Fixed-point logics on trees
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Abstract

In this thesis, we study proof-theoretic and model-theoretic aspects of some widely used modal and quantified fixed-point logics on trees.

Chapter 2 includes basics of modal logic, temporal logic, fixed-point logics, and some first-order and higher-order logics of tree structures.

In Chapter 3, we consider the class of finite node-labelled sibling-ordered trees. We present axiomatizations of its monadic second-order logic (MSO), monadic transitive closure logic (FO(TC¹)) and monadic least fixed-point logic (FO(LFP¹)) theories. Using model-theoretic techniques, we show by a uniform argument that these axiomatizations are complete, i.e., each formula which is valid on all finite trees is provable using our axioms.

In Chapter 4 we consider various fragments and extensions of propositional linear temporal logic (LTL), obtained by restricting the set of temporal connectives or by adding a least fixed-point construct to the language. Using techniques from abstract model-theory, for each of these logics we identify its smallest extension that has Craig interpolation. Depending on the underlying set of temporal operators, this framework turns out to be one of the following three logics: the fragment of LTL having only the Next operator; the extension of LTL with a least fixed-point operator $\mu$ (known as linear time $\mu$-calculus); and $\mu_{TL}(U)$, the least fixed-point extension of the “Until-only” fragment of LTL.

In Chapter 5, we focus on the logic $\mu_{TL}(U)$, that we identified in the previous chapter as the stutter-invariant fragment of the linear-time $\mu$-calculus $\mu_{TL}$. We also identified this logic as one of the three only temporal fragments of $\mu_{TL}$ that satisfy Craig interpolation. Complete axiom systems were known for the two other fragments, but this was not the case for $\mu_{TL}(U)$. We provide complete axiomatizations of $\mu_{TL}(U)$ on the class of finite words and on the class of $\omega$-words. For this purpose, we introduce a new logic $\mu_{TL}(\Diamond_{T})$, a variation of $\mu_{TL}$ where the “Next time” operator is replaced by the family of its stutter-invariant counterparts. This logic has exactly the same expressive power as $\mu_{TL}(U)$. Using known results for $\mu_{TL}$, we first prove completeness for $\mu_{TL}(\Diamond_{T})$, which then allows
us to obtain completeness for $\mu TL(U)$.

Finally, in Chapter 6 we take our style of analysis via modal and temporal fixed-point logics to games. Current methods for solving games embody a form of "procedural rationality" that invites logical analysis in its own right. This chapter is a case study of Backward Induction for extensive games. We consider a number of analyses from recent years in terms of knowledge and belief update in logics that also involve preference structure, and we prove that they are all mathematically equivalent in the perspective of fixed-point logics of trees. We then generalize our perspective on games to an exploration of fixed-point logics on finite trees that best fit game-theoretic equilibria. We end with a broader program for merging computational logics to the area of game theory.