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Intergenerational Risk Sharing in Closing Pension Funds*

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Abstract

We model intergenerational risk sharing in closing funded pension plans. Specifically, we consider a setting in which in each period, the pension fund's investment and indexation policy is the outcome of a bargaining process between representatives of the then living generations. Because some generations might be under- or overrepresented in the board, we use the asymmetric Nash bargaining solution to allow for differences in bargaining powers. In a numerical study, we compare the welfare that the generations derive from the outcome of this repeated bargaining to the welfare that they would derive if a social planner's optimal policy would instead be implemented. We find that as compared to the social optimum, older generations benefit substantially from the repeated bargaining, even if all generations are equally well-represented in the board. If older generations are relatively over-represented, as is sometimes argued, these effects are attenuated.

JEL-codes: C78, D9, G23.
Keywords: cooperative bargaining, pension fund, intergenerational risk sharing.

1 Introduction

The combination of a severe financial crisis, unanticipated longevity shocks, and tighter regulation, has implied that many Defined Benefit (DB) pension funds have decided to change their pension plans to (collective) defined contribution plans. Existing participants can then no longer accrue new rights in the DB fund, new participants can no longer enter, and it needs to be decided how the assets that were accumulated in the past are divided over the remaining generations. In a DB fund, participants or generations typically do not "own" a share of the assets, and so there is no objective way to divide the accumulated assets over the different generations. Therefore, we model a setting in which the available assets stay in the DB fund,

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until full rundown. In each period it is decided whether and how accrued rights are indexed or cut, and how the remaining assets are invested.

There is a substantial amount of literature that deals with optimal payout and investment policies in collective pension funds or PAYG pension systems (see, e.g., Enders and Lapan, 1982; Gordon and Varian, 1988; Krueger and Kubler, 2002; Ball and Mankiw, 2007; Gollier, 2008; Cui et al., 2011). Our study differs from this literature in two ways. First, we consider a closing fund, which implies that risk can no longer be shared with the "unborn" generations. Second, whereas the existing literature typically takes a welfare perspective, we consider a case where a pension fund board consisting of representatives of different generations needs to decide on the payout and investment policies in each period. Because different generations have different horizons, they typically have different preferences regarding indexation and investment policies (see, e.g., Merton, 1971; Merton and Samuelson, 1974; Blake, 1998; Teulings and De Vries, 2006; Bovenberg et al., 2007; Hoevenaars and Ponds, 2008). Therefore, the investment and payout policies that are jointly determined by representatives of different generations will depend on how well each remaining generation is represented in the board. To take this into account, we use the *asymmetric* Nash bargaining solution to model the outcome of a bargaining process in which some generations may have more bargaining power than others. We ensure time consistency by using a "period-by-period" approach in which decisions are made for the current period only, while rationally anticipating future decisions.[1] We assume that for each generation, continued participation until decease (or full run-down of the assets) is mandatory, i.e., individuals do not have the option to leave the pension fund.[2] Instead, their rights are represented by representatives in the pension fund's board, and the task of the board of a pension fund is to design a pension policy that is perceived as "fair" by the participants of all generations.

In the first part of the paper, we present the model and derive the dynamic optimization problems that need to be solved in order to determine the investment and indexation policies that the pension board would take if, in every period, the representatives of the remaining generations bargain over the policy that will be implemented in that period. We then compare the investment and indexation policies that result from this repeated bargaining with the outcome that a social planner who cares about aggregate social welfare would prefer, as in Gollier (2008). We numerically analyze the welfare effects of the optimal policy for the different generations under both approaches. We find that, as compared to the social optimum, the elderly benefit substantially from bargaining. The result is robust to changes in the calibrated parameter values.

This paper is set out as follows. In Section 2, we present the characteristics of the DB fund, and discuss the setting that we consider. In Section 3, we model the pension fund's optimal investment and benefit policies as the outcome of a bargaining process by which the members of the pension board weigh the utility that each generation derives from a particular strategy.

[1] It is well-known that in multi-period settings, the Nash bargaining solution is not time consistent for a broad class of differential games (Haurie, 1976). The importance of time consistency is shown by Pelsser and Stadje (2014) and Pelsser and Salahnejhad (2016).

[2] Ligon et al. (2002), Westerhout (2011) and Beetsma et al. (2012) show that when participation in risk sharing pools is voluntary, this generally limits the possibility to share risk efficiently.

In Section 4, we numerically analyze the welfare effects of the optimal policy for the different generations. Finally, Section 5 concludes.

2 Model

In this section we first discuss the characteristics of the defined benefit (DB) fund that we consider. We then discuss our approach to determine optimal draw down strategies for the fund after it closes.

2.1 Characteristics of the DB fund

We let date $t = 0$ be the date at which the DB fund closes for new accruals and new participants. We assume that every participant works (or has worked) for three periods and is (or will be) in retirement in the one or two periods thereafter. Hence, on date zero there are five generations in the pension fund: three generations with active workers and two retired generations. Every generation is assigned a number from the set $\{0, 1, 2, 3, 4\}$. The generation with number τ then corresponds with the generation that is in its second retirement period at the beginning of period τ.

We introduce the following notation and assumptions:

- Participants in generation τ have entered the fund at the beginning of period $\tau - 4$ and only leave the fund due to decease.
- Participants die either at the end of their first retirement period or at the end of their second retirement period.
- We allow for heterogeneous generations that may be different in size and life expectancy. Within a generation, all individuals are homogeneous with respect to accrued pension rights and survival rates. We use the following notation for generation $\tau \in \{0, \dots, 4\}$:
  - $N_\tau > 0$: the number of participants in generation τ at the beginning of their first retirement period, i.e., on date $t = \tau - 1$.
  - $p_\tau \in (0, 1]$: the deterministic fraction of participants in generation τ that are still alive at the beginning of their second retirement period.
  - $\widetilde{L}_\tau > 0$: the accrued pension right for each participant belonging to generation τ in the pension fund. These accrued rights serve as *benchmark* for the pension payment that each participant in generation τ receives at the beginning of its first retirement period, and, if alive, at the beginning of its second retirement period. The actual payment can be different due to conditional indexation or cutting of pension rights.

The participants in generation τ have accrued pension rights $\widetilde{L}_\tau$ during their active life prior to date 0. The best-estimate value of the accrued pension rights is defined as the discounted expected pension payments, where the discount rate equals the deterministic risk-free rate,

r^f. At the beginning of period 0, generation $\tau = 0$ is in its last period of the retirement. All other generations $\tau > 0$ are either still active or in their first period of retirement. Hence, the date-0 best-estimate value of the pension rights of an individual in generation $\tau \in \{0, \ldots, 4\}$ who is alive on date-0 is given by:

$$\begin{aligned} \widehat{L}_\tau &= \frac{\widetilde{L}_\tau}{(1+r^f)^{\tau-1}} + p_\tau \cdot \frac{\widetilde{L}_\tau}{(1+r^f)^\tau} \quad && \text{if } \tau > 0, \\ &= \widetilde{L}_\tau, && \text{if } \tau = 0. \end{aligned}$$

We let L_0 denote the best-estimate value of the aggregate liabilities of all five generations on date 0. On date $t = 0$, there are $p_0 N_0$ participants in generation $\tau = 0$, and N_τ participants in generation τ for $\tau > 0$. Therefore, it holds that:

$$L_0 := p_0 \cdot N_0 \cdot \widehat{L}_0 + \sum_{\tau=1}^{4} N_\tau \cdot \widehat{L}_\tau. \tag{1}$$

The asset value of the pension fund at time $t = 0$, denoted by $A_0 > 0$, is the aggregated value of the risk-free and risky assets of the pension fund. Table 1 presents a schematic overview of the balance sheet of the pension fund at time $t = 0$.

Assets	Liabilities
risk-free assets	benchmark liabilities L_0
risky assets	buffer
A_0	L_0+buffer

Table 1: Balance sheet of the pension fund.

2.2 Closing the DB fund

Because the DB fund closes on date $t = 0$, the participants no longer accrue new rights in the DB fund. However, the fund continues to exist, and the generations collectively need to agree on the investment and benefit policies until full run down of the assets. Shocks in investment returns can be shared by all generations alive, and so the generations can benefit from intergenerational risk sharing (Gollier, 2008; Goecke, 2013; Bonenkamp and Westerhout, 2014). Because the pension fund closes for new entrants, however, the degree of risk sharing is limited. Specifically, risk can no longer be shared with “unborn” generations. Moreover, different generations typically have different preferences regarding the indexation policy and the investment policy of the fund (see, e.g., Teulings and De Vries, 2006; Bovenberg et al., 2007). We model a case in which these decisions are made by the board of the pension fund, which consists of representatives of each of the remaining generations. The board of a pension fund has the task to offset the interests of all generations to determine a fair policy. We assume that the benefits of strategies to the generations are determined by means of the expected utility that each generation derives from it. We make the following assumptions.

