The foundation, planning and building of new towns in the 13th and 14th centuries in Europe: an architectural-historical research into urban form and its creation
Boerefijn, W.N.A.

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In this appendix, the theories of geometric design that are very briefly discussed in paragraph 6.4.1.4 are considered in more detail (par.B.2), and subsequently the three theories that remain as plausible options are compared to the modern town plans in detail, discussed per individual town. (par.B.3) The final paragraph contains an overview and comparison of the basic dimensions of the plans of the terre nuove.

B.1 Method of analysis of theories concerning the design method

In the following paragraphs various theories of geometric design are critically discussed. Additionally it is checked to what degree they correspond to the plans of the terre nuove. In order to do this, the optimal method would be to take all relevant measurements of the present-day town plans, as well as of older accurate plan drawings, verify that they stem from the original situation, and compare them with the essential numerical values that are given by the hypothetical methods of design. However, for practical reasons it is impossible to take and compare all those hundreds, or probably thousands, of measurements. A good alternative is to use existing accurate plan drawings, and compare them to the theoretical design methods by overlaying them. This is done in the various figures depicted below and in paragraph 6.4.2.1

For an accurate analysis of the supposed design methods of the terre nuove it is necessary to compare them to the actual plans. In doing this, it is necessary to use plans that are as accurate as possible and that optimally reflect the original situation, in order to get significant results. The best plans are, of course, those that are most scrupulously drawn on the largest possible scale, on the basis of most accurate and detailed measuring. Obviously, modern plans are best suited, because of their larger scale and greater accuracy. But a problem is that modern plans show a more recent situation, which is probably more removed from the original urban plans. So, when working with these modern plans, one must always bear in mind that elements might be different from the original design. Fortunately, it is possible to check the age of architectural elements by dating them from sight, by looking at the building material, technique and style, and to check the age of plan elements by comparison with older plans.

The most accurate town plans appeared to be found at the offices of the municipal works of the respective towns. These plans are all drawn in scale of 1:500. But they are rather diverse as to the matter of the quality of the plans, since the one is obviously measured and/or drawn more accurately and more detailed than the other.

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1 These figures are created digitally, by digitizing the most accurate paper plans that I could get hold of (see the following paragraph), and overlaying these with the geometric figures of the theoretical design methods by use of Computer Aided Drafting (Autocad). For a more detailed description of the advantages and possibilities of computer aided drafting in reconstructing design methods, see Stenvert 1991.

2 This terminology is explicitly related to non-digital mapping, which is already becoming quite obsolete at the moment. Unfortunately, I have not been able to get hold of digital maps of the towns in question, although that would be preferable.

3 I am very grateful for the helpful service of the staff of the respective uffizi urbanistiche for putting at my disposal copies of those plans. The plans that I have used are: San Giovanni: Variante al P.R.G. per il centro storico. Tav.2: Destinazioni d’uso ai piani terreni. Made by Prof. arch. Gianfranco di Pietro and arch. Grazia Gobbi, scale 1:500. Castelfranco: Programma di Fabbricazione. Tav.7, by arch. Pier Lodovico Rupi and arch. Mario Maschi, scale 1:500. Scarperia: town plan by Studio Tecnico Bertini, Firenze, 1985, scale 1:500. Terranuova: town plan 1:500, which is used by the municipal works and made by the provincial cadastre, no title, plotted from a digital map. The plan of San Giovanni is most detailed and accurate, followed by the Scarperia plan, and then the Terranuova and Castelfranco plans. The authors of the respective hypotheses on design geometry do not give information on the plans they used, but judging from their illustrations they only used (schematised) cadastral plans (1:1000 or 1:2000).
B.2 Description and analysis of various theories of lesser importance

B.2.1 Higounet

Charles Higounet proposed that the dimensions of the outline forms of the terre nuove plans are simply made up of large squares with sides of 100 or 150 brachiata (162.5 or 243.75 m.) length.4 San Giovanni originally would have measured 100 x 300 brachiata, for Scarperia and Terranuova it would have been 100 x 200, and Castelfranco would have measured 150 x 150 brachiata. Analysis of the present-day plans, however, shows that these dimensions are largely incorrect, with deviations from the actual dimensions of up to 20%.5 Besides this considerable flaw, Higounet’s theory is also rather unlikely because of the fact that contemporary Florentine documents show that, instead of the brachiata, the braccia (c. 0.5836 m.) was commonly used as the standard unit of measurement in Florentine building operations in the concerned period.6

B.2.2 Buselli

Another theory was published by Franco Buselli, in his book on the history of the town of Pietrasanta, which was founded by the city-state of Lucca in 1255 in northwest Tuscany.7 In the general sort of plan, which is rectangular with relatively long parallel rows of rather narrow house lots, this town is likely to have been a model for the terre nuove fiorentine.8 According to Buselli the plan design was made on the drawing board by use of a highly complicated geometric method, based on an 11-braccia module, connected squares, ‘golden section proportions’ and circle-geometry. (fig.b1)

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4 Higounet 1962.
5 For instance: San Giovanni measured 462.50 x 190.44 m. (see below, n.14), while according to Higounet it would be 487.5 x 162.5 m., which gives deviations of 5.4% and 17.2% (of the smallest values) respectively. Higounet does not explain on what his dimensions are based, but they clearly are wrong. For the dimensions of the other towns, see par.3.9.2.2.
6 See par.6.4.1.5, n.74.
8 See Friedman 1988, pp.87-90.
I have tried to check Buselli’s hypothesis on Pietrasanta in an accurate modern plan.9 My analysis need not be elaborately discussed here: the general conclusion is that the hypothesis is largely wrong. In Buselli’s hypothetical design method various dimensions are adopted which deviate too much from the actual dimensions.10 Another objection is that according to Buselli’s theory modular and geometric methods would have been used at the same time, in order to determine specific dimensions by two or three different methods. This is highly unlikely since there are considerable differences in the resulting dimensions, and, more importantly, it is completely against the logic of design. A further problem is that the relation of the ‘golden section’, which would have been fundamental according to Buselli, cannot be shown to have been known in the period under consideration.11 Apart from all that, Buselli’s hypothesis is also unlikely because of its extreme complexity and internal ambivalence.

In the context of this appendix Pietrasanta is not of primary interest, but it is important that Buselli claims that the Florentine new towns, except for Scarperia, were also designed by use of certain elements of the theoretical Pietrasanta design system.12 Unfortunately these claims regarding the Florentine towns are not clarified by illustrations or dimensions, so it is not easy to comprehend what Buselli exactly means. Only with regard to the plans of San Giovanni and Terranuova he gives concrete indications as to how he thinks they were designed. In figures b2, b5 and b8 the basic elements of Buselli’s theory are depicted, and in figures b3-b4, b6-b7 and b8 these elements are visually compared to the present-day town plans.

According to Buselli, the proportions of the perimeters of both towns would have been determined by the geometric relation of the so-called ‘golden section’.13 According to this theory the numerical value of the relation of the short side of the perimeter rectangle to the long side would be approximately 1 : 0.4045. In figures b3-b4 it can be observed that this proportion comes quite close to the outline of the original San Giovanni plan, but diverges considerably from the perimeter proportions of Terranuova.14 Another part of Buselli’s theory claims that the width of the central market places of both towns would have been determined by manipulating the same triangle that was used to arrive at the golden section proportions. (see fig. b5) According to this theory, the width of the market place would be related to the short side of the perimeter rectangle as 0.2679.15 The illustrations (figs.b6-b7) show quite clearly that this proportion does not fit on the San Giovanni plan, and although it conforms better to the plan of Terranuova, the difference here still is circa 3.5%, for which reason this theory does not seem very likely.16

Furthermore, Buselli claims that the exact place of the parallel front streets in the Terranuova plan is, just like in his design system for Pietrasanta, determined in relation to the positions of the central town axis and outer town walls by golden section proportions. If the distance from the town axis to the town wall is the maior, then the distance from the axis to the back street would be determined by the minor. Buselli does not mention whether the inside, the centre or the outside of this street was to be determined by this dimension; from figure b8 it becomes apparent, however, that none of these come close to the hypothetical position.17

It can be concluded that four out of five of Buselli’s concrete assertions regarding the hypothetic design

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9 This plan in the scale of 1 : 1000 is titled Provincia di Lucca, Comune di Pietrasanta, foglio n:17, levata anno 1951, riprod. anno 1954. I obtained this plan from the Ufficio tecnica of the commune. This cadastral plan seems to be quite accurate. I checked Buselli’s theory by calculating the “theoretical” dimensions given by the proposed geometric method, and comparing these to the dimensions that I measured in the plan.

10 For instance: Buselli takes the width of streets and lots as the module of 11 braccia lucchese (6.40499 m.) and the length of the lots and the width of the piazza as manifolds of it (Buselli 1970, pp.45, 48), although the actual dimensions diverge rather strongly from these theoretical dimensions (up to 10.79% for the house lots).

11 See ch.6, n.38. Buselli is conscious of this problem, but in his opinion the antique knowledge of the golden section may have returned to Italy via contacts to the Arabic world. (Buselli 1970, p.50) He also argues that the arithmetic relation between minor and maior of the ‘golden section’ was actually known in another form in the relation of the following numbers in the ‘numerical series of Fibonacci’. (Buselli 1970, p.58) Although this is basically correct, the Fibonacci series has nothing to do with the geometric manipulations Buselli proposes for the design of the Pietrasanta plan.

12 Buselli 1970, p.34.

13 According to Buselli the golden section proportions were constructed by taking the short side of the perimeter rectangle of the town as the basis of a right-angled triangle with cathetes in 1 : 2 proportion. (fig.b2) In this triangle the golden section was constructed by rotating the short side to the oblique side around point E, and rotating back the other part of the oblique side to the long cathetus, which is thus cut in two so that the short part (minor) relates to the long part (major) in the same proportion as the long part relates to the whole. According to Buselli’s theory the length of this minor, doubled, should have given the total length of the town perimeter.

14 The original perimeter of the San Giovanni plan must be reconstructed by mirroring the north half of the town, because the southermost part of the town was destroyed by a flooding in the 16th century. It is known that the plan was symmetrical from among others the 16th-century plan of Piero della Zucca. (fig.3.13) The original dimensions of San Giovanni consequently amount to circa 190.44 x 462.50 m. (based on my measurements in the modern plan (see above n.3), which gives the proportion 0.4118, diverging from the calculated theoretical value proposed by Buselli 1.8%. By mirroring the indicated dimensions in the north half of Della Zucca’s plan, the whole town would measure 328 x 790 braccia. (see fig.3.13) These dimensions relate as 1 : 0.4112, diverging 2.6% from the calculated theoretical value. For Terranuova the dimensions I measured are 158.65 x 332.70 m., which gives the relation 1 : 0.4766, resulting in a difference of 17.8% with the calculated theoretical value.

15 The width of the piazzas would have been determined by the rotation of the long side of the 1:2 rectangle over to the other long side of the town perimeter, as shown in fig. b5. The width of the piazza would thus be generated by the long side of the rectangle (=r) minus the cosine value of 30 degrees, so the width of the piazza would theoretically be related to the short side of the perimeter rectangle as 0.2679.

16 The difference between the real width of the piazza, according to my measurements and the calculated theoretical value in San Giovanni (77.11 b. at most, versus 77.44 b.) is 1.49%, and in Terranuova (68.37 versus 72.48 b.) it is 5.09%.

