the parallel streets next to the main street in San Giovanni, Castelfranco and Scarperia. (figs.6.21, 6.23, 6.24; cf. fig.6.30) This makes that the distances to the 30˚ angular points (with respect to the main axis) would not have to be doubled.178 (see fig.6.30) The main advantage of the half-scale construction, however, would probably have

---

174 Methods for making the ropes less elastic, were to treat them with oil, sulphur, hot or dry wax, keeping the ropes wholly wet or wholly dry, and only using old ropes. (Booz 1966, p.82)

175 The Prussian surveying treatise Geometria Culmensis (from 1393-1407) mentions the use of ropes of 10 rods length (c. 45 m.). According to Kiely such a rope, when treated correctly, made it possible with an inaccuracy of only 0.005%. (Pouls 1997, p.47, referring to E.R. Kiely, Surveying Instruments. Their history and classroom use. New York, 1947, p.10) To me, this seems somewhat too optimistic, however. The early 15th-century surveying treatise of Bertrand Boyset of Arles states that it is better to measure with a wooden rod than with a rope, but if a rope is used, it should be 10 or 12 destres (40.70 or 48.84 m.) long. (Guerrero 1995, p.96) It is obvious that a rod was too short to accurately set out the polygons on the full scale. Much longer ropes did exist, but it is unclear whether these were used for measuring or setting out geometric figures. Booz mentions that for the building of a church in Nürnberg a rope was used with a length of 520 ft. (c.110 m.). (Booz 1966, p.82)

176 It regards, for instance the round city of Gur, founded in 126 BC in Sassanid Persia, which has a radius of over 200 m. (Eghi 1955, pp.263-265; Johnston 1983, p.16) In Europe there are examples in the form of the fortresses that were built in the 5th and 10th centuries by the Vikings or as shelters against them. The most well-known are the Viking fortresses of Trelleborg and Aggersborg in Denmark, of around 980, which have quite regular circular layouts with radiuses of c. 85 m. resp. c. 120 m. (fig.8.4; Müller 1961, pp.102-103; Schwinekofer 1986, p.103) Circular fortifications that were probably built to provide protection against the Viking raiders were built in the coastal area of the Low Countries. The stronghold of Ost Soubourg in The Netherlands had a diameter of 125 m. (Triumpe Burger 1973; Schwinekofer 1986, p.123) The Etang de Montady, near Béziers, was reclaimed in 1248-1258. In the very centre of the radial layout there is a circular field with a radius of circa 150 m. (Friedman 1988, p.257; see also http://fr.wikipedia.org/wiki/%C3%8Etang_de_Montady)

177 Friedman not only refutes the possibility that the figures were set out on the full scale; he also rejects the possibility that they were set out on a reduced scale. His argument is, again, that there is no proof that such geometry was used by ‘medieval surveyors’ or in ‘medieval architectural geometry’. (Friedman 1988, p.263, n.6a) This is, however, arbitrary use of the argument, since there is no proof of actual practical use of the tables of chords either.

178 In n.126 above, it is mentioned that a more complex design method similar to that of San Giovanni (fig.6.21) may also have pinpointed the inner building lines of the parallel streets, and possibly also the back boundaries of the first and third rows of lots from the main street in Scarperia. This seemed not very likely, however, because it is not possible to reconstruct the exact place of the back boundaries of the lots and because, unlike at San Giovanni, the inner building lines of the original longitudinal wall
been that the ropes of half the length would make it easier to set out the figures on the ground and would probably also make the construction more geometrically accurate.