- Assets can be invested in a risk-free asset with return r^f, and in a risky asset with excess return r_t in period t. The excess returns are independently distributed over time, with $r_t > -(1 + r^f)$ for all t.

- Each participant uses a Von Neumann-Morgenstern expected utility function, u, which is the same for all participants. Moreover, $u : \mathbb{R}_+ \to \mathbb{R} \cup \{-\infty\}$ is strictly increasing and concave.

3 The dynamic bargaining problem

In this section we model the bargaining process by which the remaining generations in the DB fund decide on the investment and indexation policy, until the last generation has deceased. We first define feasible strategies. Then, we determine the utility for each generation associated with a given strategy. Finally, we use the Nash bargaining solution to model the outcome of a bargaining process by the members of the pension fund board, which act as representatives of the different generations.

3.1 Feasible strategies

At the beginning of periods $t \in \{0, 1, 2, 3\}$, the pension fund board needs to decide on the pension benefits paid to participants who are in their first or their second period of retirement. Moreover, it needs to decide on the investment strategy (i.e., the fraction of assets invested in the risky asset) for period t. If it invests part of its assets in the risky asset, then the benchmark accrued pension rights cannot be guaranteed. The accrued pension rights $\widetilde{L}_\tau, \tau \in \{0, \ldots, 4\}$ can be increased by indexation, but we also do not exclude the possibility that accrued rights are cut in case of low investment returns. Without loss of generality, we let the decision variables at the beginning of period t depend only on the asset value at that time. This yields the following definition.

Definition 1 *A pension benefit strategy at the beginning of period $t \in \{0, \ldots, 3\}$ is a function $f_t : \mathbb{R} \to \mathbb{R}^2 \times [0, 1]$ that maps the asset value at time t into period t decisions, i.e.,*

$$f_t(A_t) = (bonus_t^1, bonus_t^2, \alpha_t) \in \mathbb{R}^2 \times [0, 1],$$

where

- *$A_t \in \mathbb{R}$ is the asset value at the beginning of period t,*
- *$\alpha_t \in [0, 1]$ is the fraction of the assets invested in the risky asset in period t,*
- *$bonus_t^1 \in \mathbb{R}$ is the pension benefit payment, in addition to the accrued pension rights, to each participant who is in his first period of retirement in period t,*
- *$bonus_t^2 \in \mathbb{R}$ is the pension benefit payment, in addition to the accrued pension rights, to each participant who is in his second period of retirement in period t.*

It remains to specify the strategy of the fund at the beginning of the last period. At the beginning of period $t = 4$, only one generation is still alive (generation $\tau = 4$), and that generation starts it second (and last) retirement period. Hence, the only decision variable at time $t = 4$ is $bonus_4^2$, the bonus paid out to generation $\tau = 4$ at the beginning of its last retirement period. Because there are no future generations, we assume that all available assets at the beginning of period $t = 4$ are distributed to generation $\tau = 4$, and the fund closes. Because $p_4 \cdot N_4$ participant are still alive at the beginning of period $t = 4$, and because the pension payment to each participant equals $\widetilde{L}_4 + bonus_4^2$, the bonus paid out to each participant of generation $\tau = 4$ at the start of its last retirement period equals:

$$bonus_4^2 = f_4^*(A_4) = \frac{A_4}{p_4 \cdot N_4} - \widetilde{L}_4. \tag{2}$$

The variables $bonus_t^1$ and $bonus_t^2$ represent payments in excess of the benchmark rights for the two generations that are in retirement in period t, i.e., generations $\tau = t + 1, t$. These payments can be interpreted as indexation or cuts of accrued rights. The timing of these additional pension payments is illustrated in Figure 1.

$\tau =$ \ $t =$	0	1	2	3	4
0:	$bonus_0^2$	0	0	0	0
1:	$bonus_0^1$	$bonus_1^2$	0	0	0
2:	0	$bonus_1^1$	$bonus_2^2$	0	0
3:	0	0	$bonus_2^1$	$bonus_3^2$	0
4:	0	0	0	$bonus_3^1$	$bonus_4^2$

Figure 1: Pension benefits in excess of the benchmark right $\widetilde{L}_\tau$.

If the fund is overfunded, i.e., if $A_0 \geq L_0$, accrued rights could in principle be guaranteed by investing the amount L_0 in the risk-free asset. The buffer could then be invested so as to generate future indexation. This would ensure that $bonus_t^1$ and $bonus_t^2$ are nonnegative. However, doing so would imply that the generations can only benefit from excess returns from risky investments to a limited extent, because a large fraction of the assets needs to be invested in the risk-free asset in order to guarantee the benchmark payments. Therefore, generations may be willing to invest in the risky asset, even though this implies that their promised pension right is not ensured anymore. Therefore, we allow that $bonus_t^1$ and $bonus_t^2$ are negative.

We now determine feasibility of the strategy for period $t \in \{0, 1, 2, 3\}$, i.e., we determine feasible values for $f_t(A)$, where A denotes the asset value on date t. A strategy $f_t(A) = (bonus_t^1, bonus_t^2, \alpha_t)$ is feasible in period $t \in \{0, \ldots, 3\}$ iff: (i) individual pension payments are nonnegative, and, (ii), the aggregate pension payments do not exceed the available assets. At the beginning of period t, N_{t+1} participants from generation $\tau = t + 1$ are alive and start their first retirement period. They each receive a pension payment equal to $\widetilde{L}_{t+1} + bonus_t^1$. Moreover, $p_t \cdot N_t$ participants in generation $\tau = t$ are alive and start their second retirement period. They each receive a pension payment equal to $\widetilde{L}_t + bonus_t^2$. Therefore, the set of feasible values of $f_t(A)$, which we denote $\mathcal{F}_t(A)$, is given by

$$\mathcal{F}_t(A) = \Big\{ (bonus_t^1, bonus_t^2, \alpha_t) \mid \widetilde{L}_{t+1} + bonus_t^1 \geq 0, \widetilde{L}_t + bonus_t^2 \geq 0, \\ N_{t+1} \cdot [\widetilde{L}_{t+1} + bonus_t^1] + p_t \cdot N_t \cdot [\widetilde{L}_t + bonus_t^2] \leq A \Big\}. \quad (3)$$

Because $bonus_t^1$ and $bonus_t^2$ can be negative, the set $\mathcal{F}_t(A)$ is non-empty for any $t \in \{0, \ldots, 4\}$ and $A \geq 0$.

3.2 Utility

The pension benefit to an individual participant belonging to generation $\tau \in \{0, \ldots, 4\}$ consists of three components:

- The benchmark accrued pension rights in the DB fund, i.e., $\widetilde{L}_\tau$.
- The (possibly negative) pension income from the DB fund in addition to $\widetilde{L}_\tau$ in the first and the second retirement period, respectively, i.e., the payments $bonus_{\tau-1}^1$ and $bonus_\tau^2$.
- The (non-negative) retirement income from other sources, which we denote by I_τ^1 and I_τ^2 for the first and the second retirement period, respectively.

We first consider the date-t expected utility of the generation that is in its last period of retirement in period $t \in \{0, 1, 2, 3\}$ (i.e., generation $\tau = t$), as a function of the date t decision variables. We denote this utility by $U_{t,t}(bonus_t^1, bonus_t^2, \alpha_t)$. Here, the first subindex t refers to time period t while the second subindex t refers to generation $\tau = t$. Because generation $\tau = t$ is in its last period of retirement in period t, it will consume its retirement income in period t, which equals $\widetilde{L}_t + bonus_t^2 + I_t^2$, and will be deceased afterwards. Therefore, the date-t expected utility of participants belonging to generation $\tau = t$ equals

$$U_{t,t}(bonus_t^1, bonus_t^2, \alpha_t) \quad = \quad u\left(\widetilde{L}_t + bonus_t^2 + I_t^2\right), \text{ for } t \in \{0, \ldots, 3\}. \quad (4)$$