17 The relative difference is circa 2.5 - 6.5% from the closest actual dimension, which is the distance from the central axis to the outer side of the parallel street.
method of the terre nuove plans are found incorrect. The remaining one, concerning San Giovanni’s perimeter proportions, is more or less in agreement with the actual proportions. However, since this agreement appears to be an incident, not being part of a larger system of proportioning, and since the golden section is not likely to actually have been a proportioning method in the 13th century, this agreement must be discarded as being coincidental.18

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fig. b2: Figure representing the way in which the dimensional relationship of the golden section may be found by manipulation with compasses or ropes, starting from a right triangle with cathetes in a relationship of 1 : 2. The minor relates to the major in the same proportion as the major relates to the total of the two (c. 0.6180339 : 1). (From: Naredi Rainer 1982)

fig. b3: Graphic verification of the geometric method of determining the relation between length and width of the perimeter of San Giovanni according to Buselli, superimposed on the digitised modern plan. From the golden section construction with the 1 : 2 rectangle, of which the short cathetus is the width of the perimeter (along the original town wall), the maior is found, marked L. (as in fig. b2). Twice the length of this maior is set out along the main street of the town, starting from the northern perimeter side. From this it appears that the difference between this double maior length and the actual length of the perimeter is about 1%, which suggests that the theory is acceptable.

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18 I have also checked Buselli’s theories on the proportioning of the perimeter and the piazza on the plans of the three other towns, but there too, they clearly do not agree with the actual proportions.
fig. b4: Graphic verification of the geometric method of determining the relationship between the length and width of the perimeter of Terranuova according to Buselli, superimposed on the digitised modern plan. The principle is the same as in figure b3. From this figure it appears that the difference between the double maior length and the actual length of the perimeter is very large, almost 17%, which suggests that Buselli’s theory is highly unlikely in this case.

fig. b5: Figurative depiction of the geometric method of determining the width of the piazzas of Pietrasanta, San Giovanni and Terranuova, according to Buselli. (Based on Buselli 1970) By the rotation of the long side of the 1:2 rectangle over to the other long side of the perimeter, the width of the piazza would be generated. This would be the long side of the rectangle (=r) minus the cosine value of 30 degrees.

fig. b6: Graphic verification of the geometric method of determining the width of the piazza of San Giovanni according to Buselli, superimposed on the digitised modern plan. By the rotation of the long side of the 1:2 rectangle over to the other long side of the perimeter, the width of the piazza would be generated. In this figure the distance which is supposed to mark the width of the piazza is depicted twice: once in the place where it is actually constructed from the 1 : 2 triangle, and once transplanted to the actual place of the piazza, from which it clearly appears that the width according to Buselli’s theoretical method varies considerably (over 15%) from the actual width.
fig. b7: Graphic verification of the geometric method of determining the width of the piazza of Terranuova according to Buselli, superimposed on the digitised modern plan. (cf. figs. b5, b6) It appears that the width according to Buselli’s theoretical method varies from the actual width by about 5%.

fig. b8: Graphic verification of the geometric method of determining the placement of the street line of the parallel-streets in relation to the axis of the main street and the perimeter of the town wall at Terranuova, according to Buselli, superimposed on the digitised modern plan. The four triangles are basically the same as the one in fig. b2, so the long cathetes are divided by the golden section. It appears that the outer street lines of the parallel streets cannot be arrived at using this theoretical method. The difference is about 2.5 to 6.5%. 
In 1980 Eugenio Baldari proposed a hypothetical geometric design method for the plan of San Giovanni.\textsuperscript{19} Baldari's theory is largely based on Guidoni's design system for San Giovanni. (\textit{fig. 6.12}) But Baldari propounds a second 'executive model' next to this 'theoretical model'. (\textit{fig. 6.16}) Unfortunately, he does not explain clearly what he means by this distinction, but it seems likely that he means that Guidoni's 'theoretical model' was only used in the design stage, on the drawing board, while the 'execution model' was used as the method for the actual laying out of the town plan by the surveyor.\textsuperscript{20}

The 'theoretical model' is described in paragraph 6.4.1.1; the only difference with Guidoni is that Baldari has not adopted the proportioning of the perimeter by means of the two connected hexagons. Instead of this, he explains the perimeter proportions from his 'execution model', which seems to proceed from Guidoni's theory of the three squares that regulate the proportions of the market place. Directly connected with the two outer of these squares, are the diagonals of two larger squares that enclose the street blocks between the market place and the halfway transverse streets. The outer sides of these squares, that is the sides that mark the inner alignment of the transverse streets, are at the same time, according to Baldari's text, sides of an octagon that is centred on the centrepoint of the town.\textsuperscript{21} From here, Baldari's scheme determines the angle points of the town plan by doubling the radius of the octagon or the distance from the centre point to the outer angles of the larger squares that comprise the street blocks (which is, in fact, the same). Thus, the perimeter of the town would be determined by a large octagon, which has twice the size of an inner octagon that would determine the outer alignments of the inner street blocks.

In Baldari's illustration this all seems to fit quite well, but if one regards \textit{fig.b9}, in which this theoretical 'execution model' is lain on the present-day town plan, one may conclude that this theory is not as obvious as Baldari would have it. The angles of the town wall, which can only be located exactly on the north side of the town since the south side was destroyed in the 16th century (see \textit{fig. 3.13}), but of which the southern ones can be reconstructed, appear to be accurately determined by the larger octagon.\textsuperscript{22} But the octagon with half the radius does not coincide accurately with the outer corners of the two block-comprising squares that Baldari proposed. (\textit{fig.b9}) The corners of the octagon which are supposed to mark the inner alignments of the transverse streets lie circa 1.5\% closer to the centre point of the supposed 'executive model'. Looking closely at Baldari's figure of the 'execution model' (\textit{fig.6.16}), one can see that he had the same problem in his drawing, and therefore he has actually distorted the geometric figure in order to make the two squares comprise the house blocks to both sides of the piazza, by enlarging the circle that encompasses the imaginary inner octagon.\textsuperscript{23} Baldari's idea of squares that comprise the street blocks on both sides of the market place fits rather well on the plan.\textsuperscript{24} (\textit{fig.b9}) But the squares are located circa 1.5\% too close to the central transverse axis of the plan, since their place is theoretically determined by the corners of the inner octagon, and therefore they do not mark the inner alignments of the transverse streets and the building lines along the piazza well.\textsuperscript{25}

The three squares that Guidoni proposed as proportioning method of the piazza, can be fitted on to the modern plan of the town in such a way that they correspond to what could be assumed to be the piazza as it originally would have been. (\textit{fig.b9}) The piazza is, however, not very symmetrical or regular in detail, so the agreement is not very clear. More problematical is the fact that this construction with the different sizes of squares is not logical, or geometrically defined, since the size of the middle square is not related to the size of the other two squares in any logical way. Therefore, it seems unlikely that this was the method of design or of laying out the dimensions of the piazza.

\begin{itemize}
\item \textsuperscript{19} Baldari 1980.
\item \textsuperscript{20} This idea of a twofold design method also seems to stem from Guidoni, who writes that the design geometry often forms a starting point in the design, but that the actual dimensions may be much more pragmatically chosen. (Guidoni 1970, pp.215, 219, 222) In connection with the terre nuove (and the designs of Arnolfo di Cambio) he does, however, not suppose such a procedure. (Guidoni 1970, p.233)
\item \textsuperscript{21} In Baldari’s illustration this does not show very clearly, since it does not depict a real octagon, but only these two sides and the encompassing circle with the other angle points.
\item \textsuperscript{22} According to my measurements the town perimeter would measure 326.32 x 792.49 b. According to the 16th-century plan of Piero della Zucca it would be 328 x 790 b. Calculating from the length of the town, the relative side of the theoretical octagon would be 328.26, resp. 327.32 b., giving divergences of 0.59\% resp. 0.24\%.
\item \textsuperscript{23} The inner circle should cross through the intersection point of the diagonal lines in order to make the 'executive model' coherent.
\item \textsuperscript{24} In the width of the street blocks from one parallel street to the other, these squares do not comprise most of the original porticoes that fronted the houses on the parallel streets (particularly on the south side; on the northwestern side one of the porticoes is included). The porticoes are on average 2.57 braccia deep. It is possible that this was originally regulated as 2.5 b. Many of the porticoes were closed during the centuries in order to enlarge the interior ground floors of the houses. (see par.3.9.3.5)
\item \textsuperscript{25} The porticoes along the piazza, to both sides of the town hall in the market place, are comprised by the squares, which is illogical, as the porticoes along the parallel streets are not.
\end{itemize}
So, Baldari’s theory is not as coherent as it might seem at first sight. The difference of about 1.5% in the radius of the inner octagon may be explained by inaccuracies. But all in all, it does not seem likely that Baldari’s ‘execution model’ was actually used for the laying out (or for that case, the design) of the town plan of San Giovanni. The three diagonal squares that would have given the proportions of the piazza are most unlikely, as they do not have dimensions that are logically explainable. The two larger block-comprising squares may fit onto the street blocks quite well, but it does not seem logical that these street blocks would have been dimensioned as squares, and that the rest of the proportions of the plan do not seem to be determined by squares in any way. The same holds true for the octagons. Baldari is not very clear in his explanation of the ‘executive model’, but his figure seems to suggest that the octagons were not really constructed: a netlike structure of isosceles triangles with a sharp angle of 45º and two angles of 67.5º was constructed. The relevance of this structure is, however, rather limited: it gives the proportions of the perimeter of the town. According to Baldari it also determines the inner building lines of the transverse streets and the parallel streets. In reality there is a 1.5% deviation, but a more important objection against this idea is, that it is much more logical that the planners would have determined these building lines by simply taking half the distance of the centre point of the town to the sides of the perimeter, for which there are much easier ways.

This all makes that Baldari’s theory does not appear very likely. Only the geometric explanation of the perimeter proportions is in itself plausible. But since the geometric construction apparently does not determine other relevant proportions in the plan in a logically coherent way, the agreement between the perimeter proportions and this geometric construction might well be a coincidence.

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26 In fact, Baldari’s theory is still more complicated, also taking in consideration numerical values of distance, derived from proportions in the octagon. It would take too much space, however, to discuss this complicated theory. In my opinion, it is far-fetched and, moreover, almost impossible to handle for the designer in combination with the ‘theoretical’ and ‘execution’ models.

27 I have also checked the ‘execution model’ on the plans of the other terre nuove, but this has given no positive results.

28 An octagon consists of eight such triangles in radial arrangement. The triangles are not arranged radially in Baldari’s figure, but in netlike fashion, but nevertheless this results in the essential proportions of the regular octagon, as the long isosceles sides of the triangles implicitly have the length of the radius and the short side has the length of the sides of the octagon.
Van den Heuvel

In an article of 1983 Charles van den Heuvel suggested some further applications of Friedman’s basic theory. He claims that a polygon with a 30-degree division (that is just like a dodecagon) would show significant correspondence with a number of relevant lines in the design of the Scarperia plan. Van den Heuvel gives no exact verifications or illustrations of his suggestion; he only gives short indications in which way Friedman’s ideas could be applied differently in order to get more relevant results. In paragraph B.3.4 (and more briefly in 6.4.2) these indications will be checked in comparison with an accurate plan of the town. Van den Heuvel also proposes some adjustments to Friedman’s geometric schemes so that they would fit better on the plan of San Giovanni and also on the plan of Firenzuola. I have checked these two proposals, but no positive results have been found and therefore they will not be discussed here.

Bartoli

Giglio Fiorentino had so far escaped becoming subject of theories of geometric design, since the dimensions mentioned in the document of 1350 appear so straightforwardly arithmetically determined, as round numbers of the current unit of measurement, the braccio fiorentino. In fact, this is what probably inspired Pirillo to propose that the most important dimensions at Terranuova and San Giovanni were designed in the same way. In 2003, however, Maria Cecilia Bartoli proposed that Giglio’s plan was partly proportioned by a complex geometric method.