Another possibility is that the geometric figures were drawn in plans, from which the relevant dimensions were measured and multiplied to the full scale of the town plans. There is no proof that drawings of small-scale plans were used in the planning of new towns. It has long been thought that small-scale drawings were not used at all in architectural planning before the 15th century, since hardly any such drawings were known from the period. By now it has been clearly proven, however, that particularly since the 13th century, drawings were used in the design and building process of prominent architectural projects. For various reasons, few of these drawings have been preserved, and of urban planning projects no drawings of plan designs are known at all. Town plans did exist, however: since the early 12th century at least, plans were drawn of existing cities. But no drawings of town plan designs are known from earlier than the 15th century. A preserved drawing which is closest to a new town design, is a plan of 1306, depicting the Tuscan town of Talamone, which was re-founded and rebuilt just a few years before. The plan records who is going to occupy or who is occupying which house lot.

---

179 Research in the past few decades has shown that drawings were becoming increasingly common in the practice of highbrow architecture, such as in cathedral design, at least since about the middle of the 13th century. See Binding 1993, pp. 187-202; Harvey 1974, pp. 101-109, 241; Briggs 1974, pp. 96-98; Toker 1985, pp. 70-74; Recht 1989; Ascani 1997.

180 It regards, for instance, highly schematic plans for the depiction of famous cities such as Jerusalem or Rome, or more detailed plans that were used for the administration of public space or of urban landownership. See Lavedan & Hugueney 1974, fig. 62; Friedman 1988, pp. 51-52; Braunfels 1953, pp. 99; Lobel 1969; Schulz 1978, pp. 440, 445; Shelton & Harvey 1986.

181 The first drawings of urban ground plan designs that have been preserved are from the famous theoretical treatises and urban ideal models of the 15th century, such as those by Francesco di Giorgio Martini and Filarete. (see pars. 8.4, 10.3)

182 Braunfels 1953, pp. 77-78. In the plan, which is not on scale and which does not appear to be drawn by a professional draftsman, the names of the future settlers are written in the various house lots.
This drawing may not be an urban plan design; it certainly illustrates that the concept of a ground plan drawing of a new town was not unknown to the period. For the creation of new urban structures planners generally must have had an idea of the plan form of the settlement. A large-scale spatial urban structure that was to be planned must in the first stage have been conceived as a specific piece of land, within which different areas were destined for different ownership and for different functions, such as roads, market places, moats and house lots. In order to have any overview, the planners must have had an idea of a ground plan. This ground plan must at least have been a concept in the mind of the planner, and was not necessarily drawn, although that would seem likely for the cases where various people were involved in the planning. The accurate description of the plan of Giglio Fiorentino clearly shows that the concept of the town plan was quite clear to the planner(s) of that town.183 (see appendix A and fig.3.27)

With regard to the terre nuove it appears highly probable that drawings must have been made in the design stage. If it is correct that they were designed by use of the proposed polygonal geometry, which is rather complex, it seems almost impossible that it was conceived and recorded only for the mind’s eye, without the help of some sort of depiction.

It seems possible that the actual town plan was set out on the basis of dimensions that were measured from the small-scale drawing and multiplied to the full scale. This method would only have been chosen if the plan drawing was accurate and on a specific scale that was considered large enough to take measurements from. According to Toker a similar method was used in the design for the Palazzo Sansedoni in Siena around 1340.184

As far as I can see, there are no disqualifying arguments against the possibility that the geometric figures were set out, either on the site or in a drawing, and were used, directly or indirectly, to arrive at the relevant dimensions in the town plans. The most likely option appears to be that the figures were set out at half scale on the site. But it is also possible that the figures were set out on a smaller scale or in a drawing (not on the site itself), and it is not impossible either, that they were set out on the full scale.

183 Similar but less accurate descriptions for new urban structures of roughly the same period regard the new town of Fontanetto Po (1323; see fig.8.8) in Lombardy and the exact location of streets that were to be laid out in the new extension of the city of Breccia (1237), also in Lombardy. See par.8.6.3; Panero 1979; Guidoni 1992 (II), pp.84, 96, 354-357; Friedman 1988, pp.31, 53-54, 137-143; Boereijn 2005.

