Fixed-point logics on trees
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Citation for published version (APA):

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Chapter 7

Conclusion

In this thesis we studied model-theoretic and proof-theoretic aspects of widely used logics on trees, including fixed-point extensions of first-order logic and linear-time temporal logic and $\mu$-calculus.

In Chapter 2, we gave an overview of the area by presenting the different classes of structures and logics that we consider in the thesis. We discussed the issues of expressive power, decidability and complete axiomatization.

In Chapter 3 we presented complete axiomatizations for MSO, $\text{FO}(\text{TC}^1)$ and $\text{FO}(\text{LFP}^1)$ on finite node-labelled sibling-ordered trees. In order to prove completeness, we developed model-theoretic tools specifically geared towards Henkin models. We believe that these tools are of independent interest. Indeed, since the original publication of the results, both our axiomatizations and our proof techniques have been applied in other settings, namely in the context of coalition logic [27] and of the $\mu$-calculus [40].

In Chapter 4, we essentially identified $\text{LTL}(X)$, $\mu\text{TL}$ and $\mu\text{TL}(U)$ as the three only temporal fragments of $\mu\text{TL}$ that satisfy Craig interpolation (moreover, they satisfy uniform interpolation). In this sense, our results singled out these three temporal logics as being exceptionally well-behaved. On finite and $\omega$-words, $\mu\text{TL}$ and $\mu\text{TL}(U)$ have the same expressive power as MSO and its stutter-invariant fragment (with respect to initial semantics), and therefore these results can also be seen as identifying fragments of MSO that satisfy uniform interpolation on words. Well-behaved logics on trees are often characterized with respect to the trade-off that they provide between expressive power and complexity, but interpolation deserves to be explored further, as it is another interesting criteria to compare and classify these logics. In particular, extending our results to the case of MSO on trees and branching-time temporal logics is an interesting challenge.

In Chapter 5, we gave a complete axiom system for $\mu\text{TL}(U)$, which was identified in the previous chapter as one of the three fragments of $\mu\text{TL}$ with Craig interpolation, and which, it appears, has not been studied before. The results we gave were obtained through $\mu\text{TL}(\bigcirc_U)$, a new logic that has the same expressive
power as $\mu_{TL(U)}$, but which is syntactically extremely close to $\mu_{TL}$. We believe that $\mu_{TL(\Diamond_T)}$ could be reused as a tool to easily transfer results from $\mu_{TL}$ to $\mu_{TL(U)}$ and to other logics characterizing the stutter-invariant fragment of MSO on words.

In Chapter 6, we turned to an important special case of tree structures: finite extensive games with perfect information. Such games are finite trees enriched with additional relations representing the preferences of players. We showed how various fixed-point logics can define standard solution concepts from game theory while staying faithful to the underlying solution procedures. More generally, by making a link with current dynamic-epistemic logics of knowledge update and plausibility change, we showed how this approach can provide “dynamic foundations” for game analysis that supplement the usual style of thinking. In doing all this, we also studied fixed-point logics that exploited the special well-founded orders available in game trees. In addition, we touched upon the issue of the complexity of such logics, since one wants to know how the expressive power needed for game solution balances with the potential undecidability of trees with additional structure beyond the successor order. Finally, we identified a great number of further issues that arise when we take our fixed-point logic perspective to more sophisticated parts of game theory.

To conclude, let us emphasize a few general points and questions related to fixed-point logics on trees that became more visible through the results obtained in this thesis. First of all, the search for well-behaved systems appears as a recurring theme. Various criteria for identifying them were studied: expressivity, decidability, axiomatization, interpolation and, in connection with games, procedural aspects. Secondly, a general back and forth between modal and first-order languages also characterized the perspective adopted in the thesis. As we explained in Chapter 2, this is a distinctive feature of the landscape of fixed-point logics on trees. Indeed, as we mentioned above, the work described in Chapter 3 has inspired further work in modal logic [27, 40], whereas the results in Chapter 4 and 5, while formulated mostly in modal terms, shed light on well-behaved fragments of MSO. For instance, we identified $\mu_{TL(U)}$ as the stutter-invariant fragment of $\mu_{TL}$, which is also the stutter-invariant fragment of MSO on words. Chapter 6 explicitly dealt with both modal and quantified logics. The issue of finding a balance between expressive power and complexity was especially highlighted, as the additional preference or knowledge structure carried by game trees typically increase complexity and call for the identification of well-behaved modal fragments of the usual logics on trees. As a final note, let us mention that similar issues are being addressed in the context of XML query languages, where one also need to add rich additional features to basic tree structures. Finite trees indeed serve as the standard theoretical abstraction of XML documents, but in this context it is often important to enriching the trees with additional “data structure” consisting of data values from an infinite alphabet, and enriching the logics with
means to compare the data values associated to different nodes of a tree. This increases complexity dramatically, and FO is for instance no longer decidable on such structures. Some work has been done in order to identify decidable fragments of usual logics on such enriched tree structures, also known as data trees (see in particular [28, 65]). It might be interesting to take inspiration from the results obtained in this area in order to characterize interesting decidable fragments of fixed-point logics on finite extensive game trees.