For generation $\tau = t$, the date-t utility does not depend on how period-t choices affect bonus payments in future periods. In contrast, generations $\tau \in \{t+1, \ldots, 4\}$ have both retirement periods forthcoming. Therefore, for these generations, the utility that they derive from the date-t choices $(bonus_t^1, bonus_t^2, \alpha_t)$ depends (in part or fully) on how these date-t choices affect the bonus payments that occur in the two periods in which generation τ will be retired, i.e., the bonus payments $bonus_{\tau-1}^1$ and $bonus_\tau^2$ in future periods $\tau - 1$ and τ. The probability distribution of these future bonus payments depends also on the future decisions of the fund. In principle, the five generations that are alive on date 0 could negotiate a complete investment and benefit policy for the full rundown of the assets. However, this solution might not be time consistent. Once some generations have died, the remaining generations may wish to renegotiate the policy for the remaining periods.[3] Therefore, we consider the case where at the beginning of period t, the generations that are still alive negotiate the investment and benefit policies for that period only, i.e., they choose values for $(bonus_t^1, bonus_t^2, \alpha_t)$, taking into account that the investment and benefit policies for future periods will be renegotiated by the generations that are then alive. We denote $U_{t,\tau}(bonus_t^1, bonus_t^2, \alpha_t | A, F_{t+1}^*)$ for the date-t utility that generation $\tau \in \{t+1, \ldots, 4\}$ derives from date-t choices $(bonus_t^1, bonus_t^2, \alpha_t)$, given that the asset value on date t equals $A_t = A$, and given that generation τ rationally infers the future strategies $F_{t+1}^* = \{f_{t+1}^*, \ldots, f_4^*\}$. Then:

$$\begin{aligned} U_{t,\tau}(bonus_t^1, bonus_t^2, \alpha_t | A, F_{t+1}^*) = E\Big[u\left(\widetilde{L}_\tau + bonus_{\tau-1}^1 + I_\tau^1\right) \\ +\beta \cdot p_\tau \cdot u\left(\widetilde{L}_\tau + bonus_\tau^2 + I_\tau^2\right)\Big| A_t = A, F_{t+1}^*\Big], \end{aligned} \qquad (5)$$

where β denotes the time preference parameter.

At time t, the first and second retirement period bonus payments to generation $\tau > t$, i.e., the variables $(bonus_{\tau-1}^1, bonus_\tau^2)$ in the right-hand-side of (5), are random variables because they depend jointly on asset returns and strategies in the periods until they retire. Specifically, given the date-t decisions $(bonus_t^1, bonus_t^2, \alpha_t)$, the probability distribution of $(bonus_{\tau-1}^1, bonus_\tau^2)$ for generations $\tau > t$ follows recursively from the strategies

$$\begin{aligned} &(bonus_s^1, bonus_s^2, \alpha_s) = f_s^*(A_s), \qquad \text{for } s \in \{t+1, \ldots, 3\}, \\ &bonus_4^2 = f_4^*(A_4), \end{aligned}$$

combined with how these strategies affect the level of assets (A_s) available at the beginning of every future period $s \in \{t+1, \ldots, \tau\}$. At the beginning future period s, the level of assets increases with the return over the previous period, and decreases with payments made to generations that are either in their first or their second retirement period. We assume that these pension benefits are paid out at the beginning of the period. Then, the asset dynamics are as follows:

$$\begin{aligned} &A_t = A, \\ &A_{s+1} = \left(A_s - N_{s+1} \cdot [\widetilde{L}_{s+1} + bonus_s^1] - p_s \cdot N_s \cdot [\widetilde{L}_s + bonus_s^2]\right) \\ &\quad \cdot (1 + r^f + \alpha_s \cdot r_s), \qquad \text{for } s \in \{t+1, \ldots, 4\}. \end{aligned} \qquad (6)$$

Now, for any given date-t asset value $A_t = A$, date-t choices $(bonus_t^1, bonus_t^2, \alpha_t)$, and, future strategies $F_{t+1}^* = \{f_{t+1}^*, \ldots, f_4^*\}$, the probability distribution of the bonus payments to generation τ in their first and second retirement period can be determined from (6), and the utility associated with the date-t choices can then be determined from (5).

[3] We will use the (asymmetric) Nash bargaining solution to model the pension payments in excess of the benchmark accrued rights as the outcome of a bargaining process between the generations. It is well-known that the Nash bargaining solution is not time consistent for a broad class of differential games (Haurie, 1976). The importance of time consistency is shown by Pelsser and Stadje (2014) and Pelsser and Salahnejhad (2016). We ensure time consistency by using a "period-by-period" approach in which decisions are made for the current period only, while rationally anticipating future decisions.

3.3 The optimal strategies

Recall that the optimal strategy for period $t = 4$ is given by (2). Now suppose the optimal strategies for periods $\{t+1,\dots,4\}$ are given, i.e., $F^*_{t+1} = (f^*_{t+1},\dots,f^*_4)$ is given. We then determine the optimal strategy for period t as a function of the total available assets at the beginning of period t, $A_t = A$. Because the generations continue to share risk after the fund is closed, they need to agree on a policy. It is well-known that different generations have different preferences regarding investment strategies. The extent to which a generation will perceive a policy as sufficiently "fair" will depend on the extent to which they benefit from the policy in utility terms, as compared to other generations. We use the asymmetric Nash bargaining solution to model the pension fund board's policy choices as the outcome of a bargaining process between the generations that are represented in the board.[4] The (asymmetric) Nash bargaining solution trades off the utility gains of the generations. The utility gain of a generation is measured as the difference between the utility derived from the policy, and the so-called disagreement utility. Specifically, we denote

- $d_{t,\tau}(A)$: the disagreement utility of generation τ. It represents the utility level below which a generation would feel that the policy is so unfair that it no longer wishes to continue to share risk with the other generations, i.e., the negotiations would fail. We assume that the minimum required utility level is the expected utility that the generation obtains when it receives its benchmark rights, corrected for the funding ratio in case of underfunding, i.e.,

$$d_{t,\tau}(A) = u\left(\delta_t(A)\cdot\widetilde{L}_\tau + I^1_\tau\right) + \beta\cdot p_\tau\cdot u\left(\delta_t(A)\cdot\widetilde{L}_\tau + I^2_\tau\right),\ \text{for } \tau > t \tag{7}$$

$$d_{t,t}(A) = u(\delta_t(A)\cdot\widetilde{L}_t + I^2_t), \tag{8}$$

where

$$\delta_t(A) = \min\left\{\frac{A}{L_t}, 1\right\}, \tag{9}$$

and where L_t denotes the date-t best-estimate value of the aggregate liabilities of the pension fund. At the beginning of period t, there are $p_t\cdot N_t$ participants in generation $\tau = t$, and N_τ participants in generation τ, for $\tau\in\{t+1,\dots,4\}$. Therefore, it holds that:

$$L_t = p_t\cdot N_t\cdot\widetilde{L}_t + \sum_{\tau=t+1}^{4} N_\tau\cdot\left[\frac{\widetilde{L}_\tau}{(1+r^f)^{\tau-t-1}} + p_\tau\cdot\frac{\widetilde{L}_\tau}{(1+r^f)^{\tau-t}}\right]. \tag{10}$$

- $\Delta U_{t,\tau}(bonus^1_t, bonus^2_t, \alpha_t|A, F^*_{t+1})$: the utility gains as compared to the disagreement utility i.e.,

$$\Delta U_{t,\tau}(bonus^1_t, bonus^2_t, \alpha_t|A, F^*_{t+1}) = U_{t,\tau}(bonus^1_t, bonus^2_t, \alpha_t|A, F^*_{t+1}) - d_{t,\tau}(A). \tag{11}$$

[4] The (asymmetric) Nash bargaining solution is originally characterized by Nash (1950) and Kalai (1977) by means of properties. This characterization is based on a static setting, in which the goal is to allocate a vector of goods. Rubinstein (1982) shows that in a bilateral non-cooperative game with perfectly patient players, the equilibrium allocation converges to the symmetric Nash bargaining solution. Moreover, Van Damme (1986) shows that the Nash bargaining solution constitutes the unique equilibrium if two firms have different opinions about what is the appropriate solution concept to use for determining allocations.

To ensure that all generations are willing to cooperate, the strategy chosen by the board in period t, i.e., $f_t(A) = (bonus_t^1, bonus_t^2, \alpha_t)$, has to be *individually rational* in the sense that it yields weakly higher utility than the disagreement utility for each generation that is alive in period t, i.e.,

$$\Delta U_{t,\tau}\left(f_t(A)|A, F_{t+1}^*\right) \geq 0, \tag{12}$$

for all $A > 0$ and $\tau \in \{t, \ldots, 4\}$. We will say that $F_{t+1}^* = \{f_{t+1}^*, \ldots, f_4^*\}$ is individually rational if and only if for all $s \in \{t+1, \ldots, 4\}$, f_s^* is individually rational given F_{s+1}^*. The following theorem shows that in each period, there exists a feasible and individually rational strategy.

Theorem 1 *It holds that:*

(i) Strategy $f_4^* : A \to f_4^*(A)$ from (2) is feasible and individually rational.