184 Franklin Toker studied a drawing in a building contract for the palazzo. The drawing appears to have been a copy of the actual design drawing, both in the scale 1 : 48 (this actually was 1/4 crazia : 1 braccio). Dimensions are written in the drawing, which probably were rounded off from irrational dimensions determined by geometric manipulations. (Toker 1985, pp.30-39) So, the dimensions must have been measured in quarter crazia (just over one cm.) in the original drawing and be scaled up to braccia. If the same scale was chosen for the design of the plan of San Giovanni, the radiuses of the polygons would have measured up to 2.3 m. This would not be impossible, knowing that architectural designs were not necessarily made on parchment or paper but also on drawing floors. (see literature n.179 above).
The motive for the use of design geometry in the \textit{terre nuove} plans

Now it is time to ask the crucial question why such a complex method of design would have been used. As mentioned earlier, this question is left largely unanswered or even completely unasked in much of the literature. Only a few scholars in the broader field of architectural design in the period under consideration have addressed this question in their publications. Based on their work, a possible motive for the use of complex design geometry will be proposed in this paragraph.

In the final paragraph of section 6.4.3.1 it has been argued that the use of polygons or regularly divided circles in the proposed design method may have been inspired by the symbolic meaning of such figures. This is already a reference to a motive for the use of the specific form of the geometry. According to the relevant scholarly literature, however, there is also a more general explanation for the use of geometry in architectural design of the concerned period: in contemporary thought geometry was regarded as the divine method of the creation of the universe, and this method was imitated in order to achieve harmony with the divine creation.\footnote{186}

From the philosophical writings of the 11th to 13th centuries it clearly appears that great importance was attached to the liberal art of geometry. This attitude is represented among others by the ideas of philosophers such as Robert Grosseteste, who found that only by the study of ‘[…] lines, angles and figures, the universe can be understood’\footnote{185}, or by the well-known image of God as the creator of the universe making use of dividers.\footnote{187} \cite{fig.6.3.2} Great importance was attached to the biblical text from the \textit{Liber Sapientiae}, which says that God ‘has ordered all things by measure and number and weight.’\footnote{188} Inspired by this text, church father Augustine wrote in the early 5th century ‘order is the means which determines everything that is created by God’ and ‘nothing exists outside of the divine order’.\footnote{189} Since around the 11th century, scholars in the highly important cathedral schools of Paris and Chartres were inspired by this idea, thinking they could gain crucial knowledge of the divine creation, and thereby of divine thought, by studying the mathematical laws of the natural world. It is in this context that the above-quoted phrase from Grosseteste must be seen.\footnote{190} In the 13th century, Bonaventura wrote that nothing in the universe is unordered because God created it.\footnote{191} So, the arithmetic and geometric order was considered to be an essential link between God and the natural world, which was (partly) apprehensible for mankind through the study of mathematics.\footnote{192}

Mathematical order was also thought of as a precondition for beauty and fairness in art and human creations in general. Unlike in the present day, the goal of art generally was not considered to be the communication of personal or collective feelings, but rather the imitation of the, essentially perfect, divine creation.\footnote{193}

The scholars that have proposed the use of complex geometry in ‘medieval architectural design’ mostly seem to have been inspired by this kind of thought, although they often did not explicitly mention this.

Now, is this supposed motive for the use of design geometry also valid for the design of the \textit{terre nuove}? This question is all the more relevant since the \textit{terre nuove} form the only proposed case of complex design geometry in new town planning in the period under consideration that has been found truly probable so far.