According to Bartoli the basis of the design was the piazza, which measured 70 x 70 braccia. In the document of 1350 the piazza is described as measuring 90 x 70 braccia. However, this probably was including the two lateral parallel streets that were 10 b. wide, so if these are subtracted from the space of the piazza it would indeed measure 70 x 70 b. From this square the length and width of the town, described as 470 x 246 b. in the document, would have been determined by geometric manipulation. The length was determined as the length of the piazza plus four times its diagonal, with which the $\sqrt{2}$ relation of the ‘rotated squares’ is once again proposed as a method of determining proportions. This would theoretically lead to a length of 465.98 b., but according to Bartoli the diagonal of the 70 x 70 b. square was approximated as 100 b., so that a length of 470 b. was the result. Apparently Bartoli does not mean that the geometric manipulation as depicted in her illustration (fig.6.17) was actually carried out as such: the dimension of 470 b. would have been determined arithmetically as the width of the piazza plus four times the ‘common’ approximation of the diagonal of the 70 x 70 square.

The width of the town would subsequently have been determined by taking twice the diagonal (200 b.) as the height of an equilateral triangle. The sides of this triangle would theoretically measure 230.94 b. According to Bartoli this was approximated as 232 b., to which 14 b. were added for the width of the main street, thus providing the 246 b. width of the town. The other dimensions that were mentioned in the 1350 description would have been determined arithmetically, according to Bartoli.

In my opinion, this way of determining the length and width of the town are quite unlikely. If one is ready to accept the approximations as tolerance, the dimensions may be taken for fitting, but the method is inconsistent and illogical. The subsequent steps in the process are not logically related. Also, the idea that the square piazza was the basis of the whole plan is difficult to reconcile with the description of 1350, in which the piazza is described as measuring 90 x 70 b. Furthermore, it seems most illogical that the width of the main street is to be added to the side of the equilateral triangle to arrive at the width of the town. All in all, Bartoli’s proposed proportioning method for Giglio Fiorentino does not appear to be a valuable contribution to the discussion on the design of the terre nuove plans. It remains more likely that the dimensions given in the description of the Giglio project of 1350, were determined arithmetically, largely based on practical considerations, symmetry of layout and preferably numbers rounded off at tens.

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29 Van den Heuvel 1983, p.42. Van den Heuvel actually reacted to Friedman’s article of 1974, in which he already made some rather vague suggestions about possible further applications of the hypothetical geometric design method. (Friedman 1974, p.242)
30 I have checked Van den Heuvel’s suggestions concerning San Giovanni and Firenzuola for my doctoral thesis on the terre nuove fiorentine. (Boerefijn 1994, pp.152-154)
31 See appendix A.
32 Bartoli 2003, pp.16-17.
33 With these dimensions, not all traffic space is subtracted from the piazza, as the two intersecting main streets (both 14 b. wide) are still included.
In the same book in which Bartoli proposed her theory, Stefano Bertocci suggested a proportioning method for some aspects of San Giovanni’s plan. As many others did before him, Bertocci postulates that the proportional relation of the side and the diagonal of the square (√2) are crucial in ‘gothic’ architecture. Hence, in his opinion, the outer perimeter of the town must have been proportioned by taking the short side as the side of a square to which the diagonal of the square was added to get the long side of the perimeter. According to my measurements the outline dimensions of San Giovanni were 462.50 x 190.44 m. According to Bertocci’s method the short side of 190.44 m. must have lead to a long side of 459.76, which gives a deviation of just 0.60%.

The inner perimeter of the town was dimensioned in another way according to Bertocci. Apparently, he believes that the inner perimeter was not simply the outer perimeter minus the width of the town wall, which would normally seem logical. In his opinion the internal perimeter was proportioned by taking the short side – he does not mention how this was related to the outer perimeter, but one would guess that it was the short side of the outer perimeter minus twice the width of the town wall – making a square of it and rotating the diagonal in two opposite directions, thus creating overlapping rectangles (with proportions a : √2a) of which the diagonals were rotated in order to reach the length of the inner perimeter. According to two 16th-century plans the town wall was 1.5 b. wide. So the inner perimeter must have been 459.50 x 187.44 b., based on the outer dimensions that I measured. Starting from 187.44 b. as the side of the square, the long side of the inner perimeter theoretically must have been 461.87, which gives a deviation of 0.52%.

Bertocci’s third figure is meant to illustrate the way the wall towers were spaced along the perimeter. It’s basis is the same square that formed the basis of the supposed method of determining the inner perimeter, which is divided into nine equal small squares. Both diagonals of each of the four corner squares are rotated outward, by which they are supposed to mark the place of eight of the 24 original wall towers. In this way the side-length of the small squares is transplanted along the long sides of the town perimeter. This method seems very unlikely to actually have been used to determine the place of the towers, since it only regards eight out of 24 towers. It seems much more likely that they were simply spaced evenly along the perimeter sides. This is not a known fact, however, since the towers along the long sides have all been torn down long ago, and their place is not exactly known. For this reason it is also impossible to check whether Bertocci’s proposed method actually marks the place of the eight towers in question.

All in all, Bertocci’s proposed design method is not convincing. The third element (fig. 6.18C) is least likely and cannot be checked with the real plan. The first two elements are harder to judge. The deviations of theoretical and actual perimeter dimensions are small enough to be tolerated. A big problem, however, is that this method of design seems most illogical from a designer’s point of view. If the walls had the same dimensions on all four sides, which they appear to have had, it is most unlikely that outer dimensions had been determined in such a complex way apart from the inner dimensions, which were determined in another, even more complex, way. It is much more likely, therefore, that only one of the two methods had been used, and the first of the two would seem more probable then, since it is less complex. However, this method for determining the outer perimeter dimensions still does not appear very probable, since this design method had no other significance to the structure of the town plan, the square not appearing to have played a role in the design process other than providing the dimension of the diagonal, and the same holds true for the method for determining the inner perimeter. In my opinion it is, therefore, only coincidental that the proposed methods for determining the outer and inner perimeter correspond so well to the (partly reconstructed) actual dimensions.

34 Bartocci 2003, pp.84-85. The book was published as a supplement to the exhibition Città e architetture. Le metrice di Arnolfo (San Giovanni Valdarno 29-11-2003 – 14-3-2004), in commemoration of the death of Arnolfo di Cambio, the supposed designer of San Giovanni and Castelfranco, 700 years earlier.
35 The four small rectangles in the figures are probably meant to indicate that the original Palazzo Pretorio, the church of San Giovanni and double house lots on the main street had the same dimensions. This is not wholly correct, since the church was considerably longer.
36 See above, n.14.
37 The figure may also contain elements that are meant to indicate how the place of the innermost alleys and the inner boundary of the lateral wall streets were determined. This is not mentioned in Bertocci’s text or captions, however, and therefore it will not be discussed in detail here. Even when this method would appear to mark the relevant lines in the plan accurately, it would not seem very likely, since it is only determines a limited number of the parallel lines that mark the alleys and streets.
38 It regards the plans by Della Zucca and maestri Gentile and Batista, in which various dimensions are inscribed in numbers of braccia. (Friedman 1988, pp.10, 11, 147-150. Archivio di Stato di Firenze, Piante dei Capitani di parte, cartaone XVIII, no.28 ; Archivio di Stato di Firenze, Cinque conservatori del Contado, 258, fol. 602 bis). I measured 1.87 b. on average, but it is obvious that with such a small dimension, their measurements must be more accurate than mine from the 1:500 plan.
B.3 Detailed analysis of the theories of Guidoni, Friedman and Pirillo in comparison to the terre nuove plans

The theories of Guidoni, Friedman and Pirillo, which are described in paragraphs 6.4.1.1 to 6.4.1.3, remain as possible methods of design for the terre nuove plans. Below, they are compared to the modern town plans in figures as well as in measurements. I have measured the relevant dimensions in the modern plans and listed them in tables, in order to make it easier to compare the measured dimensions and the dimensions according to the different theories. In the following paragraphs, the comparison between theoretical figures and modern plans and between theoretical and measured dimensions is discussed per individual town. The text in paragraph 6.4.2 is a brief summary of this detailed analysis.

B.3.1 Terranuova Bracciolini

Regarding the case of Terranuova, there is a strong correspondence between the modern plan and the design system proposed by Friedman. (fig.6.14) This is demonstrated by figure 6.19, in which the theoretical figure is laid over the digitised modern plan. Friedman’s theory, which is an adaptation of Guidoni’s roughly worked out proposal (fig.6.13), fits the plan very well, unlike the other relevant theories of design methods by Higounet and Buselli.

Below, the graphic representation of the hypothesis (fig.6.19) is supplemented with numerical data of the various dimensions that play a relevant role in the theory, which are compared to the dimensions of Pirillo’s theory of arithmetic design, in order to establish which theory is more likely.

Friedman’s hypothesis is that the design of the plan of Terranuova is determined by a geometric figure that lays down many relevant dimensions in the plan. As explained in paragraph 6.4.1.2 the idea is largely based on Guidoni’s rather rough hypothetic figure of 1970. (fig.6.13) There are some differences, however. Guidoni’s figure seems to suggest that the piazza originally was square, being somewhat wider, and having been proportioned by a diagonal square. There is however, no reason whatsoever that the piazza was indeed square, and even when it would have been square, it is most unlikely that a diagonal square was used for its design or for setting out the plan. Another difference is that Friedman describes the theoretical geometry as a circle with a radius of 142.91 braccia (83.4 m.), the circumference of which is divided into 24 equal parts with arcs of 15 degrees. (fig.6.14) The relevant dimensions would have been calculated as sine-values of manifolds of the 15-degree angles of this circle.

The length of the town (from town wall to town wall) would have been determined by taking two times the radius of the circle.

Friedman has taken measurements on the spot, which he compared to the dimensions calculated from his theory. He did not take measurements from a plan or try a graphic check by overlaying the theoretical figure on a plan. Therefore, I checked his measured dimensions and his theoretical dimensions with the dimensions that I measured in the paper plan, in order to establish whether the plan that I obtained corresponds with Friedman’s measurements and his theory.

In order to check Friedman’s conclusions, I have measured the relevant dimensions around twelve times each, in different places in the plan, from which the averages were calculated. These averages are recorded in the first row of the table below. In this table they can be compared to the dimensions that Friedman

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39 Additionally, a suggestion by Van den Heuvel as to the application of Friedman’s theory on Scarperia (see par.B.2.4) is also still to be considered.
40 I have measured every individual theoretically relevant dimension in several places in the plan, since dimensions, such as for instance the width of a street, may vary slightly along its course. In most cases I have taken eight measurements for the width of every street and the width of every row of lots (that is the length of the lots), and two to four measurements of other relevant dimensions. From these, the averages are calculated.
41 See pars.6.4.1.1 and 6.4.1.2. See also Boerefijn 1994, pp.141-173.
42 The fact that he depicted the diagonal square may well have to do with the idea of rotated squares as a common principle in ‘medieval architectural design’ (see pars.6.3.2-6.3.3).
43 Friedman, 1974, pp.242-243; Friedman 1988, pp.122-123; See par.6.4.3.2.
44 Friedman 1988, pp.122-123.
45 Friedman took his measurements on the ground, for which reason I believe they must be relatively accurate. His measurements were probably taken in the streets, so actually measuring the width of streets and the length of blocks on the outside of the walls of the various buildings. My measurements come from a paper plan, so there is more chance that they are inaccurate, because the plan was probably made by taking measurements on the ground, probably combined with graphic information from aerial photography, after which the data were drawn in the plan, which necessarily means a loss of accuracy. Subsequently this plan was copied, and then I took the measurements from it. Therefore, it is probable that my measurements are less accurate than Friedman’s.
46 These are the distances from the centre-line of the plan, in the middle of the main street, to the parallel boundaries of house lots and streets, which the theoretical geometric system (fig.6.14) is supposed to lay down.
measured and the theoretical values according to Friedman’s hypothetical design method.