(ii) Let $t \in \{0, \ldots, 3\}$. Then, for any F_{t+1}^* that is feasible and individually rational, there exists a strategy $f_t : A \to f_t(A)$ that is feasible and individually rational given $F_{t+1}^* = \{f_{t+1}^*, \ldots, f_4^*\}$.

The proof of this theorem is delegated to Appendix A.

Now let F_{t+1}^* be feasible and individually rational. Then, among the set of period t strategies that are feasible and individually rational given F_{t+1}^*, the (asymmetric) γ-Nash bargaining solution selects a strategy by trading off the utility gains of the different generations, taking into account their bargaining powers. Specifically, the (asymmetric) γ-Nash bargaining solution for period t satisfies:[5]

$$\begin{aligned} f_t^*(A) = \quad & \text{argmax} \quad \textstyle\prod_{\tau=t}^4 \left[\Delta U_{t,\tau}(bonus_t^1, bonus_t^2, \alpha_t|A, F_{t+1}^*)\right]^{\gamma_\tau} \\ & s.t. \quad (bonus_t^1, bonus_t^2, \alpha_t) \in \mathcal{F}_t(A), \\ & \qquad \Delta U_{t,\tau}(bonus_t^1, bonus_t^2, \alpha_t|A, F_{t+1}^*) \geq 0, \text{ for all } \tau \geq t, \end{aligned} \tag{13}$$

where $\gamma_\tau > 0$ denotes the relative bargaining power of generation τ, for $\tau \in \{t, \ldots, 4\}$.

If $\gamma_\tau = 1$ for all $\tau \in \{0, \ldots, 4\}$, the γ-Nash bargaining solution corresponds with the symmetric Nash bargaining solution of Nash (1950). However, it is possible that different generations have different bargaining powers, e.g., due to a different size of their representation in the pension fund board. If larger generations are better represented, one can let γ be an increasing function of N_τ. For instance, if the bargaining power of a generation is proportional to its size, it holds that $\gamma_\tau \propto N_\tau$ for $\tau \in \{0, \ldots, 4\}$.

The complete strategy of the pension fund board is given by

$$(bonus_s^1, bonus_s^2, \alpha_s) = f_s^*(A_s), \qquad \text{for } s \in \{0, \ldots, 3\}, \tag{14}$$

$$bonus_4^2 = f_4^*(A_4), \tag{15}$$

where the dynamics of the asset value follows from (6), with $A_0 = A$.

[5] We note that $\Delta U_{t,t}(bonus_t^1, bonus_t^2, \alpha_t|A, F_{t+1}^*) = \Delta U_{t,t}(bonus_t^1, bonus_t^2, \alpha_t)$ because the date-t utility of generation t does not depend on A and F_{t+1}^*.

Before proceeding to the numerical analysis, we note that in the setting that we consider the extent to which generations can benefit from risk sharing is affected by limited commitment. Existing literature (e.g., Ligon et al., 2002; Westerhout, 2011; Beetsma et al., 2012) has shown that voluntary participation limits the extent to which parties can benefit from risk sharing. Specifically, Beetsma et al. (2012) show that in situations with low funding ratios, younger generations could be tempted to leave the funds, while in situations with high buffers older generations may be better off by leaving the fund. Although in our setting generations cannot leave the fund, there is limited commitment in the sense that if in a policy the utility for a generation is lower than the disagreement utility, the negotiations break down. Hence, although generations cannot leave the fund, the degree of risk sharing is limited because some policies are "unacceptable" to some generations.

4 Numerical analysis

In this section, we numerically analyze the welfare effects of the γ-Nash bargaining solution. We use backward induction with a linear interpolation function to determine the optimal strategies, using (2), (6), and (13)-(15).[6] As is common in the literature on optimal investing over a life-cycle (see, e.g., Hansen and Singleton, 1983; Hubbard et al., 1995; Cocco et al., 2005; Gollier, 2008), we assume a Constant Relative Risk Aversion (CRRA) utility function given by:

$$u(x) = \begin{cases} \frac{x^{1-\lambda}}{1-\lambda}, & \text{if } \lambda \neq 1, \\ \log(x), & \text{if } \lambda = 1, \end{cases} \tag{16}$$

for all $x > 0$, and $u(0) = -\infty$ if $\lambda \geq 1$, and $u(0) = 0$ otherwise. Here, $\lambda > 0$ is the parameter of relative risk aversion.[7]

4.1 The approach

We will illustrate the effect of the bargaining powers, the survival probabilities, and the degree of risk aversion, on the welfare that the different generations derive from the policy that results from repeated bargaining (i.e., the policy given by (14)-(15)). We also compare this policy with the policy that is optimal from the viewpoint of a social planner who cares about the aggregate expected discounted utility of all generations. Following Gollier (2008), we consider a social planner who determines the optimal pension fund policy by maximizing

[6] Due to the non-linear objective function, we cannot formulate and apply a Bellman equation. Therefore, period t decisions are conditional on the pension fund's decisions in all future periods.

[7] We note that the utility function takes negative values when $\lambda > 1$, which implies that the utility in case of survival is lower (strictly negative) than the utility in case of decease (zero). Bommier et al. (2016) show that with utility specifications of the form $\phi^{-1}(p \cdot \phi(u(c)) + (1-p) \cdot \phi(u_d))$ with $\phi(\cdot)$ strictly concave, this property is undesirable because it affects life-cycle consumption and investment decisions. However, in our case $\phi(x) = x$, and so the value of the utility in case of decease does not affect the optimal policy. In our setting, replacing the utility function by $\tilde{u}(c) = u(c) - u(c_{\min})$ for some $c_{\min} > 0$ and setting $u_d = 0$ suffices to ensure that the utility in case of decease is lower than in case of survival and leads to the same optimal policies if $c_{\min}$ is sufficiently small. In our case, this holds true for any $c_{\min} \leq 1$.

the discounted aggregate expected utility of all participants. Specifically, the pension benefit strategy corresponding to the social optimum is determined as follows:

$$\begin{aligned}
\max \quad & \textstyle\sum_{t=1}^{3} \omega^t \cdot N_t \cdot E\left[u\left(\widetilde{L}_t + bonus_{t-1}^1 + I_{t-1}^1\right) + p_t \cdot \beta \cdot u\left(\widetilde{L}_t + bonus_t^2 + I_t^2\right)\right] \\
& \qquad +p_0 \cdot N_0 \cdot E\left[u\left(\widetilde{L}_0 + bonus_0^2 + I_0^2\right)\right] \\
\text{s.t.} \quad & A_0 = A \text{ and } (6), \\
& (bonus_t^1, bonus_t^2, \alpha_t) \in \mathcal{F}_t(A_t), \ \text{for all } t \in \{0,\ldots,3\}, \\
& bonus_4^2 = A_4/(p_4 N_4) - \widetilde{L}_4,
\end{aligned} \tag{17}$$

where $A_0 > 0$ is the initial asset value, and $\omega \in (0,1]$ a subjective discount factor. As the objective function is formulated from the point of view of a social planner, we refer to the solution as the *first-best* pension benefit policy. The approach in (17) differs from that in Gollier (2008) in that we distinguish two retirement periods and allow for heterogeneity in the size and survival rates of the different generations.

Both for the Nash bargaining solution, and for the first-best solution, we quantify the effects of the policy by determining the mean and standard deviation of the cumulative indexation in the first and in the second retirement period, respectively. The cumulative indexation is defined as the percentage deviation of the first (second) retirement period DB income from the benchmark payment, $\widetilde{L}_\tau$, i.e.,

$$\beta_\tau^1 = (\widetilde{L}_\tau + bonus_{\tau-1}^1)/\widetilde{L}_\tau - 1, \ \text{for } \tau \in \{1,\ldots,4\}, \tag{18}$$

$$\beta_\tau^2 = (\widetilde{L}_\tau + bonus_\tau^2)/\widetilde{L}_\tau - 1, \ \text{for } \tau \in \{0,1,\ldots,4\}. \tag{19}$$

Moreover, for each generation, we also determine the cumulative indexation corresponding to the certainty equivalent pension income. The certainty equivalent pension income is defined as the fixed income, $\bar{L}_\tau$, that solves:

$$\begin{aligned}
& E[u(\widetilde{L}_\tau + bonus_{\tau-1}^1 + I_\tau^2) + \beta \cdot p_\tau \cdot u(\widetilde{L}_\tau + bonus_\tau^2 + I_\tau^2)] \\
& \quad = u(\bar{L}_\tau + I_\tau^1) + \beta \cdot p_\tau \cdot u(\bar{L}_\tau + I_\tau^2),
\end{aligned} \tag{20}$$

for $\tau \in \{1,\ldots,4\}$, and

$$u(\widetilde{L}_0 + bonus_0^2 + I_0^2) = u(\bar{L}_0 + I_0^2).$$

The cumulative indexation corresponding to the certainty equivalent pension income, which we will refer to as *the certain equivalent cumulative indexation*, is given by:

$$\bar{\beta}_\tau = \bar{L}_\tau/\widetilde{L}_\tau - 1, \ \text{for } \tau \in \{0,\ldots,4\}. \tag{21}$$

4.2 Results for a benchmark case

In our benchmark case, we make the following assumptions:

- *Retirement age and length of a period*: We assume that a period corresponds to ten years, and all participants retire at age 65. Hence, the youngest generation has age 35 on date $t = 0$.