What the proposed design method of the \textit{terre nuove} plans mainly does, is determine the diminishing length of the lots, with more or less detail in the different towns according to the different variations of the design method. There was, however, no inert need to establish these dimensions by way of such a complex geometric method. This is clearly demonstrated by the case of Giglio Fiorentino, where the length of the lots...

\begin{thebibliography}{10}
\setlength{\itemsep}{0pt}
\footnote{185}{
\footnote{186}{
Binding 1993, p.352; Binding 1996, p.429.}
\footnote{187}{
Block Friedman 1974; Zahlten 1979, pp.153-156; Kraft 1985, p.38. This attitude can also be found clearly formulated in a poem by Alanus of Lille (c. 1128-1202).
(Tatarikewicz 1970, p.290)
Wagner 1983.
}
\footnote{188}{
}
\footnote{189}{
‘Ordnung ist das Mittel, durch das alles bestimmt wird, was Gott festgelegt hat’ (Augustinus, De Ordine I/10.28; see also II/10.11; II/7.21), ‘nichts steht außerhalb der göttlichen Ordnung’ (Augustinus, De Ordine II/241 Cited in Naredi-Rainer 1982, p.19. See also Gelernter 1995, p.75.}
\footnote{190}{
This attitude was also very important in the work of, among others, Thierry of Chartres, Hugh of St. Victor and Roger Bacon. Concerning the aspect of geometry which was believed to be part of divine thought, see Von Simson 1995, pp.25-31; Binding 1996, pp.189-200, 424-429; Edgerton 1991, pp.45-47, 288; Naredi-Rainer 1982, pp.19-20; Masi 1983, pp.115-130; Binding 1996, pp.189-193.}
\footnote{191}{
Bonaventura, \textit{Commentarii in quator libros sententiarum}, Lib.II, dist.6, art.3, quest.6; citat in Tatarikewicz 1970, p.60. In the work of, among others, William of Conches the same idea can be recognized.
}
\footnote{192}{
Masi 1983, p.159. This was still an actual thought in the late 16th century, as can be read in the work of among others Johannes Kepler (Dijksterhuis, 1977, p.339), and in the early 17th century, as can be read in Galileo Galilei’s \textit{The Assayer}, which states that ‘Philosophy is written in this grand book, the universe, […] which is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures, without which it is humanly impossible to understand a single word of it.’ (Crosby 1997, p.240; Gelernter 1995, p.122) Note the correspondence between this idea and the phrase from Grosseteste cited above.
}
\footnote{193}{
}
\end{thebibliography}
Possibly, the designer of the geometric scheme was also aiming at personal reward and fame, or at least emancipation of his profession. Geometry was a science that stood in high esteem, for being one of the seven liberal arts. With geometric figures and to do things in ways that seem unnecessarily complex to the pragmatical mind.

Another possible motive for the use of a complex geometric design method may have been more mundane. It is a cosmological idea that motivates the use of such proportioning methods and that inspires their forms. Much like these modern versions, the idea behind the design geometry of the Florentine new towns may have been that it made the structure harmonious in its dimensions, in order to make life in and around it better, as the structure is in better harmony with the cosmos that the designer has in mind. So, it was not so much a question of trying to make the design beautiful, which in some modern cases may be an important facet of the motive, but rather of making it harmonious, in itself and with everything around it and in it.197

At first sight, it may seem strange that the geometric scheme remains invisible in reality and even in the plan, but was only used as a proportioning device in order to dimension a plan which has a shape that is completely different from the polygonal figure. This seems less strange, however, when it is considered that architects from other periods and from the near past also used proportioning systems, such as the golden section or the modulor, without making them explicitly visible in the resulting design.198 They mostly did so because they believed that the product would thus be more harmonious in itself as well as in relation to nature. In essence, it is a cosmological idea that motivates the use of such proportioning methods and that inspires their forms. To some 'modern' people with a rationalist view of the world, such may seem a naive and magical, almost superstitious, way of acting. For the terre nova, but it seems likely. The fact that the form was chosen in this way clearly proves that a sort of magical cosmological thought did play a role in the foundation of the terre.

Another possible motive for the use of a complex geometric design method may have been more mundane. Possibly, the designer of the geometric scheme was also aiming at personal reward and fame, or at least emancipation of his profession. Geometry was a science that stood in high esteem, for being one of the seven arts liberales, which made up the curriculum that according to tradition would be the path leading to wisdom.198 Therefore, the designer of the plan, whatever his professional background was, may have used the geometric scheme in the design to show his learned skill, in order to emancipate his position towards his patrons or to his social environment in general.199 The least one may conclude, is that the designer must have enjoyed to 'play' with geometric figures and to do things in ways that seem unnecessarily complex to the pragmatical mind.