It appears that Friedman’s measured values diverge 0.82% on average with the theory (with a radius of 142.91 b.), while the dimensions that I measured diverge 0.59% on average. The difference between measured and theoretical dimensions can even be reduced, when the theoretical radius is chosen 143.75 b. (83.89 m.) instead of Friedman’s theoretical radius of 142.91 b. (83.4 m.). This would reduce the average percentile difference between theoretical dimensions and my measurements to 0.43%. Similarly, for Friedman’s measurements the difference between theoretical dimensions and actual measurements are reduced from 0.82% to 0.71% when the theoretical radius is adjusted to 143.75 b.47 The theoretical relevant dimensions calculated with this radius are given in the fourth row of the table.

<table>
<thead>
<tr>
<th>Table I: distances from centre of main street to outer alignments of rows of lots and longitudinal wall streets (all values are in Florentine braccia of 0.5836 m.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st lot</td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>measurements Boerefijn</td>
</tr>
<tr>
<td>measurements Friedman</td>
</tr>
<tr>
<td>theoretical with $r=142.91$ b.</td>
</tr>
<tr>
<td>theoretical with $r=143.75$ b.</td>
</tr>
</tbody>
</table>

This adjustment of the theoretical radius also fits well with one of the basic assumptions of Friedman’s theory, which is that the theoretical radius, stretching from the centre point of the town in the middle of the piazza, also determines the length of the house rows and the whole town along the axis of the main street. (see fig. 6.14) Friedman measured 83.4 m. from the centre point to the halfway cross streets, 168 m. to the point where the southwest gate must have been, and 167.05 m. to the northeast gate.48 From the last two dimensions one could calculate an average radius of 83.76 m. = 143.52 b. This dimension is actually closer to the theoretically ideal radius of 143.75 m. (difference 0.16%) than to the 142.91 b. radius which Friedman proposed (difference 0.59%). So, on this point the longer radius also fits the actual dimensions better.49

It seems illogical, however, that the radius, as theoretically generative value of the whole design, would be 143.75 b. or, for that case, 142.91 b. It would be more logical that the length of radius was determined at a rounded number in braccia. Therefore, it seems more likely that the radius was chosen to be 144 braccia (which is only 0.17% more), since that would actually be a very obvious number.50 The number 144 was a very self-evident number back in those days, because it is 12 times 12 and because the duodecimal system was as common in quantification and calculation as the decimal system is now. Even today the number 144 stands for a more or less commonly used quantity as the ‘gross’. Apart from that, the number 144 also bore important implications of a symbolic nature, and the dimension of 144 braccia may even have referred to the biblical description of the Heavenly Jerusalem.51

When the radius of 144 b. is accepted, Friedman’s idea that the radius also determined the place of the secondary transverse streets and the length of the whole town becomes a little less likely: the difference would become 0.33% with Friedman’s average measurements and 0.55% with mine.

The theories of Friedman and Guidoni also propose that the place of the secondary cross streets and the length of the town were determined by the diameter of the circle (or polygon). The distance between the cross streets would be equal to this diameter, and the length of the town would be equal to twice the

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47 This difference could be reduced further to 0.55%, if it is assumed that the 75-degree sine value did not pinpoint the inner alignment of the town wall, but the outer one, 81.15 m. from the axis of the main street according to Friedman’s measurements. (Friedman 1988, p.123).
49 In my measurements the difference between the relevant values is somewhat larger: I measured an average of 144.79 b. for the relevant dimension along the axis of the main street, which makes a difference of 0.72% with the adjusted radius. But, as explained above, I consider Friedman’s measurements to be more exact than mine.
50 With this theoretical radius the difference between Friedman’s measurements and the theoretical relevant sine values would even be reduced to 0.74% (0.53% if the 1.46 b. of the town wall is included). The average difference with my measurements would be 0.48%.
51 The number twelve had a high symbolic value in contemporary numerology (see par.8.1.2), so 12 x 12 would have it just as much or even more. The dimension of 144 braccia may have referred to the Heavenly Jerusalem, as described in the bible in the Revelations of St. John (21:17), where an angel measures the walls, which are 144 ells (whether this is the width or the height is not mentioned). This dimension must already have been a symbolical dimension in the bible, since it is unlikely that the height or the width of a real city wall would have been 144 ells.
This part of the theories is not included in table I. From fig. 6.19 it appears that the theory fits quite well for the distance between the cross streets, but that the length of the town is not marked with great accuracy, the difference being circa 2%. However, if Friedman’s measurement of the length is accepted, as it is likely to be more accurate than the digitised plan, the difference is considerably less.\textsuperscript{53}

In conclusion, one can confirm, on the basis of the measurements from the plan drawing of Terranuova, that Friedman’s theory (and with that Guidoni’s) seems likely indeed, all the more so with the small adjustments made to the length of the radius.

But, with that, this hypothesis of geometric design cannot be unconditionally accepted yet. First, Pirillo’s hypothesis of arithmetic proportioning in rounded numbers of braccia still needs to be checked. The dimensions from Pirillo’s hypothesis can easily be checked by comparing the dimensions that he postulates with the averages of the measurements that I took in the plan, and, for extra comparison, with Friedman’s measurements.\textsuperscript{54} The relevant dimensions of width of streets and length of lots are given in table II.

It appears that the differences between my measurements and Friedman’s, are relatively greater than in the measurements of the larger dimensions in table I. In fact, this is well explainable, since the relative inaccuracy in drawing a plan and measuring it gets larger as the separately measured elements get smaller.\textsuperscript{55}

Looking at the values in the table, it may seem at first sight that Pirillo’s theory is largely right. Regarding the width of the main street it seems more likely that it was 14 b. instead of 15, as he assumes, but the depths he suggests for the respective house lots, seem acceptable. My measurements for these four dimensions diverge on average 2.28\% (three times larger and once smaller), while Friedman’s diverge 3.2\% (all his measurements being larger). The difference between my measurements and Friedman’s is 3.39\% on average.

So, it seems possible that the lots in Terranuova were 30, 25, 20 and 15 b. deep. Since the lots were 10 b. wide, these dimensions of the lots would be chosen very rationally, with all dimensions taken as multiples of five braccia. The width of the streets, on the other hand, would seem to be less ‘logical’ in this context.

Table II: widths of streets and lengths of lots (in Florentine braccia)

<table>
<thead>
<tr>
<th></th>
<th>main street</th>
<th>1\textsuperscript{st} lot</th>
<th>1\textsuperscript{st} back street</th>
<th>2\textsuperscript{nd} lot</th>
<th>2\textsuperscript{nd} back street</th>
<th>4\textsuperscript{th} lot</th>
<th>wall street</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pirillo</td>
<td>15</td>
<td>30</td>
<td>25</td>
<td>20</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Boerefijn</td>
<td>14.53</td>
<td>30.3</td>
<td>8.35</td>
<td>25.58</td>
<td>10.18</td>
<td>20.53</td>
<td>8.53</td>
</tr>
<tr>
<td>Friedman</td>
<td>14.06</td>
<td>30.5</td>
<td>8.05</td>
<td>25.7</td>
<td>9.97</td>
<td>20.39</td>
<td>8.02</td>
</tr>
</tbody>
</table>

* I could not measure the width of the town wall separately from the plan, but according to Friedman (1988, p. 122) the town wall is 85 cm. (1.46 b.) thick.

52 The relevance of the diameter to the distance between the cross streets is clearly marked by Friedman’s figures (figs. 6.14, 6.15), but less clearly so by Guidoni’s (fig. 6.13). Since Guidoni does not describe his theory, it is not clear whether he meant this element to be relevant. The relevance of twice the diameter marking the length of the town is clear from Guidoni’s figure and from Friedman’s text (Friedman 1988, p. 122). The difference is, however, that according to Guidoni’s figure the distance between the inner alignments of the wall streets is marked, while according to Friedman’s text it regards the distance between the outer alignments. In fig. 6.19 the theoretical distance appears to mark places somewhere in between, more or less halfway the wall streets.

53 For the reliability of Friedman’s measurements in comparison to the digitised 1:500 plan of fig. 6.19, see n. 45 above. According to Friedman’s measurements the distance between the outer alignments of the wall streets would have been 334 m. (twice 167 m.). (Friedman 1988, pp. 122-123) This measured length fits very well with Friedman’s proposed radius of 83.4 m., which would make a theoretical length of 333.6 m., thus giving a difference of only 0.1\%. The adapted radius of 83.89 m. would make a theoretical length of 335.56 m., giving a difference of 0.5\%, and the radius of 144 b. (84.9384 m.) would make a theoretical length of 330.5 m., giving a difference of 0.6\%. All these relative differences are within reasonable limits of tolerance.

54 Not all relevant dimensions are given directly by Friedman. It is only by combining the information from his article (1974, p. 239) and his book (1988, pp. 122-123) that I was able to calculate all relevant dimensions from his measurements.

55 A large part of this rather great difference lies in the measurements for the fourth row of lots and the wall street. This outer row and the longitudinal streets by the town wall are difficult to measure because this street was built-over and annexed by the holders of the plots in the fourth row. Actually, the fourth row was only partly occupied by houses, as can be seen in the 19\textsuperscript{th}-century cadastral plan. (fig. 3.23) So, the greatest part of the present houses in the fourth row must have been built relatively recently, with immediate use of the town wall as back-wall for the houses. Consequently, there are only few places where the original building line, separating the fourth row from the wall street, still clearly shows. The measurements at different places of these two elements showed relatively great deviations of up to 30 \% for the outer blocks and 40\% for the wall street. This may be due to less accurate measuring of the relevant former boundary between the fourth block and the former wall street by the surveyors for the plan drawing, as well as by Friedman, which is explicable because these former boundaries are built in the private structures.

If the length of the fourth lot and the width of the wall street are taken together as one measurement, it would be 27.16 b. according to my measurements and 27.93 b. according to Friedman’s, which would make the average difference between the two come down to 2.0\%.

It is striking to see that my measurements are larger when it comes to the width of streets, but smaller when it comes to the length of house lots, with a maximal difference of 0.70 b. (0.41 m.). Obviously, this is at least partly due to the difference of measuring in reality (from outer edge of house lot or wall to the next outer edge) and measuring from a plan with ink lines representing these outer edges (I measured from the centre of the one line to the centre of the other).
The main street would be 14 b., the back streets 8, the parallel streets 10, and the longitudinal wall streets 12 braccia wide. These different widths also form an arithmetic series (although not arranged accordingly in the plan): 8, 10, 12, 14.

In order to check this hypothesis, one can consider the total width of the whole town. If these hypothetical dimensions would be right, the total would measure 270 b., from the inner edge of the town wall to the opposite inner edge. My measurement in the plan is 276.34 including town walls, which are 85 cm. (1.46 b.) thick according to Friedman, so that would give a difference of 1.16%. Friedman measured 276.04 b., excluding the town walls, so this makes a difference of 2.24% with the hypothetical total dimension and 0.98% with my measurement.

However, when a choice must be made between Friedman’s theory or Pirillo’s on the basis of the correspondence with the measurements, then Friedman’s would be most obvious to choose. My measurements differ 2.47% on average from the five dimensions according to Pirillo, whereas Friedman’s diverge 3.91%. Disregarding the 15 b. width that Pirillo suggests for the main street, this would be 2.28% respectively 3.2%. But the divergence of the five relevant dimensions in the geometric theory compared to my measurements is only 0.48%, and 0.74% to Friedman’s.56

So, from this it may be concluded that the hypothesis of complex geometric design of Friedman (and Guidoni) fits better with the actual dimensions measured in Terranuova. Therefore, it seems more likely to be right. That is, if one accepts that the ‘designer’ did not simply choose for the easiest and most obvious method. It would be too easy, however, to conclude from this that the theory of arithmetical dimensioning in rounded numbers of braccia is completely wrong.