- *Risk free rate, asset return and subjective discount factor*:
  - The risk-free rate is 2% per year, and so $r^f = (1.02)^{10} - 1 \approx 0.22$.
  - Both the subjective discount factor of the social planner and the subjective discount factor of the pension fund participants equal 2% per year, and so $\omega = \beta = (1.02)^{-10} \approx 0.82$.
  - The excess return of the risky asset equals $r_t = r_H = 50\%$ per period with probability 0.5 and $r_t = r_L = -25\%$ per period with probability 0.5, independent and identically distributed over time t.[8]

- *Characteristics of the generations*:
  - The relative risk aversion parameter equals $\lambda = 5$.
  - The size of generation $\tau \in \{0, \dots, 4\}$ at the beginning of their first retirement period is $N_\tau = (1.0087^{10})^\tau \approx 1.09^\tau$ (see Beetsma and Bucciol, 2015).
  - The relative bargaining power of generation τ is proportional to N_τ. Without loss of generality, we set $\gamma_\tau = N_\tau$.
  - The probability of being alive at the beginning of the second retirement period, p_τ, equals the best-estimate of the 10-year survival probability of a 65-year old belonging to generation τ, estimated using the Lee and Carter (1992) model. The resulting ages of the participants and survival probabilities are as follows:[9]

τ	0	1	2	3	4
Age	75	65	55	45	35
p_τ	0.81	0.84	0.86	0.89	0.92

(22)

- *Accrued rights and funding ratio*: We assume that generations have accrued 1 unit of pension rights during each active period. Moreover, generations that are not yet retired at the time the fund closes will accrue one unit of pension income in every future active period, i.e., $\widetilde{L}_\tau = \min\{4-(\tau-1), 4\}$, and $I_\tau = 4 - \widetilde{L}_\tau$. With $r^f \approx 0.22$, this implies that the time-0 best estimate value of the liabilities equals $L_0 \approx 20.02$. In the benchmark case, we let the initial asset value be such that the date-0 funding ratio is 120%, i.e., $A_0 = 1.2 \cdot L_0 \approx 24.02$.

The parameter values are summarized in Table 2.

We compare the pension benefits resulting from the γ-Nash bargaining solution to those of the first-best solution. There are several reasons why the outcome of the bargaining process can deviate from what a social planner would prefer. First, the social planner cares about aggregate expected utility of the retirement income of all the generations. In contrast, when pension board representatives bargain over the policy to be implemented, the outcome is

[8] Because a period corresponds to 10 years, r_H corresponds to a yearly excess return of +4.14%, while r_L corresponds to a yearly excess return of −2.84%.

[9] We use data of Dutch males for the period from 1977 until 2009 from the Human Mortality Database.

Symbol	Description	Calibration
r^f	Real risk-free rate	0.22
FR_0	Funding ratio on date 0	120%
$\widetilde{L}_\tau$	Benchmark liabilities of generation τ	$\min\{4-(\tau-1),4\}$
I_τ^1	Other pension income of generation τ	$4-\widetilde{L}_\tau$
I_τ^2	Other pension income of generation τ	$4-\widetilde{L}_\tau$
N_τ	Size of generation τ	1.09^τ
β	Subjective individual discount factor	0.82
γ_τ	Relative bargaining power of generation τ	N_τ
ω	Social planner parameter	0.82
λ	Parameter of constant relative risk aversion	5

Table 2: Summary of the calibrated parameters in the benchmark case.

	$\tau=0$	$\tau=1$	$\tau=2$	$\tau=3$	$\tau=4$
$\mu(\beta_\tau^1)$	-	10.3%	16.7%	30.5%	72.2%
$\mu(\beta_\tau^2)$	20.3%	23.2%	34.4%	53.8%	80.2%
$\sigma(\beta_\tau^1)$	-	0	13.7%	22.7%	54.7%
$\sigma(\beta_\tau^2)$	0	14.3%	22.4%	38.5%	63.5%
$\bar{\beta}_\tau$	20.3%	14.1%	20.0%	31.8%	59.7%
	$\tau=0$	$\tau=1$	$\tau=2$	$\tau=3$	$\tau=4$
$\mu(\beta_\tau^1)$	-	5.7%	44.6%	70.0%	151.8%
$\mu(\beta_\tau^2)$	8.0%	7.5%	12.7%	25.2%	66.3%
$\sigma(\beta_\tau^1)$	-	0	13.9%	25.2%	58.1%
$\sigma(\beta_\tau^2)$	0	9.6%	16.1%	29.2%	67.1%
$\bar{\beta}_\tau$	8.0%	6.0%	24.9%	39.3%	87.2%

Table 3: The mean and standard deviation of first and second period cumulative indexation, as well as the certain equivalent cumulative indexation. The top (bottom) panel corresponds to the γ-Nash bargaining solution (first-best solution).

determined by how many generations are still alive at the time the decisions need to be made. As time passes by, there are fewer generations in the defined benefit fund, which affects the outcome of the bargaining process. Second, in the bargaining process generations can block a redistribution if it yields utility lower than their disagreement utility.

Table 3 displays the mean of first and second retirement period expected cumulative indexation, as well as the certain equivalent cumulative indexation, for the benchmark case. The top panel presents results for the γ-Nash bargaining solution. The bottom panel presents results for the first-best solution. The fraction of the assets that is invested in the risky asset is 37% in each period in the γ-Nash bargaining solution, and 34% in each period in the first-best solution. The table shows that for both solutions, the expected cumulative indexation in both retirement periods is positive and increasing in τ for $\tau > 0$. The latter effect occurs because younger generations (higher τ) have a longer investment horizon, and so they can benefit more from excess returns on risky investments. This in turn implies that also the risk (as measured by the standard deviation of cumulative indexation) is higher for younger generations.

To investigate the welfare effects of the Nash bargaining solution, we compare the pension benefits to the first-best pension benefits. Figure 2 shows that as compared to the first-best, active generations in expectation receive higher cumulative indexation in the first retirement period and lower expected cumulative indexation in the second retirement period. In Figure 3 we display the certain equivalent cumulative indexation (left panel) and the certain equivalent total retirement income (right panel) under the two methods. The figure shows that as compared to the first-best solution, active generations ($\tau \in \{2, 3, 4\}$) receive lower certain equivalent pension income in the γ-Nash bargaining solution, while retirees ($\tau \in \{0, 1\}$) receive higher certain equivalent pension income. Stated differently, from a welfare perspective retirees benefit from Nash bargaining while active participants are worse off.

4.3 Sensitivity analysis

In this section we analyze the effect of changes in the parameter values on the certain equivalent pension benefits resulting from the Nash bargaining solution. We also show that the result that older generations benefit from Nash bargaining (as compared to the first-best) is robust to changes in parameter values. We consider the following alternative parameter values:[10]

- initial funding ratio: 100% and 140%.
- return on assets: we consider four alternative distributions, each with an "up" state (r_H) and a "down" state (r_L) with equal probability. The excess returns in the two states are chosen such that we have return distributions with the same mean as the benchmark excess return but higher variance and lower variance, respectively; as well as excess returns with the same variance, but higher and lower mean, respectively. The distributions are displayed in Table 4.

[10] We have also performed a sensitivity analysis with regard to the survival probabilities of the generations. We considered an upward and a downward shock of 20% per period to the mortality probabilities, i.e., $\widehat{p}_\tau^{\text{down}} = 1-(1-p_\tau)\cdot 0.8^\tau$, and $\widehat{p}_\tau^{\text{up}} = 1-(1-p_\tau)\cdot 1.2^\tau$, for all $\tau \in \{0, \ldots, 4\}$, with p_τ as displayed in (22). The effects on the optimal policies were very small. Moreover, the main qualitative conclusions hold true for these alternative probabilities. Details are available from the authors upon request.