---

194 The question why the house lots have different lengths will be treated in par. 8.5.2.1.
195 See par. 6.4.3.1.
197 To some 'modern' people with a rationalist view of the world, such may seem a naive and magical, almost superstitious, way of acting. For the terre nova, but it seems likely. The fact that the form was chosen in this way clearly proves that a sort of magical cosmological thought did play a role in the foundation of the terre.
199 A similar desire for personal reward and professional emancipation via the application of geometry can be recognised in the so-called 'sketchbook' of Villard d'Honnecourt of c. 1230. In the introduction Villard describes that the manuscript teaches among others 'the art of drawing, or portraiture, as the science of geometry commands and teaches'. (Bechmann 1991, p. 7) In several drawings one can indeed find geometric constructions. But many of these are rather un-pragmatic: they seem to have been introduced in order to give the drawings, for instance of animals and human figures, a greater validity and the knowledge of its maker a sort of absoluteness. According to Bechmann (1991, pp. 512-56) the figures are secret mnemonic devices that stand for complex geometric methods used in architectural design, for instance of window tracery or church plans. This is, however, rather implausible. Unlike the geometry needed to design for instance vaults of polygonal apses, this geometry is not practical: it seems to serve primarily to emancipate Villard's (professional) status and to validate his knowledge. (see Barnes 1989, n. 46)
6.5 Complex geometry versus simpler methods of planning, and modern ideas on ‘medieval design’

After the foregoing paragraphs, it is important to keep in mind that complicated geometric methods of design were not a common feature in urban planning in the period under consideration. As far as I know, Castelfranco, San Giovanni, Scarperia and Terranuova are the only cases found so far where the use of such a method is really likely. Moreover, one should also keep in mind that many urban structures that were newly created were not even laid out on a geometrically regular plan. As has been described in chapters 1 and 2, many towns had rather irregular forms, in which the only regularity was that the lots were for the most part more or less rectangular and possibly of more or less the same size originally, and that the streets and plot boundaries generally tended towards straightness as long as they were not determined by older structures or hampered by obstacles.

For the many newly planned towns of the high-period of town foundation that have more regular forms, however, various scholars have proposed theories of complex geometric planning. With this, they favoured complicated geometric methods of design over other possible methods, such as arithmetic (or simple geometric) design with rational dimensions and proportions. In paragraph 6.3.3 the case of Grenade-sur-Garonne has been taken as an example since it featured prominently in literature on ‘geometric design of town plans in the middle ages’, but it may safely be assumed that in many other cases similar conclusions could be drawn after close study of the actual dimensions in the plans.

The problem here, in my opinion, lies in the inspiration of the various theories of complex geometric design: in most cases the hypotheses seem to have been inspired not so much by the form or the dimensions of the plans, but rather by preconceived ideas on ‘medieval architectural design’. It seems that many scholars have been eager to identify complex geometric design methods, not because they had concrete indications of the use of such methods, but rather because they were convinced that such methods must have been used. This conviction seems to have been largely based on the idea that in ‘the middle ages’ geometry was regarded as the key to the understanding of the divine creation, as described in the previous paragraph. More in general, it is commonly believed that ‘the medieval mind’ thought symbolically and mystically, which is regarded as a further indication that geometric methods must have been the basis of architectural design.

Although there is no such thing as ‘the medieval mind’, the generally held idea of the importance of symbolic thought is correct up to a certain point. But this does not mean that the planners in the 12th to 14th centuries could not think pragmatically or rational up to a certain extent, and that they could not lay out a town plan by simply measuring out rationally determined dimensions.