Friedman’s (and Guidoni’s) hypothesis does not explain all relevant dimensions: it only pinpoints the inner alignments of the streets parallel to the main street, as well as the position of the longitudinal parts of the town wall. This geometric method determines dimensions that contain the length of the house lots together with the width of the streets. So, the exact place of the other lines of division between these must have been determined in another way. Here it would seem likely that dimensions were chosen in round numbers of braccia. If this is right, it is possible that the planners chose the lengths of the house lots in the series of 30, 25, 20 and 15 b., so that the width of the streets would be the ‘rest value’. In Friedman’s measurements, as given in table III, however, the street widths are closer to rounded numbers of braccia than are the lot lengths. In my measurements there is no clear contrast between the two in this respect. Thus, it seems as though the widths of the streets in Terranuova were determined in whole numbers of braccia, and the lengths of the house lots were given by the rest values.

Assuming that the hypothetical 144 b. radius was used, the relevant dimension would be: 37.27 b. for the half of the main street and the length of the adjoining lot; 34.73 for the back street and the second row of lots; 29.82 for the next street and lot; 22.88 for the next street and lot, and 14.39 b. for the wall street (and possibly also the wall itself; which is difficult to determine since Friedman’s and my measurements contradict each other on this point). This would result in the next possibilities: division in dimensioned lot and rest street, and division in dimensioned street and rest lot. Table III shows the resulting dimensions.

Table III: widths of streets and lengths of lots (in Florentine braccia)

<table>
<thead>
<tr>
<th></th>
<th>main street</th>
<th>1st lot</th>
<th>back street</th>
<th>2nd lot</th>
<th>parallel street</th>
<th>3rd lot</th>
<th>back street</th>
<th>4th lot</th>
<th>wall street (and wall?)</th>
</tr>
</thead>
<tbody>
<tr>
<td>with lots dimensioned</td>
<td>14.5</td>
<td>30</td>
<td>9.73</td>
<td>25</td>
<td>9.82</td>
<td>20</td>
<td>7.88</td>
<td>15</td>
<td>14.39</td>
</tr>
<tr>
<td>with streets dimensioned</td>
<td>14</td>
<td>30.27</td>
<td>8</td>
<td>26.73</td>
<td>10</td>
<td>19.8</td>
<td>8</td>
<td>14.88</td>
<td>14.39</td>
</tr>
<tr>
<td>measurements Friedman</td>
<td>14.1</td>
<td>30.5</td>
<td>8.05</td>
<td>25.7</td>
<td>9.97</td>
<td>20.4</td>
<td>8.02</td>
<td>15.94</td>
<td>11.99 no wall</td>
</tr>
<tr>
<td>measurements Boerefijn</td>
<td>14.5</td>
<td>30.27</td>
<td>8.35</td>
<td>25.58</td>
<td>10.18</td>
<td>20.5</td>
<td>8.53</td>
<td>14.53</td>
<td>13.05 incl. wall</td>
</tr>
</tbody>
</table>

56 That is, with the ‘adjusted’ theory, with r=144 b.
Analysing these numbers, it appears that the difference between the dimensions in the first row with Friedman's measurements is 6.70% on average. But if the measured dimension of the wall street would be increased with the 1.46 b. for the town wall (the width according to Friedman), this dimension would be 13.45 and the average difference would be reduced to 5.25%. The difference can be further reduced by taking Friedman's measurements for the fourth lot, the wall street and the wall together (cf. note 56 above): 4.25%.

If the same is done for the second row, compared to Friedman’s measurements, the average difference comes to 4.04%, and with the 1.46 b. for the town wall 2.60%, and with the fourth lot, the wall street and the wall together 1.21%. It is clear that these percentages are considerably lower, with the widths of the streets dimensioned in whole braccia, than with the lots dimensioned in rounded numbers. Therefore, it appears more likely that the design was made accordingly.

Finally, when my measurements are compared to the first row, the difference is 5.05%, which is reduced to 4.81% if the entries in the last two columns are taken together. Comparing my measurements to the second row, the difference is 4.15%, respectively 3.85%. So, here too, it seems more likely that the street widths were dimensioned in whole braccia. In my measurements (and somewhat less in Friedman’s if the width of the town wall is added) it is obvious that especially the dimensions in the last column diverge rather strongly from the theoretical value. In Friedman’s measurements there is also a rather great difference with the theoretical dimension regarding the fourth lot. It seems likely that this is due to the difficulty of measuring these dimensions, because of the building-over of the wall street.

Thus, it seems as though the widths of the streets of Terranuova were laid out in rational numbers of braccia, and that the lengths of the house lots were given by the rest values. In fact, this is more logical, because by taking a fixed width in a specific number of braccia, it is easier to lay out the street with the two building lines completely parallel, which is important for the aesthetic of the street as well as for guarding public space from being encroached upon by private structures.

### B.3.2 San Giovanni

For the design of the plan of San Giovanni Friedman suggested a geometric design method which is based on the same principle, but which is less advanced. According to Friedman only the lines determining the inner side of the parallel streets and the inner side of the longitudinal wall streets are pinpointed by the cords of the sines of 30 and 60 degrees of the radius of 96.68 m. (165.66 b.). (see fig. 6.15) This theoretically results in dimensions of 48.34 and 83.72 m. In reality Friedman measured 48.34 (on which he based the theoretical radius) and 84.05 m. on average, which makes a difference of only 0.39%. My average measurements come to 48.94 and 84.37 m., diverging 1.24% and 0.78% respectively. With a slightly larger theoretical radius of 97.65 m., however, the resulting differences would be just 0.23% and 0.23%. It is not possible to say whether the differences between my measurements and Friedman’s stem from faults in the paper plan I measured, from faults in my measurements, or if it is because I measured on different spots in the plan than Friedman did. In any case, it seems very well possible that this geometric method of proportioning was indeed used in the design, all the more so since the same principle fits very well on the plan of San Giovanni’s sister town of Terranuova.

When a small adjustment is made to this geometric system by shortening the polygon radius (or hypotenuse in Friedman’s geometric description) so that the angle-points mark the lines between original house facades and porticoes (or street, where porticoes are absent), the outer angle-points of the dodecagon mark the alignments of the original longitudinal town walls. However, when Friedman’s system is adjusted in this

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57 See n.56. It is obvious that Friedman’s measurements fit better to the theoretical values, with street widths dimensioned in rational numbers, than mine. Nonetheless, the accuracy is less than in the comparison with just the relevant sine values that are given directly by the theoretical geometric system, which pinpoint five of these nine relevant lines in the plan (see table I). This is quite natural, however, since it regards much smaller dimensions and because the average percentile divergences are not calculated according to the relative weight of the individual dimensions within the set of dimensions, but by just taking all percentages and calculating their average.

58 Although it may be that the numbers of braccia for the street widths were chosen so, that the dimensions of the lots are at least close to the arithmetic series.

59 For the contemporaneous aesthetics of the street, see pars. 8.6.2, 8.6.3.

60 The oldest documented regulations of building lines and fixed widths of streets are from Strasbourg in the 12th century. (see Braunfels 1953, pp. 88, 99-104, 110-115; and pars. 8.6.2, 8.6.3)

61 Friedman 1988, pp. 121-122.

62 Those two outer points of the dodecagon would in Friedman’s description be termed the 90-degree-sine or total hypotenuse-length. While with Friedman’s hypotenuse-length the outer points would be circa 1.4 - 3.2% outside the outer wall-alignment, it would be only 0 - 1.4% in the reduced form.
way, it becomes basically the same as a part of Guidoni’s geometric system for San Giovanni, because the angle-points in this dodecagon mark the same spots in the plan as the intersection points of radiiuses and hexagon sides in Guidoni’s figure.

Friedman’s hypothesis for Terranuova is practically similar to Guidoni’s. His theory for San Giovanni, however, is a strongly reduced version of Guidoni’s. (compare figs. 6.12 and 6.15). Friedman does not explain why he made these changes, but it is likely that he assumed that the rest of Guidoni’s theory did not correspond with the plan very well.

Nonetheless, it seems worthwhile to compare Guidoni’s theoretical figure with the town plan. It is already explained in paragraph B.2.3 that the three diagonal squares, which would have determined the proportions of the piazza according to Guidoni, make no sense at all. From figure 6.20, in which the theoretical figure is laid over the digitised modern plan, it appears that there is quite a strong correspondence between the modern plan and the design system proposed by Guidoni. The geometric figure seems to mark the centre of the main street, the inner and outer alignments of the parallel streets, the backside of the third and fourth rows of lots from the main street and the outside of the longitudinal sides of the town wall. The figure also seems to mark the relation between length and width of the town, just like Guidoni suggested. But this is not entirely logical, as will be explained below.

It appears, however, that an addition can be made to the geometric figure so that it also comes to mark the backside of the house rows right next to the main street. (see fig.6.21) This addition is made up by the lines that cross through the centre point and through the four intersection points of the sides of the two rotated hexagons closest to the main street. The four points where these lines touch the outer circle mark the backsides of the first rows of lots. Actually, these points can be seen as the corners of a 24-sided polygon, as at Terranuova. This polygon is not depicted as such in the figure, but it would have the same radius as the hexagons, in this case 189 b.

In order to make a more accurate comparison between theoretical and actual dimensions, I measured the relevant dimensions in the plan, of which the averages are listed in the first row of table V. The distances are measured from the axis of the main street to the outer alignments of the elements mentioned in the headers of the columns. It is to be noted that I measured the distances to the original ground floor building lines of the house rows, whereas Friedman measured up to the line of the porticoes, which were often filled in on ground floor level afterward. These porticoes are generally circa 1.4 m. deep.

From the dimensions that I measured in the plan I calculated that the best fitting theoretical radius would be 189 b. The theoretical dimensions calculated with this radius are given in the second row of the table, while the third row displays the relative differences between actual measurements and theory.

<table>
<thead>
<tr>
<th></th>
<th>1st lot</th>
<th>2nd lot</th>
<th>parallel street</th>
<th>3rd lot</th>
<th>4th lot</th>
<th>wall street</th>
<th>town wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>measured by Boerefijn</td>
<td>48.92</td>
<td>81.47</td>
<td>95.36</td>
<td>119.91</td>
<td>142.16</td>
<td>161.62</td>
<td>163.16</td>
</tr>
<tr>
<td>theoretically with r=189b</td>
<td>48.92</td>
<td>81.84</td>
<td>94.5</td>
<td>119.74</td>
<td>141.75</td>
<td>163.67</td>
<td>165.67</td>
</tr>
<tr>
<td>divergence</td>
<td>0%</td>
<td>0.12%</td>
<td>0.91%</td>
<td>0.14%</td>
<td>0.29%</td>
<td>0.31%</td>
<td></td>
</tr>
</tbody>
</table>

From these numbers one may conclude that there is a strong agreement between the measurements and the theoretical dimensions. It is unlikely that this agreement is coincidental. The biggest difference is the divergence of 0.91% for the distance from the centre of the main street to the outer building line of the parallel streets, which does not seem to be problematically large. But it appears that there is an objection with respect to this element, as this divergence makes that the theoretical width of the parallel street relatively far removed from the width I measured. This is shown in table VI, which does not give the ‘sines’, as Friedman calls these dimensions, but the individual dimensions of the different elements (which add up to the sines).63

63 These average dimensions are calculated from my measurements of the individual elements of the plan, not from my measurements of the cumulative dimensions (or ‘sines’). Although they ought to be the same when the individual elements are cumulated, they are not in practice, because of small inaccuracies in taking dimensions from the plan.
Table VI: widths of streets and lengths of lots (in Florentine braccia) and divergence between measurements and theory

<table>
<thead>
<tr>
<th>measured by Boerefijn</th>
<th>main street/1st lot</th>
<th>alley/2nd lot</th>
<th>parallel street</th>
<th>3rd lot</th>
<th>alley/4th lot</th>
<th>wall street/wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>theoretically</td>
<td>49</td>
<td>32.31</td>
<td>13.9</td>
<td>24.47</td>
<td>22.02</td>
<td>21.31</td>
</tr>
<tr>
<td>divergence</td>
<td>0.16%</td>
<td>1.89%</td>
<td>10.98%</td>
<td>3.15%</td>
<td>0.09%</td>
<td>2.91%</td>
</tr>
</tbody>
</table>

It is clear that the width of the parallel street disagrees strongly with its theoretical value. The inner alignment is 0.37 b. further inward, while the outer alignment is 0.86 b. further outward. It is possible that this difference is made by inaccuracies in the design or the laying out of the plan. On the other hand, however, it might be that the designers or surveyors of the plan did not use the outer building line of the parallel street which is given by the geometric system, but chose instead, to make the street wider, possibly 14 b. If this theoretical line is discarded and the measured dimensions of the street and the third lot are added up, they cumulate to 38.37 b., which is much closer to the two added theoretical dimensions: 37.90 b., giving a divergence of 1.24%.