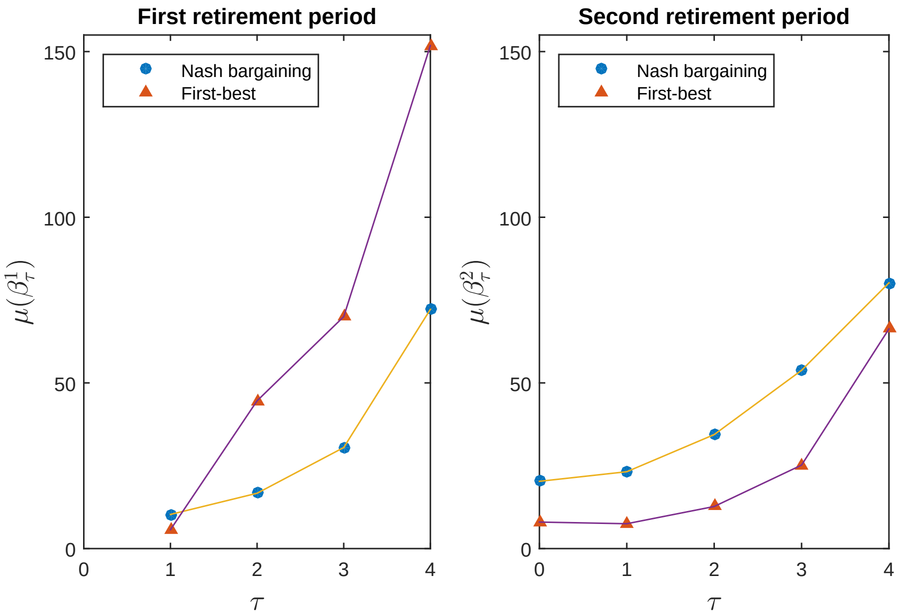

Figure 2: The mean of first period (left panel) and second period (right panel) cumulative indexation for the γ-Nash bargaining solution and for the first-best solution.

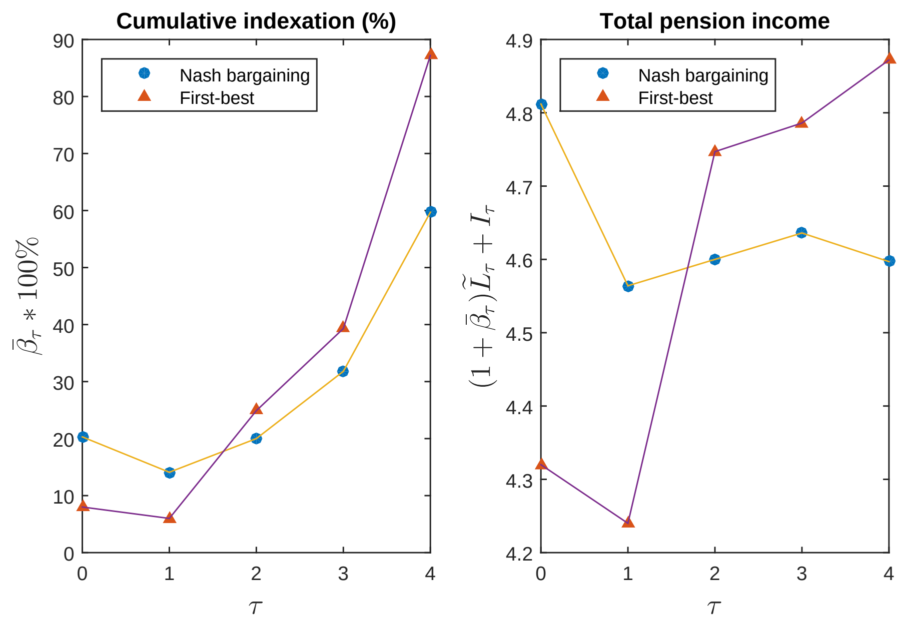

Figure 3: The certain equivalent percentage cumulative indexation ($\bar{\beta}_\tau * 100\%$, left panel) and the certain equivalent total pension income ($(1+\bar{\beta}_\tau)\widetilde{L}_\tau + I_\tau$, right panel) for the γ-Nash bargaining solution and for the first-best solution.

	r_L	r_H	$E[R_i]$	$\sigma(R_i)$
R_1	-50%	75%	12.5%	39%
R_2	-12.5%	37.5%	12.5%	6.3%
R_3	-12.5%	62.5%	25%	14%
R_4	-35%	40%	2.5%	14%

Table 4: Alternative excess return distributions. The first (second) column displays the periodic excess return (i.e., over a period of 10 years) in the down (up) state. The last two columns display the mean and the standard deviation of the return. The risk free rate is the same as in the benchmark case.

- relative bargaining power of the generations: we consider as alternatives the case of equal bargaining power for each generation, i.e., $\gamma_\tau = 1$ for all $\tau \in \{0, \ldots, 4\}$, higher representation of older participants, $\gamma_\tau = (0.6)^\tau$ for all $\tau \in \{0, \ldots, 4\}$, and higher representation of younger participants, $\gamma_\tau = (1.6)^\tau$ for all $\tau \in \{0, \ldots, 4\}$.
- risk aversion parameter λ: we consider $\lambda \in \{1, 3, 9\}$.

In Figure 4, we display the effects of these parameter changes on the certain equivalent cumulative indexation under the γ-Nash bargaining solution. In each case, one parameter changes, while all other parameters are set equal to the benchmark value.

The top panels display the effects of the funding ratio (left) and of the returns distribution (right). As expected, the certain equivalents of all generations increase when: (i) the initial funding ratio increases; or (ii) the mean of the asset return increases while the variance is unchanged; or (iii) the variance of the asset return decreases while the mean is unchanged. These effects are largest for the youngest generation. The bottom left panel shows the effect of changes in the relative bargaining powers of the generations. As compared to the case where $\gamma_\tau = 1$ for all τ, letting $\gamma_\tau = 0.6^\tau$ ($\gamma_\tau = 1.6^\tau$) implies that the relative bargaining power of generations $\tau \in \{0, 1\}$ increases (decreases) while the relative bargaining powers of generations $\tau \in \{3, 4\}$ decreases (increases). A higher (lower) relative bargaining power increases (decreases) expected pension benefits. Finally, the bottom right panel shows the effect of the degree of risk aversion. A higher degree of risk aversion leads to more conservative investment, i.e., a lower fraction of the assets is invested in the risky asset in each period.[11] As can be seen in the bottom right panel, this in turn implies that the certain equivalent income decreases for all generations. As was the case for the initial funding ratio and the asset returns, the effect is largest for the youngest generation.

We conclude with a sensitivity analysis for the difference in certain equivalent total pension income between the γ-Nash bargaining solution and the firs-best solution. Figure 3 showed that in the benchmark case, retirees benefit from the Nash bargaining solution as compared to the social optimum, while active participants are worse off. Figure 5 displays the percentage difference between the certain equivalent pension income under the Nash bargaining

[11] For instance in the first period, the fraction invested in the risky asset equals $\alpha = 100\%, 64\%, 37\%, 24\%$ for constant relative risk aversion parameter $\lambda = 1, 3, 5, 9$.