The idea of the great importance of geometry in the methods of design in ‘medieval architecture’ has often been wrongly understood. In the previous paragraph it is described how geometry was regarded as an essential source of knowledge of the Divine creation in contemporary philosophy. There are also written sources that testify of the importance that was attached to geometry as important element in the practice of architectural design. From these sources, it appears that simple geometric methods were used to determine the dimensions of elements of buildings such as pinnacles, jambs and the width of piers and walls, in order to achieve good proportions and statically sound constructions. Actually, to understand the importance of knowledge of geometry one does not need to know these sources, since it is perfectly clear that basic knowledge of geometry was necessary to design and construct polygonal buildings such as church apses, vaults (particularly the complex vaultings of churches in central and western Europe in the 14th to 16th centuries), window tracery or spiral staircases. Without the use of geometric methods, those features could never have been built with the regular forms that they have.

But, looking at the regular forms of buildings of that period, it is also perfectly clear that these buildings were often made up of repeating units of equal form, and from closer analysis it appears that many...
dimensions were determined arithmetically, as round numbers in the current units of measurement.\textsuperscript{203} From the case of Giglio Fiorentino it clearly appears that this method was also used in the design of urban structures.\textsuperscript{204} In paragraph 6.3.3 it is argued that the plan of Grenade-sur-Garonne was also designed in this way, and in my opinion it is likely that most newly founded towns with regular orthogonal plans were dimensioned in a similar way.\textsuperscript{205}

This method may be simpler than are design by use of complex geometry, but this does not automatically mean that it is therefore entirely pragmatical and without symbolical significance. The regular order of the orthogonal grid, based on a much simpler sort of geometry, may also have carried symbolical meaning. And when the dimensions were not chosen in random numbers, but in numbers of specific significance or in specific numerical proportions, it seems even more likely that there was some deeper meaning. In paragraph 8.6 this subject will be elaborated on, but here it can already be mentioned that this deeper meaning was largely similar to the meaning that is supposed for architectural design geometry in the period in general: reference to the order and harmony of the God-created universe.

For a long time it has been generally assumed that in ‘the middle ages’ geometry was the basis of architectural design, and that this basis was taken over by arithmetic in the ‘renaissance’.\textsuperscript{206} Bucher, for instance, followed this idea in the article in which he proposed the complex geometric design method for the plan of Grenade-sur-Garonne.\textsuperscript{207} He wrote that it was only in ‘late medieval theory’, that proportional relations came to be used in ‘rules of thumb’, as for instance, for the thickness of walls and piers of buildings in relation to their height. This proportional theory would have had nothing to do with the ‘generative or aesthetic design process’, which essentially worked with more complex geometric methods: ‘Linear measurements within the highly modular design system were thus used only as master values for the main modules.’ With this, Bucher means that from a module with a specific dimension, other dimensions were mutated only by geometric manipulation.\textsuperscript{208} As described in paragraphs 6.3.3 and 6.4.2, this is contrary to the evidence of Grenade-sur-Garonne and Giglio Fiorentino.\textsuperscript{209} Against such ideas as those of Bucher, convincing arguments based on exact measurements of contemporary buildings and drawings, are brought up by Hecht and others more recently.\textsuperscript{210}

In my opinion, however, the supposed contrast between geometry and arithmetic as being completely different design methods of necessarily different periods has been exaggerated too much. This idea sprang from the more or less explicit view of history as divided into different periods, in this case especially regarding the difference between ‘the modern period’, as essentially characterised by rationality and clarity, and ‘the middle ages’, which are regarded as mystical, dark and mysterious. This view is, of course, a gross simplification which does no justice to historical reality. In chapter 11 it will be extensively discussed how this view of history stood at the basis of a largely wrong perception of the history of urban planning.

### 6.6 Conclusion

Close examination of various theories of town plan design by complex geometric methods in the period under consideration has shown that most of them appear rather unlikely, because they do not correspond well to the actual dimensions, they are too complex to have been performed at the time or they lack apparent logic. In fact, only the plans of the terre nuove Castelfranco, San Giovanni, Scarperia and Terranuova seem

---

\textsuperscript{203} See Hecht 1969-1971.