Thus, it seems that the designers had five relevant building lines that were pinpointed by this rather complex geometric system. Four more building lines and the width of the town wall were, just like in Terranuova, established in another way, probably by just taking rounded numbers of braccia in a pragmatic way, so that width of streets and length of lots would be convenient. So, following my measurements, the first theoretical dimension of 49.92 b. was divided in half the width of the main street (9.5), and the length of the first lot (39.42). The second theoretical dimension (32.92 b.), was divided in an alley (3.75) and the second lot (29.17); the third dimension (37.9 b.) was divided in the parallel street (14) and the third lot (23.9); the fourth dimension (22 b.) was divided in an alley (2.75) and the fourth row of house lots (19.25); the fifth dimension (21.93 b.), finally, was occupied by the town wall (1.5 b. wide) and the wall street (20.43).

In this way the different theoretical dimensions given by the geometric system show strong correspondence to the added dimensions of: half the width of the main street and the length of the first lot; the width of the alley and the length of the second lot; the width of the parallel street and the length of the third lot; the width of the alley and the length of the fourth lot; the width of the wall street and the width of the town wall. This is analogous to the geometric design of Terranuova according to both Guidoni’s and Friedman’s theories, although there are back streets instead of alleys there.

The radius of 189 b. is not determined by an obvious symbolical number, as at Terranuova. The only special dimension of this number, as far as I can see, is that it is three to the fifth power.

A further element of Guidoni’s theory regarding San Giovanni, considers the proportions of the perimeter of the town, which would be determined by two hexagons of the same radius as the central two rotated hexagons. (see fig.6.12) In this theory it is not very logical that the width is taken from town wall to town wall, whereas the length is taken on the inner alignments of the transverse wall streets. It is difficult to check this theory, since the original structure at the southern end of the town has been washed away by a flood in the mid-16th century. It is most likely, however, that the plan was symmetrical, so that the southern half of the town was just as long as the northern half. According to my measurements the town perimeter would measure 326.32 x 792.49 b. then, while according to the 16th-century plan of Piero della Zucca it would be 328 x 790 b. I measured a width of the transverse wall street of 13.36, Della Zucca of 14 b. So, subtracting this dimension twice (one for either of the sides of the town, the length would be 765.77, resp. 762 b. If one calculates the theoretical length of the town from the width, it would be 753.60, resp. 757.48 b., so the difference between measurements and theory is 1.61%, resp. 0.60%. These divergences are not too great to make the theory implausible, but it is problematic that the principle is rather unobvious, since the related dimensions reach the outer side of the town wall in the one direction, but only reach the inner side of the wall street in the other direction. In my opinion, this objection is serious enough to make this element of Guidoni’s theory rather unlikely to be correct.

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64 I will return on this problem below.
65 According to two 16th-century plans (see below) the wall was 1.5 b. wide, while I measured 1.87 b., but it is obvious that with such a small dimension, their measurements must be more accurate than mine from the 1:500 plan.
66 The plan by the surveyor Piero della Zucca was made after he was assigned to assess the damage caused by a flood in 1553. (Friedman 1988, pp.11, 348-350. Archivio di Stato di Firenze, Cinque conservatori del Contado, 258, fol. 602 bis)
Pirillo’s theory concerning San Giovanni is that the house lots are 40, 30 (including the width of the back alley), 25 and 20 braccia (including the back alley) deep. The main street and the cross street would be 20 b. wide.\textsuperscript{67}

Fortunately, the 16\textsuperscript{th}-century plan of Della Zucca and another 16\textsuperscript{th}-century plan of maestri Gentile and Batista\textsuperscript{68} are of help for the case of San Giovanni, since they give contemporary measured values of the relevant dimensions. In both plans, measured dimensions in braccia are inscribed. The relevant measurements from these plans are given in table VII, which also contains Pirillo’s hypothetical dimensions, Friedman’s measurements and my measurements.\textsuperscript{69} In the last row of the table the dimensions are given that can be formed from the theoretical dimensions with r=189 b., as calculated above.

<table>
<thead>
<tr>
<th></th>
<th>main street</th>
<th>1\textsuperscript{st} lot</th>
<th>alley</th>
<th>2\textsuperscript{nd} lot</th>
<th>parallel street</th>
<th>3\textsuperscript{rd} lot</th>
<th>alley</th>
<th>4\textsuperscript{th} lot</th>
<th>wall street</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gentile &amp; Batista</td>
<td>19.83/20</td>
<td>38</td>
<td>3.5</td>
<td>29.33</td>
<td>15</td>
<td>4</td>
<td>24.5</td>
<td>2.5</td>
<td>19</td>
</tr>
<tr>
<td>Della Zucca</td>
<td>21</td>
<td>39</td>
<td>4</td>
<td>28/28.5</td>
<td>15.33/15.66</td>
<td>24</td>
<td>3</td>
<td>20</td>
<td>17.875/18</td>
</tr>
<tr>
<td>Friedman</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>19.34</td>
<td>19.44</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pirillo</td>
<td>20</td>
<td>40</td>
<td>30</td>
<td>25</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>theoretically</td>
<td>19</td>
<td>39.42</td>
<td>3.75</td>
<td>29.17</td>
<td>14</td>
<td>23.9</td>
<td>2.75</td>
<td>19.25</td>
<td>20.43</td>
</tr>
</tbody>
</table>

It may be deduced from this table that either one or both of the 16\textsuperscript{th}-century plans are not very accurate, as their measurements diverge rather strongly.\textsuperscript{70} It appears that Pirillo is not too far from the real dimensions with his hypothesis. But still, the dimensions that he suggests are round numbers, and the 16\textsuperscript{th}-century surveyors did not measure the same round numbers, although they also tended to round off their measurements. Only the width of the main street according to Gentile and Batista, and the length of the fourth house lot according to Della Zucca correspond to Pirillo’s dimensions. Therefore, it does not seem very likely that Pirillo is right. The lot sizes given by the ‘theoretical’ last row are closer to my measurements than to Pirillo’s theoretical dimensions (of course, while the division of these theoretical dimensions given by the ‘sines’, is inspired by my measurements, as explained above). These lot sizes, however, are on average also closer to the dimensions given by the 16\textsuperscript{th}-century surveyors, than to Pirillo’s. This would be even more true when the dimensions from the two old maps would be taken instead of mine, and when it is supposed that the main street would be 20 b. wide, which would theoretically leave 38.92 b. for the first lot.

It is obvious, however, that there are two rather problematical elements in this theory. The first one concerns the width of the parallel street, which was measured more than a braccia wider in the 16\textsuperscript{th} century compared to my measurement. Making it wider in theory would mean, however, that the third lot became

\textsuperscript{67} Pirillo 1989, p.18.
\textsuperscript{68} The plan by maestri Gentile and Batista is probably from the first half of the 16\textsuperscript{th} century. (Friedman 1988, pp.10, 347–348. Archivio di Stato di Firenze, Piante dei Capitani di parte, cartone XVIII, no.28)
\textsuperscript{69} Friedman 1988, pp.70, 75. The dimensions Friedman cites are in rounded numbers. It is not clear whether they come from his own measurements, and to what degree they are rounded off.
\textsuperscript{70} It is difficult to determine whether Batista and Gentile’s, or Della Zucca’s measurements are more accurate. Comparing their measurements with mine, the indications are rather ambiguous, but over all, the first come slightly closer to my measurements than the second. One would expect, of course, that these plans reveal dimensions that are closer to the original situation than my measurements because of their early date. However, comparing with my measurements, one might ask if their measurements are as accurate as mine, because I believe that I have taken more measurements in different places. From the differences between their measurements it seems obvious that, then as well as in the present day, different dimensions were measured in different places.

On the whole, all differences per column in the table added, the measurements between Gentile and Batista’s plan and Della Zucca’s diverge circa 0.25 b., while the first diverge 3.41 from mine, and Della Zucca’s 6.12 from mine (which is 3.66%, 3.16% and 3.58% respectively, of the cumulative of my measurements). The cumulative dimensions are, respectively, 171.14, 172.69 and 170.96 (and 171.67 for the last row). This makes a difference of 0.91% between the two 16\textsuperscript{th}-century plans, 0.10% between the first and mine, and 1.01% between the second and mine.
too short to fit the length that I measured. The second problem is the width of the wall street, which was at least a braccia narrower in reality than in theory.\textsuperscript{71} I have no explanation for these divergences.

Hence, it is difficult to come to solid conclusions on the basis of these different dimensions. It does not seem very likely, however, that Pirillo is right with his theory that the lots were dimensioned in a series of fivefold numbers of braccia. Friedman’s theory of geometric proportioning of the distances from the town-axis to the inside of the parallel street and the wall street corresponds well with the actual dimensions, but its significance for the dimensions of the various constituent elements is rather limited. Therefore, Guidoni’s theory has more significance, pinpointing eleven (and in slightly extended version thirteen) relevant lines in the plan, against five for Friedman’s theory. The drawback is, however, that its geometric principle is rather complicated, and although it is not too difficult to make the geometric construction, it does not seem to be a very obvious method of designing an orthogonal town plan.\textsuperscript{72} A further weak point is, that the resulting individual dimensions are not always close to my measurements (as is the case with the width of the wall street) or the measurements on the 16\textsuperscript{th}-century plans (as is the case with the widths of the wall street and the parallel street).

Nonetheless, the small divergences in the dimensions of the ‘sines’ (the overall dimensions from which the dimensions of the individual elements can be calculated by subtraction) strongly suggest that this geometry does indeed underlie the design.\textsuperscript{73} Another argument that speaks for this theory is the analogy to Terranuova, where the geometric figure is different but the basic principle is quite similar, and two more of the terre nuove, as will be described below.

\textbf{B.3.3 Castelfranco di Sopra}

For the plan of Castelfranco di Sopra Guidoni proposed a simple geometric figure underlying its design.\textsuperscript{74} (fig.6.11) He assumed that the piazza had been designed as a geometric square, and not as the rectangle that it presently is.\textsuperscript{75} It may safely be assumed, indeed, that symmetry was sought after in the design. It seems more likely, however, that the piazza was longer (c.102 b.) instead of shorter, as it generally was more common that public space was made smaller, being encroached upon, instead of larger.\textsuperscript{76} In any case, even if the piazza was originally planned square, it is most unlikely that a diagonal square, as depicted in Guidoni’s figure, was used for its design or for setting out the plan.\textsuperscript{77}

Guidoni also proposed that the inner blocks of the town were laid out as a square, or rather as a parallelogram because the angle between the main axes of the town is not 90° but circa 88/92°. Comparison with the 1:500 plan of the town shows, however, that even when this irregularity is taken in consideration, the square does not fit the actual dimensions very well, because it gives differences of circa 2 - 5%. (fig.6.22) Therefore, it does not seem very likely that this square was part of the geometry underlying the design of the plan.