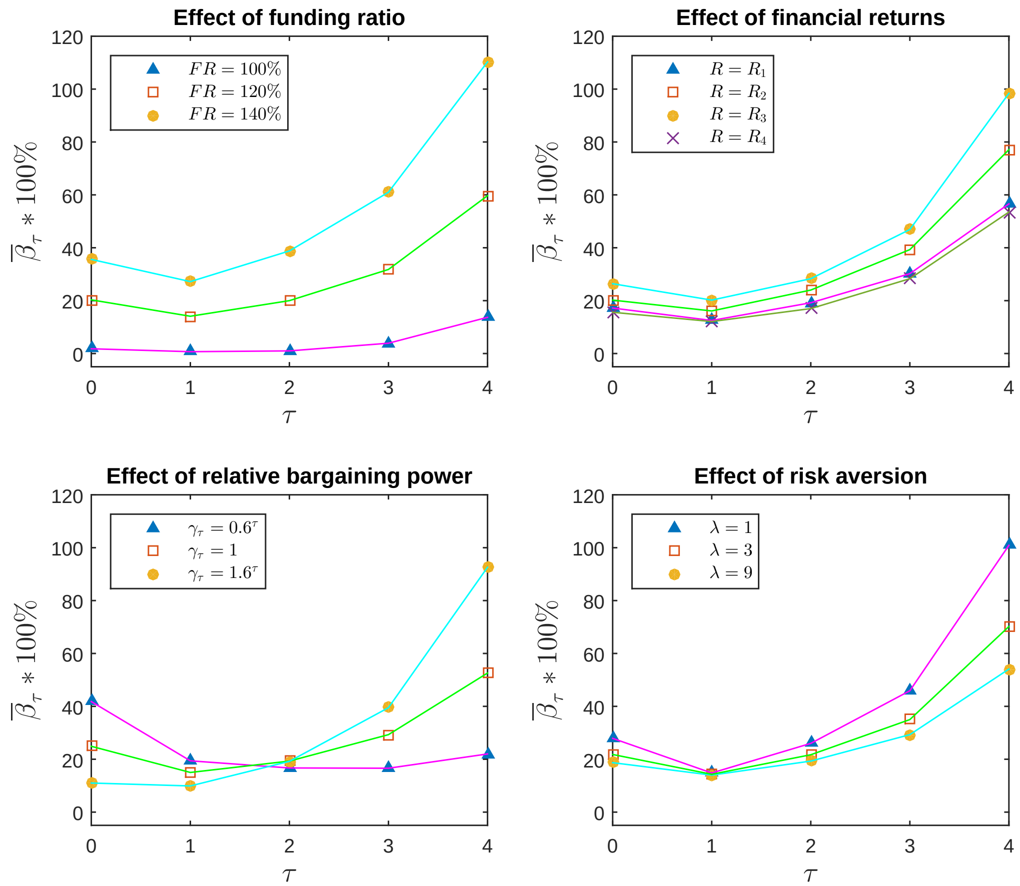

Figure 4: Sensitivity analysis for the certain equivalent percentage cumulative indexation ($\bar{\beta}_\tau * 100\%$) under the γ-Nash bargaining solution. In each case, one parameter changes, while all other parameters are set equal to the benchmark value. Top left panel: effect of the initial funding ratio; top right panel: effect of returns distribution; bottom left panel: effect of relative bargaining powers; bottom right panel: effect of degree of risk aversion.

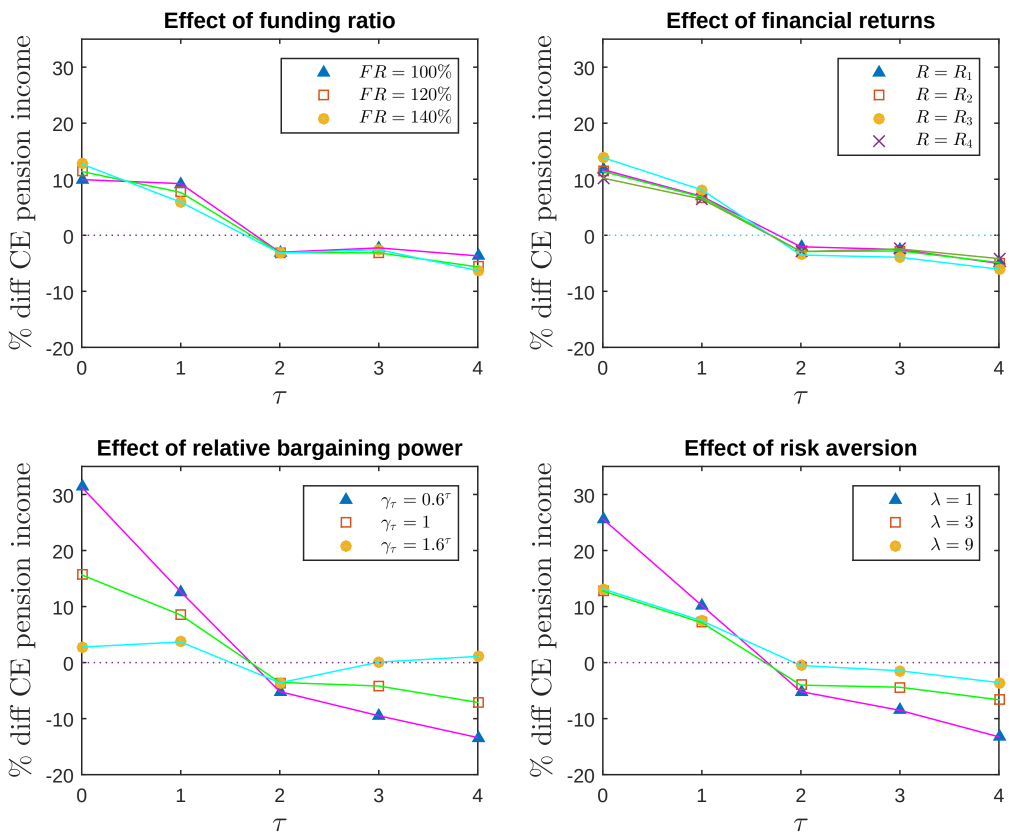

Figure 5: Sensitivity analysis for the percentage difference in certain equivalent total pension income between the γ-Nash bargaining solution and the first-best solution. In each case, one parameter changes, while all other parameters are set equal to the benchmark value. Top left panel: effect of the initial funding ratio; top right panel: effect of returns distribution; bottom left panel: effect of relative bargaining powers; bottom right panel: effect of degree of risk aversion.

solution and the certain equivalent pension income under the first-best solution. A positive (negative) value indicates that the generation benefits (loses) from bargaining as compared to the first-best. The figure shows that the result that retirees benefit from the Nash bargaining solution (as compared to the social optimum) while active participants are worse off, is robust to changes in the initial funding ratio (top left panel), asset returns (top right panel) and the degree of risk aversion (bottom right panel). The bottom left panel shows that if older generations are relatively over-represented ($\gamma = 0.6^\tau$), as is sometimes argued, these effects are attenuated. If the per capita bargaining power of the active participants is sufficiently high ($\gamma = 1.6^\tau$), the differences become small.

Finally, we note that our results also illustrate the potential impact of limited commitment on risk sharing. Consider, for example, the case where the funding ratio on date zero is 100 percent. In this case, retired generations will break up the negotiations whenever the proposed policy implies that their pension liabilities are cut. This occurs because the disagreement utility in this case requires that the pension payment is at least equal to the nominal accrued pension rights $\widetilde{L}_\tau$.[12] In principle, it is possible to give each generation their nominal accrued rights when the funding ratio is 100 percent. However, for future generations, receiving their nominal accrued rights means a cut in pension benefits in real terms. Hence, from the perspective of a social planner, it may be optimal to impose a (small) cut on the pension rights of current retirees so as to avoid large cuts (in real terms) for future retirees. In our results, it is indeed the case that the socially optimal policy when the initial funding ratio is 100 percent involves a cut in pension rights for current retirees.[13] This policy, although being Pareto optimal, is excluded in the bargaining setting. Therefore, the limited commitment that arises from the fact that parties can break up the negotiations if their utility is below the disagreement utility can rule out socially desirable policies.

5 Conclusion

In this paper, we model intergenerational risk sharing in a closing funded pension plan as a bargaining process between generations. Existing literature typically takes the viewpoint of a social planner who aims to maximize aggregate welfare over all generations. In practice, however, a pension fund's investment and payout policies are typically determined by a pension fund board consisting of representatives of different generations. We have analyzed decisions that the pension fund board would take if, in each period, representatives of the then living generations bargain over the policy that will be implemented. We use the asymmetric Nash bargaining solution to characterize the investment and payout policies that will result from repeated bargaining, allowing for the possibility that not all generations are equally well represented. In a numerical study, we compare the welfare that different generations derive from the outcome of this repeated bargaining to the welfare that they would derive if the social planner's optimal policy would instead be implemented. Our results suggest that when the distribution of the remaining assets in a closing DB fund is determined via repeated bargaining between the remaining generations, the oldest generations benefit substantially as compared to the distribution of assets that a social planner would prefer. This goes at the expense of younger generations. The result holds true even if younger generations have relatively high bargaining power, and is attenuated if older generations are relatively over-represented, as is sometimes argued. Moreover, we find that the restriction imposed by limited commitment (i.e., via the disagreement utility) implies that some policies that are socially desirable may be ruled out. To avoid these outcomes, regulators could consider to impose constraints on the extent to which generations can block a policy that is socially desirable.

[12] If the funding ratio on date zero is 100 percent, it follows from (9) that $\delta_0(A) = 1$, and so the disagreement utility for generation $\tau = 0$ on date $t = 0$ equals $d_{0,0}(A) = u(\widetilde{L}_0)$.

[13] Under the FB solution, it holds that $\bar{\beta}_\tau < 0$ for generation $\tau \in \{0, 1\}$ when the initial funding ratio is 100%. It follows from (21) that $\bar{\beta}_\tau < 0$ represents a cut in pension rights for generation τ.