\textsuperscript{204} See also pars.6.4.2.

\textsuperscript{205} See also pars.6.1, 9.6.1. For other examples of towns of roughly the same period for which it is argued that they were planned in a more or less similar way, see literature in n.4.

\textsuperscript{206} This idea was spread among others by Rudolph Wittkower’s influential book Architectural principles in the age of humanism of 1949 (Wittkower 1949), in which it is claimed that architects from the ‘renaissance period’ mainly used the arithmetic proportions of the musical harmonies. This theory, however, has been proven wrong by among others Mitrović (2001). Mitrović also demonstrates that the ‘renaissance architect’ Palladio used geometric proportions, particularly 1:1.2. Note that this is exactly the relation that Bucher and others believed to be specifically important in ‘medieval design’, but which has appeared unlikely for various hypotheses on urban design in pars.6.3.2 and 6.3.3. Özural argues that the arithmetic proportions of the musical harmonies were also used in architectural design in the 14th century. ( Özural 2002, p.235)

\textsuperscript{207} See Hecht 1969-1971, esp.1971, pp.221-222. Hecht even goes as far as to deny the importance of geometry for contemporary architectural design almost completely: ‘On the building site as well as behind the drawing board, he [the gothic architect] used dimension and number as his only reliable tools’ (‘An der Baustelle wie am Reisbrett benutzte er [der gotische Architekt] als einzig verlässliche Hilfsmittel Mass und Zahl’). In my opinion this is too bold, however, since I found that geometric methods of proportioning were most probably used in the plans of the terre nuove. More recent arguments for the use of arithmetically-determined dimensions can be found in the collection of essays on ‘the practical application of geometry in medieval architecture’ Wu 2002. (pp.7-8, 65-68, 127, 169-174, 219-228).
to have been dimensioned by use of complex geometric methods, which were based on the dimensions of polygons. Most other regular orthogonal town plans seem to have been planned by use of relatively simple straight lines, right angles and arithmetical dimensioning in round numbers of current units of measurement. Irregular non-geometric plans seem to have involved much less careful spatial design, but most newly created towns of the period do contain at least some elements of planned spatial regularity.

The design method that appears to have been used for the *terre nuove* may have been derived from a contemporary method of dimensioning palazzo facades, and the underlying geometric figures may have been inspired by compass roses, geometric figures on astrolabes or circular models of the cosmos that were radially divided into twelve parts. Unfortunately there are no solid indications as to this matter. The designs were most probably first made in drawings. After that, the regular polygons may have been set out on reduced scale on a drawing board or a floor, from which the dimensions were measured and multiplied. But it seems more likely that they were set out on the site on a reduced scale (most probably half the scale) from where the relevant dimensions were geometrically multiplied. The polygons were probably not set out on the full scale because the long ropes that were needed probably were too elastic and too difficult to handle in order to accurately set out the geometric figures.

The motive for the use of the complex geometric design method of the *terre nuove* plans probably was to make them auspiciously harmonise with the order of the universe, which was believed to have been designed by God, making use of geometry. The fact that the geometric figures bear likeness to circular cosmic symbols and depictions of the Heavenly Jerusalem may also be interpreted as indication to this meaning. An additional motive may have been that the designer aimed at personal reward and fame or emancipation of his profession, as geometry was a very highly valued art. Unfortunately it is impossible to discern what the relative importance of the different possible motives was and to what degree people (planners, settlers and the general public) were conscious of them.

It appears that the importance of the use of complex geometrical methods in architectural design in the period has been generally overestimated. It seems that this has largely been caused by an over-generalised view of history in which complex geometry was the basis of architectural design in the essentially mystical ‘middle ages’ and in which arithmetic design became the common method in the essentially rational ‘renaissance’. This view of history as basically divided into different periods is, however, a gross simplification that obstructs the view of the actual historical developments in the past.