Apart from that, Guidoni proposed a hexagon that describes the relation between the width and the length of the perimeter of the town, as it originally must have been. Unfortunately, this cannot be verified in the northwestern half of the plan, because there the original perimeter form has become blurred during the centuries, as the town wall has largely been demolished since the 17\textsuperscript{th} century. It may safely be assumed, however, that the part to the northwest of the main street was symmetrical to the other half, in which more of the old structure is preserved, especially in the southern quarter. (see fig.3.6) It appears that the hexagon does indeed mark the perimeter proportions quite accurately, just as Guidoni suggested. It can be objected, however, that this is not very logical, as the length of the town is given between the outer alignments of the wall streets on the northeast and southwest sides, whereas the width is given between the inner alignments of the wall streets on the other two sides (theoretically; only on the southeast side still verifiable).

\textsuperscript{71} From the dimensions in table V, it may be deduced that, according to my measurements, the wall was built on average about half a braccia closer to the main axis of the town, than it would be in theory. The outer alignment of the fourth row, however, was built 0.41 b. further outward.

\textsuperscript{72} In this context, the consistency of the geometric principle is weakened by the fact that the 90°-sine value of the 189 b. radius, with the relatively large divergence of 0.91\%, was obviously not used directly in laying out the plan, while the parallel street was made considerably wider than the theoretical width of 12.66 b. according to the hypothetical geometric system. (see above)

\textsuperscript{73} It should be noted, however, that the entry in the third column of table V, the sine of 30°, probably was not taken as a relevant dimension by the designers. (see above)

\textsuperscript{74} Guidoni 1970, fig.116.

\textsuperscript{75} This was already suggested by Richter (1940, p.369). See also the comparison of the terre nuove piazzas in par.B.4.

\textsuperscript{76} Friedman 1988, p.342, n.34.

\textsuperscript{77} The fact that he depicted the diagonal square may well have to do with the idea of rotated squares as a common principle in ‘medieval architectural design’. (see par.6.3.2-6.3.3)
The distance from the axis of the main street to the outer alignment of the lateral wall street in the southern quarter is 201.98 braccia. If this outer alignment would be taken as relevant line for the side of the hexagon to mark, the length of the hexagon along the axis of the main street would be 466.43 braccia. In fact, the length of the town between the outer alignments of the northeastern and southwestern wall streets is 451.16 braccia, resulting in a difference of 15.27 braccia, or 3.38%, which is more than would be tolerable on this scale. But when the dimensions in the plan closely are studied, there appear to be other remarkable dimensions that ask for attention. The outer building line of the second parallel street is at 117.46 braccia from the main axis, and the outer building line of the wall street near to the southeast gate is at 201.98 braccia from it. This suggests that these dimensions might have been designed as the sines of 30° and 60° of a dodecagon with a radius of 234.07 braccia. (fig. 6.23) This would be similar to the situation in San Giovanni and the other terre nuova of Scarperia, as will appear in the next paragraph.

Table VIII: distances from centre of main street to outer alignments of longitudinal streets (in Florentine braccia) and divergence between measurements and theory

<table>
<thead>
<tr>
<th></th>
<th>outside of 2nd parallel street</th>
<th>outside of wall street</th>
</tr>
</thead>
<tbody>
<tr>
<td>measurement Boerefijn</td>
<td>117.46</td>
<td>201.98</td>
</tr>
<tr>
<td>theoretically with r = 234.07 braccia</td>
<td>117.04</td>
<td>202.71</td>
</tr>
<tr>
<td>difference</td>
<td>0.36%</td>
<td>0.36%</td>
</tr>
</tbody>
</table>

Thus, it seems quite well possible that this geometry was used in the design of the plan for Castelfranco. There are not many relevant dimensions given by the geometric figure of the dodecagon - it regards just two dimensions that are related in this way - but the similarity of this geometry to that found for San Giovanni, Scarperia (see the following paragraph) and Terranuova appears to confirm that this geometry really played a role in the design of the dimensions. If a special significance is to be found in the number 234 (rounded from the dimension of 234.07 braccia) for the length of the radius, the only option I can think of is that the figures of 2, 3 and 4 follow one another as in the elemental numerical series.

Apart from the geometrically determined dimensions in the plan of Castelfranco, it may be helpful to consider other dimensions of streets and house lots or street blocks, in order to compare them later on, to the corresponding dimensions in the other terre nuova. (see par.B.4) Most probably, the lots were 10 braccia wide originally, as in San Giovanni and Terranuova. Further measurements taken from the plan have lead to the average dimensions shown in table IX. In this table the first row contains the width of the complete blocks, while the second row has this element cut up in the separate dimensions of depth of lot / width of alley / depth of lot. These last three cumulated, diverge somewhat from the complete street block dimensions, because the alleys are only left in some places, for which reason the three could not be measured separately in the same places where the complete width of the block was measured. A further difference is made by the fact that in this way the inaccuracy from measuring is tripled. The third row contains rounded values, derived from the measured dimensions. These are significant, while here, where a smaller number of relevant lines seem to be determined by the design geometry than in San Giovanni and Terranuova, it seems more obvious that these dimensions were chosen as numerical values in whole braccia.

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78 See par.B.4.
79 The number of 234 can be subdivided as $2 \times 3 \times 3 \times 13$, which does not seem to be significant in any way.
The most remarkable conclusion to be drawn from the dimensions in this table, is that the diminution of the lot length is very gradual between 3rd / 4th, 5th / 6th and 7th / 8th row. A further remarkable feature is the fact that the 3rd parallel street is a little wider than the 1st and 2nd parallel streets.\(^{80}\) There is no arithmetic system recognisable in the relation between the different dimensions.

### B.3.4 Scarperia

For the terra nuova of Scarperia I have also checked to what degree the theoretic geometric design methods encountered above correspond to the plan. Guidoni, Friedman and Pirillo did not extend their theories to this town. According to Guidoni this town is much less interesting because of an unspecified ‘loss of tension’ in comparison to Castelfranco, San Giovanni and Terranuova, and Friedman assumes that similar methods of design as he proposed for San Giovanni and Terranuova were not used here because only the house lots on the main street are larger than the three rows of lots behind them.\(^{81}\) Nonetheless, it seems possible that a similar method of design was used here, as suggested by Van den Heuvel.

The plan of Scarperia is rather less regular than those of Terranuova and San Giovanni, and of Castelfranco as it was until about the 17th century. Particularly the outline is less regular.\(^{82}\) On the west side the plan came to follow the irregular contour of the steep ridge on which the town was built. This allowed for additional plots to be laid out there, for which reason the plan is much less symmetrical than the other terre nuove. On the other three sides, the circuit of the town wall was at least since the 17th century made up of straight stretches that, however, did not make up a real rectangular form. (fig. 3.19) The irregular contours of the town, together with the fact that the fourth row of lots was left largely vacant on the eastern side and in the southwestern quarter, make that the second parallel street, or what would have been the wall street had the town got a regular rectangular outline, is not recognisable as such. Only in the northwestern quarter it is clearly present, apart from a very small tract of dead end street in the southeastern part of the town. Further irregularity in the plan is caused by the fact that there are no clear continuous boundaries dividing the house rows at their backsides. Therefore, it is difficult to find the original length of the house lots. In fact, the backside boundaries of the rows of lots facing the main street are so irregular, that no clearly relevant dimension has been found directly. The other rows have more lots

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\(^{80}\) I suspect that this feature was not really intended in the original design, because this third parallel street does not seem to have had a special function, nor is it located in a place where it could have had some higher symbolic importance as an axis of symmetry.

\(^{81}\) Guidoni 1970, p.229. According to Friedman, the lots on the main street were circa 13 to 16 m. (22.27-28.44 b.) deep, while the other three originally planned rows contain lots of circa 11 m. (18.85 b.) deep. (Friedman 1988, p.120) This is confirmed by cadastral plans of the 19th and 20th centuries. (see appendix B, table XI) Two documents from the 14th century describe the assignment of lots measuring 12 x 16 and 12 x 18 b. (Friedman 1988, pp.314-318, docs. 5 and 7) The 18-braccia lot is still quite close to the hypothetical originally planned lots of 18.85 b., and its reduced length may be due to unclear lot boundaries or inaccurate measuring. The 16-braccia lot is more problematic in this context. Possibly, it lay on the western side of the town, in the area between the ridge and the outermost street, where the depth of the lots varied according to the irregular contour of the ridge.

with more or less consistent dimensions. But still, the various dimensions show stronger variation than in the other terre, for which reason the averages are less likely to reflect the original dimensions accurately.

Looking at the average measurements of the relevant boundaries in the plan, it appears that the outer alignment of the parallel streets is on average at 62.33 b. from the main axis, while the outer building line of the ‘wall street’ is at a distance of 109.75 b. This suggests that these dimensions may have been designed as the sines of 30˚ and 60˚ of a dodecagon with a radius of 125.68 b. (see fig. 6.24 and table X)

Table X: distances from centre of main street to outer alignments of longitudinal streets (in Florentine braccia) and divergence between measurements and theory

<table>
<thead>
<tr>
<th>distance from centre of main street to:</th>
<th>outside parallel street</th>
<th>outside wall street</th>
</tr>
</thead>
<tbody>
<tr>
<td>measurement Boerefijn</td>
<td>62.33</td>
<td>109.75</td>
</tr>
<tr>
<td>theoretically with ( r = 125.68 ) b.</td>
<td>62.84</td>
<td>108.81</td>
</tr>
<tr>
<td>difference</td>
<td>0.82%</td>
<td>0.86%</td>
</tr>
</tbody>
</table>

As at Castelfranco, this geometry is relatively limited in its relevance for the plan as a whole, but the method corresponds closely to it, and thereby also to the design systems of San Giovanni and, less directly, of Terrana. of 234.07 b. (cf. figs. 6.23, 6.21, 6.19)

The radius of 125.68 b. is calculated as best fitting to the measurements, but actually 125 b. would be the logical dimension to suppose for this radius. Although the number is not known for any specific symbolic meaning, it may have been relevant for the choice of this radius that 125 is 5 x 5 x 5.

Considering the other relevant dimensions of plan elements in Scarperia, it appears that, unlike the other terre nuove, the width of the lots was 12 b. This is evidenced by two 14th-century documents regarding the assignment of lots to settlers83, as well as by the width of many present-day lots.84 Other relevant dimensions are listed in table XI.

Table XI: widths of streets and lengths of lots (in Florentine braccia)

<table>
<thead>
<tr>
<th></th>
<th>width main street</th>
<th>length 1st lot</th>
<th>length 2nd lot</th>
<th>width parallel street</th>
<th>length 3rd lot</th>
<th>length 4th lot</th>
<th>width ‘wall street’</th>
</tr>
</thead>
<tbody>
<tr>
<td>measured as blocks</td>
<td>12.77</td>
<td>47.63</td>
<td>8.57</td>
<td>39.58</td>
<td>6.92</td>
<td></td>
<td></td>
</tr>
<tr>
<td>measured</td>
<td>12.77</td>
<td>undeterminable</td>
<td>20.31</td>
<td>8.57</td>
<td>19.19</td>
<td>19.28</td>
<td>6.92</td>
</tr>
<tr>
<td>rounded</td>
<td>13</td>
<td>27.5</td>
<td>20</td>
<td>8.5</td>
<td>19</td>
<td>alley 1.5</td>
<td>19</td>
</tr>
</tbody>
</table>

The existence of alleys dividing the two rows of lots within a block is proven by the statutes from the 1420’s.85 Only in the northwestern block one can find relics of such an alley. It is difficult to determine how wide it originally was (particularly because it is only a small dimension which is difficult to measure in a plan drawing in this scale), but it seems that it was circa 1.5 b. The alleys between the first and the second rows have completely disappeared in the present-day as well as in the 19th-century cadastral plan, so one of these two rows of lots, or both, must have been less deep, possibly also 1.5 b.