We conclude by discussing an interesting direction for future research. Given the recent increased trend in closing down DB funds, the analysis on this paper focused on a fund in run-off. However, also for funds that are not in run-off, investment and benefit policies in practice typically result from negotiations. An interesting issue in this case is that while the "unborn" generations clearly are affected by the pension fund's policy, their stakes may be underrepresented when pension policies are negotiated between active and retired generations. Future research could focus on how this affects the welfare of different generations.

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A Proof of Theorem 1

First, consider $s = 4$. It follows from (9) and (10) that

$$\delta_4(A) \cdot \widetilde{L}_4 = \min\left\{\frac{A}{L_4}, 1\right\} \cdot \widetilde{L}_4 = \min\left\{\frac{A}{p_4 N_4}, \widetilde{L}_4\right\}. \tag{23}$$

Now let $f_4(\cdot) = f_4^*(\cdot)$, with $f_4^*(\cdot)$ as defined in (2). It follows from (4), (8), and (23) that

$$U_{4,4}\left(f_4^*(A)\right) = u\left(\frac{A}{p_4 N_4} + I_4^2\right) \geq u(\delta_t(A) \cdot \widetilde{L}_4 + I_4^2) = d_{4,4}(A).$$

Hence, strategy f_4^* is feasible and individually rational.

Now let $t \in \{0, \ldots, 3\}$, and suppose that F_{t+1}^* is feasible and individually rational. We will show that there exists a strategy $f_t(\cdot)$ that is feasible and individually rational. Let

$$f_t(A) = \left(bonus_t^1, bonus_t^2, \alpha_t\right) = \left((\delta_t(A) - 1) \cdot \widetilde{L}_{t+1}, (\delta_t(A) - 1) \cdot \widetilde{L}_t, 0\right). \tag{24}$$

We first show that the strategy is feasible, i.e., $f_t(A) \in \mathcal{F}_t(A)$, for all $A > 0$. With this strategy, the aggregate payment in period t equals

$$\begin{aligned} & N_{t+1} \cdot [\widetilde{L}_{t+1} + bonus_t^1] + p_t \cdot N_t \cdot [\widetilde{L}_t + bonus_t^2] \\ = \; & \delta_t(A) \cdot \left[N_{t+1} \cdot \widetilde{L}_{t+1} + p_t \cdot N_t \cdot \widetilde{L}_t\right] \\ \leq \; & A, \end{aligned}$$

where the inequality follows from the fact that $\delta_t(A) \leq A/L_t$ and $L_t \geq N_{t+1} \cdot \widetilde{L}_{t+1} + p_t \cdot N_t \cdot \widetilde{L}_t$. Moreover, since $\delta_t(A) > 0$ for all $A > 0$, we get $\widetilde{L}_{t+1} + bonus_t^1 = \delta_t(A) \cdot \widetilde{L}_{t+1} > 0$ and $\widetilde{L}_t + bonus_t^2 = \delta_t(A) \cdot \widetilde{L}_t > 0$. Hence, $f_t(A) \in \mathcal{F}_t(A)$.

We now show that the strategy is individually rational. It follows from (6) that the value of the assets at the beginning of period $t + 1$ are given by

$$A_{t+1} = (1 + r^f)\left(A - \delta_t(A) \cdot \widetilde{L}_t^{agg}\right), \tag{25}$$

where $\widetilde{L}_t^{agg}$ is the aggregate benchmark payment in period t, i.e.,

$$\widetilde{L}_t^{agg} = N_{t+1} \cdot \widetilde{L}_{t+1} + p_t \cdot N_t \cdot \widetilde{L}_t.$$

Moreover, it follows from (10) that the best-estimate value of the liabilities at the beginning of period $t + 1$, equals

$$L_{t+1} = (1 + r^f)\left(L_t - \widetilde{L}_t^{agg}\right),$$

Combined, this implies that

$$\frac{A_{t+1}}{L_{t+1}} = \frac{(1+r^f)\left(A - \delta_t(A)\cdot \widetilde{L}_t^{agg}\right)}{(1+r^f)\left(L_t - \widetilde{L}_t^{agg}\right)} \geq \frac{A - \frac{A}{L_t}\cdot \widetilde{L}_t^{agg}}{L_t - \widetilde{L}_t^{agg}} = \frac{A}{L_t},$$

i.e., the funding ratio at the beginning of period $t+1$ is higher than the funding ratio at the beginning of period t. With (9), this implies

$$\delta_{t+1}(A_{t+1}) \geq \delta_t(A). \tag{26}$$

and, moreover, $A_{t+1} > 0$. We now show that (12) is satisfied for $f_t(A)$ defined in (24).

- Generation $\tau = t$: it follows from (24) that the aggregate payment from the DB fund in period t (benchmark + bonus) equals

  $$\widetilde{L}_\tau + bonus_t^2 = \widetilde{L}_\tau + (\delta_t(A) - 1)\cdot \widetilde{L}_t = \delta_t(A)\cdot \widetilde{L}_t.$$

  Because generation $\tau = t$ receives its last payment in period t, it follows from (4) and (8) that

  $$U_{t,t}\left(f_t(A)\right) = u\left(\widetilde{L}_t + bonus_t^2 + I_t^2\right) = u\left(\delta_t(A)\cdot \widetilde{L}_t + I_t^2\right) = d_{t,t}(A),$$

  for all $A > 0$.

- Generation $\tau = t+1$: this generation is in its first retirement period in period t, and it follows from (24) that the aggregate payment from the DB fund in period t equals

  $$\widetilde{L}_{t+1} + bonus_t^1 = \widetilde{L}_{t+1} + (\delta_t(A) - 1)\cdot \widetilde{L}_{t+1} = \delta_t(A)\cdot \widetilde{L}_{t+1}. \tag{27}$$

  Moreover, this generation is in its second retirement period in period $t+1$, and the aggregate payment from the DB fund in period $t+1$ equals

  $$\widetilde{L}_{t+1} + bonus_{t+1}^2,$$

  where $bonus_{t+1}^2$ follows from

  $$\left(bonus_{t+1}^1, bonus_{t+1}^2, \alpha_{t+1}\right) = f_{t+1}^*(A_{t+1}),$$

  with A_{t+1} given by (25). Because the generation receives its last payment in period $t+1$, it holds that

  $$\begin{aligned} u\left(\widetilde{L}_{t+1} + bonus_{t+1}^2 + I_t^2\right) &= U_{t+1,t+1}\left(f_{t+1}^*(A_{t+1})\right) \\ &\geq d_{t+1,t+1}(A_{t+1}) \\ &= u\left(\delta_{t+1}(A_{t+1})\cdot \widetilde{L}_{t+1} + I_{t+1}^2\right) \\ &\geq u\left(\delta_t(A)\cdot \widetilde{L}_{t+1} + I_{t+1}^2\right), \end{aligned} \tag{28}$$

where the first inequality follows from the fact that (12) is satisfied for $s = t + 1$, and the second inequality follows from (26). Combining (5), (27), and (28) yields

$$\begin{aligned}
& U_{t,t+1}\left(f_t(A)|A, F^*_{t+1}\right) \\
= \ & u\left(\widetilde{L}_{t+1} + bonus^1_t + I^1_{t+1}\right) + \beta \cdot p_{t+1} \cdot E\left[u\left(\widetilde{L}_{t+1} + bonus^2_{t+1} + I^2_{t+1}\right)\middle| A_t = A, F^*_{t+1}\right] \\
\geq \ & u\left(\delta_t(A) \cdot \widetilde{L}_{t+1} + I^1_{t+1}\right) + \beta \cdot p_{t+1} \cdot u\left(\delta_t(A) \cdot \widetilde{L}_{t+1} + I^2_{t+1}\right), \\
= \ & d_{t,t+1}(A),
\end{aligned}$$

for all $A > 0$.

- Generations $\tau > t + 1$: for these generations, the first pension payment occurs at the earliest in period $t + 1$. Therefore,

$$\begin{aligned}
U_{t,\tau}\left(f_t(A)|A, F^*_{t+1}\right) &= E\left[U_{t+1,\tau}\left(f^*_{t+1}(A_{t+1})|A_{t+1}, F^*_{t+2}\right)|A_t = A, F^*_{t+1}\right] \\
&\geq E\left[d_{t+1,\tau}(A_{t+1})|A_t = A, F^*_{t+1}\right] \\
&\geq E\left[d_{t,\tau}(A)|A_t = A, F^*_{t+1}\right] \\
&= d_{t,\tau}(A),
\end{aligned}$$

for all $A > 0$, where the first inequality follows from the fact that (12) is satisfied for $s = t + 1$, and the second inequality follows from the fact that (7) and (26) imply that $d_{t+1,\tau}(A_{t+1}) \geq d_{t,\tau}(A)$.