The totals are as follows: from the axis of the main street to the outer alignment of the ‘wall street’, it is 109.75 b. when the average total is measured; 108.81 b. theoretically with \( r = 125.68 \) b.; 109.08 b. when the average measurements of the individual elements are added up; and 109 b. when the hypothetical rounded dimensions are added up. These divergences are negligible.

83 See Friedman 1988, pp. 314, 317, docs. 5 and 7.
84 It seems, however, that a number of the corner plots may have been 14 to 15 b. wide, as may be observed in 14 out of 47 possible cases. Only in five of the remaining 33 corner plots the width is circa 12 b.; in the rest of the cases the lots have been subdivided or amalgamated.
85 Romby & Diana, 1985, p. 34 (A.S.F., Statuti Comuni Soggetti 831 - Lega di Scarperia (1413-1427)).
B.4 Comparison between the terre nuove

Now that the individual terre nuove plans have been examined, it is time to compare the dimensions found in the various plans. In order to make them easily comparable, they are listed in the tables that are depicted below.

Considering the basic layout of the towns, it is a very obvious difference that Castelfranco had 16 rows of houses while the other four towns had just eight in the original designs. Another relevant difference lies in the width of the original lots (not included in the table) which is ten braccia at Castelfranco, San Giovanni, Terranuova and Giglio, and 12 b. in Scarperia. Table XII contains the length of the lots in the distinct rows in the different towns. For easy perception of the data, the values are rounded off from my measurements, which does not imply that they were actually laid out as such.

Table XII: lengths of lots (in Florentine braccia), rounded off from measurements

<table>
<thead>
<tr>
<th>Town</th>
<th>1st lot</th>
<th>2nd lot</th>
<th>3rd lot</th>
<th>4th lot</th>
<th>5th lot</th>
<th>6th lot</th>
<th>7th lot</th>
<th>8th lot</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Giovanni</td>
<td>39</td>
<td>29</td>
<td>24</td>
<td>19</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Castelfranco</td>
<td>30</td>
<td>26</td>
<td>19</td>
<td>19</td>
<td>18</td>
<td>18</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>Scarperia</td>
<td>26*</td>
<td>20</td>
<td>19</td>
<td>19</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Terranuova</td>
<td>30</td>
<td>26</td>
<td>21</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Giglio</td>
<td>28</td>
<td>20</td>
<td>20</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*not measured but following from the length of the block minus the length of the 2nd lot and alley

Analysing the numbers in table XII, it appears that the first lot was in most cases circa 30 b. long, the second twice 20 and twice 26, the third circa 20, and the fourth lot circa 19 b. in three out of the five towns. San Giovanni is the obvious exception with much longer lots in the first, second and third rows. The 15 b. lot in Terranuova and the 10 b. lot in Giglio in the fourth row are clearly smaller than the rest. The intervals between the different lots are most regular in Terranuova.

Table XIII contains the measured widths of the longitudinal streets and alleys.

Table XIII: widths of streets (in Florentine braccia), rounded off from measurements

<table>
<thead>
<tr>
<th>Town</th>
<th>main street</th>
<th>alley/ back-street</th>
<th>parallel street</th>
<th>alley/ back-street</th>
<th>2nd parallel street</th>
<th>3rd parallel street</th>
<th>wall street</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Giovanni</td>
<td>19</td>
<td>4</td>
<td>14</td>
<td>3</td>
<td></td>
<td></td>
<td>19</td>
</tr>
<tr>
<td>Castelfranco</td>
<td>12</td>
<td>2*</td>
<td>7</td>
<td>2</td>
<td>7</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Scarperia</td>
<td>13</td>
<td>1.5</td>
<td>9</td>
<td>1.5</td>
<td></td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>Terranuova**</td>
<td>14</td>
<td>8</td>
<td>10</td>
<td>8</td>
<td></td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>Giglio</td>
<td>14</td>
<td>10</td>
<td>10</td>
<td>8</td>
<td></td>
<td></td>
<td>10</td>
</tr>
</tbody>
</table>

* All alleys in Castelfranco are taken to be about 2 b. wide. The alleys in the third and fourth block are not separately entered in this table.
** For Terranuova these dimensions are rounded from Friedman’s measurements, since they are probably more accurate than mine.
A clear difference is that Terranuova and Giglio have back streets, while there are alleys at the rear of the lots in the other towns. In San Giovanni the alleys actually were accessible (and recently opened up again), whereas at Castelfranco and Scarperia they seem to have been not much more than narrow gutters. Just like with the lots, the streets are by far the largest in San Giovanni, while they are narrowest in Castelfranco. The differences between these two towns are the more remarkable, since they were founded in the same instance and were built in the same period, quite near to each other, for which reason one would expect a greater similarity. Such chronological logic is, however, difficult to find in the ground plan designs of the terrane. There is no clear evolution in the plans; only between the last two towns of Terranuova and Giglio one may recognise certain similarities that more or less set them off against the rest, among which the phenomenon of the back streets is most obvious.

Another significant similarity between Terranuova and Giglio are the dimensions of the piazzas. It was explicitly stated in the foundation document for Giglio, that the piazza was to measure 70 x 90 b., which is almost the same as what I measured in the plan of Terranuova (90.52 x 68.97 b.).86 These almost equal dimensions are, however, made up by different sub-dimensions of the plan elements that flank the piazza, and that actually determine its dimensions, considering the piazza as a space left open by leaving lots vacant. The 90 b. length of the piazza is made up out of a 14 b. main street, two 30 b. lots and two 8 b. back streets at Terranuova, while in the case of Giglio it is made up out of a 14 b. main street, two 28 b. lots and two 10 b. back streets. Thus, at Giglio the lots were made a bit shorter and the back streets a little wider. The same is true for the width of the piazza: in Giglio the cross street was probably planned 14 b. wide, while at Terranuova I measured 11.39 b. on average, whereas Friedman claims that it was 10 b. wide.87

It is possible that the similarity between the dimensions of the two piazzas is accidental. After all, the piazza at Terranuova is not precisely 90 x 70 b. But it is also possible that the piazzas were explicitly planned with these dimensions. In that case, one would expect these dimensions to have had some sort of special relevance; but I have not been able to trace such a special significance of the values of 70 x 90 b.

There is, however, another way of looking at the open space of the piazzas of the two towns. If the streets are regarded as extending over the piazza, then four imaginative squares would be left as open spaces, not used for traffic. These four squares would be 28 x 28 b. in Giglio, and if Friedman’s measurements are right, circa 30 x 30 b. in Terranuova. The total dimensions of the piazzas may thus have been arrived at by squaring the length of the house lots that face the main streets.88 All in all, it remains difficult to establish the way the piazzas were designed. They may have been proportioned, either as a total of 70 x 90 b., possibly with a special significance in these dimensions, or as four open squares separated by trough-going, intersecting streets. It is also possible that the similarity of the dimensions of the two piazzas is coincidental, and that the piazza at Terranuova is only the result of leaving open a number of house lots in the centre of the town. This also seems to have been the case with the piazzas of the other towns, which have quite different dimensions. At San Giovanni it had a completely different shape, elongated and transverse in direction. Its original dimensions are difficult to determine, but the 16th-century plans of Della Zucca and Gentile and Batista mention 248 x 80 b. and 246.40 x 81.86 b. (accumulated).89 The piazza at Castelfranco measures approximately 70 x 86 b. and in Scarperia the dimensions are circa 69 x 46 b.90

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86 Friedman also claims that the piazza at Terranuova measures 70 x 90 b. (Friedman 1988, p.79).
87 Friedman 1988, p.146-147. This probably means that, to arrive at the 70 b. width at Giglio, not all house lots facing the square would be 10 b. wide, although that is not mentioned in the document. Friedman supposes that four of them may have been either 8 or 18 b. wide. (see fig.3.27) If my measurements are right, then, similarly, at Terranuova some lots must have had a width other than 10 b. in order to arrive at the measured 68.97 b. or the rounded dimension of 70 b. If Friedman is right (his measurements are probably more accurate than mine; see note 45 above), the lots and the cross street at Terranuova were all 10 b. wide.
88 When the space of the back streets that border the piazza is subtracted, it would measure 70 x 70 b. at Giglio and 74 x 70 b. at Terranuova. Possibly, there is a similarity with the original design for the piazza at Castelfranco, which might have been proportioned as a real square of circa 70 x 70 b. This was suggested by Guidoni (1970, fig.116, see fig.5.11), following Richter (1940, p.369). But it seems more likely that the piazza was wider, being circa 102 b. long, as it was more common that public space was made smaller, being encroached upon, instead of larger. (Friedman 1988, p.242, n.34)
89 Friedman 1988, p.79; Friedman 2004, p.149. According to Friedman (2004, p.149) the piazza was purposely given the proportions 3:1.
90 On Castelfranco, see par B.3.3. The piazza at Scarperia is very different from the other piazzas, not lying in central position with the main street as its axis, for which reason it is not really comparable. The length is almost 70 b. and thereby more or less analogous to the piazzas of Terranuova and Giglio, but it is likely that this is accidental, as the overall form is so different.
The dimensions of the outlines of the terre vary from town to town, and so do the relative proportions between length and breadth.\textsuperscript{91} It is obvious that the dimensions of the outlines are not chosen as round numbers, and apart from the case of Terranuova, they do not seem to have been determined by geometric proportioning methods.\textsuperscript{92} It seems that the outline dimensions were mainly the result of the dimensions in the inner structure\textsuperscript{93} and, of course, the planned number of house lots.\textsuperscript{94}

\textsuperscript{91} My measurements in braccia partly result from reconstructions of the original rectangles: Castelfranco 451.16 x 403.96; San Giovanni, 792.49 x 326.32; Terranuova 570.08 x 271.85; Scarperia, circa 520 x 220 (to the outside of the second parallel street, which was presumably meant as wall street in the original design). For Giglio the document mentions 470 x 246 b. For Firenzuola the foundation document mentions 635 x 342 b., but (at least) since the late 15th century the town’s dimensions are circa 356 x 322 b. It is rather strange that these are the only dimensions given in this document. It suggests that the layout was already designed in detail, but was not found necessary to describe in an official document, as was done 13 years later for Giglio. Alternatively, the design may have been recorded in another document, which has been lost.

\textsuperscript{92} On Terranuova, see par. B.3.1. See also pars. B.1 and B.3.2, B.3.3.

\textsuperscript{93} This was not the case with the Florentine new town that was to be founded on the Consuma pass in 1329 (see par. 3.3), which has not left any traces in the present-day landscape. This town was to measure 300 x 600 braccia, according to the document of legislation for its foundation. (Richter 1940, p.381; Friedman 1988, pp.327-329) It is probable that the layout for this town was not really planned into detail yet, so that these dimensions were just given as a rough indication of its future size. A considerable difference between this town and the towns that are amply described in chapter 3, is that this town was to receive two gates instead of four.

\textsuperscript{94} Friedman has done research after the number of settlers who could be expected for the foundation of Giglio. He came to a number of between 176 and 330 families. The plan provided for circa 338 lots, in the space of which the church and town hall also needed to be accommodated. (Friedman 1988, p.76) It is not impossible, however, that the diameters of the polygons played a role in determining the length of the towns of Castelfranco, San Giovanni (doubled) and Scarperia (doubled), just like at Terranuova. (see fig.6.19) At Castelfranco the diameter reaches to the outer side of the gates (fig.6.22) and at San Giovanni, conversely, only to the inside of the wall streets (see fig.6.20). The later option may also have been planned for Scarperia, but this is conjectural, as it is unknown in what form the northern end was originally planned. (fig.6.24) Possibly, the planners used the (doubled) diameters roughly for guiding points, from which they diverged according to the number of house lots that were needed.